

## Restructuring the Mathematical Power of Students and Teachers: A Case Study in the Misconceptions of Parallelogram Teaching

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**Abstract: Restructuring the Mathematical Power of Students and Teachers: A Case Study in the Misconceptions of Parallelogram Teaching. Objectives:** This research aims to (a) investigate cases of misconceptions about understanding parallelograms in mathematics learning and (b) restructure the mathematical power of teachers and students in understanding the concept of parallelograms. **Methods:** This research is a case study that analyzes in-depth cases of misconceptions in parallelogram learning. The data collection methods are surveys and interviews. This survey involved 120 students and ten teachers as respondents. In the final data analysis, the researcher conducted confirmation and triangulation to ensure the credibility of the findings and conclusions. **Findings:** The study's findings indicate ontological misconceptions about parallelograms in mathematics learning in elementary and high schools. Students and teachers experience two types of misconceptions, namely preconception errors and modeling errors. **Conclusion:** The conclusion states that students and teachers have successfully corrected misconceptions through knowledge or mathematical power restructuring so that students and teachers can understand the concept of parallelograms. The process of restructuring mathematical power is characterized by cognitive conflict, scaffolding, and cognitive balance in their thinking processes (teachers and students). The implication is that teachers must correctly instill conceptual knowledge about parallelograms through project-based or inquiry-based learning strategies so that students can build their knowledge based on their learning experiences.

**Keywords:** case study, mathematical power, misconception, parallelogram.

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## ■ INTRODUCTION

Learning mathematics is learning about abstract objects. In learning mathematics, a systematic way of thinking is needed so that students can solve problems and understand the concepts. Thinking involves using information mentally by forming concepts, solving problems, making decisions, and showing them critically and creatively (Çelik & Özdemir, 2020; Jaarsveld & Lachmann, 2017).

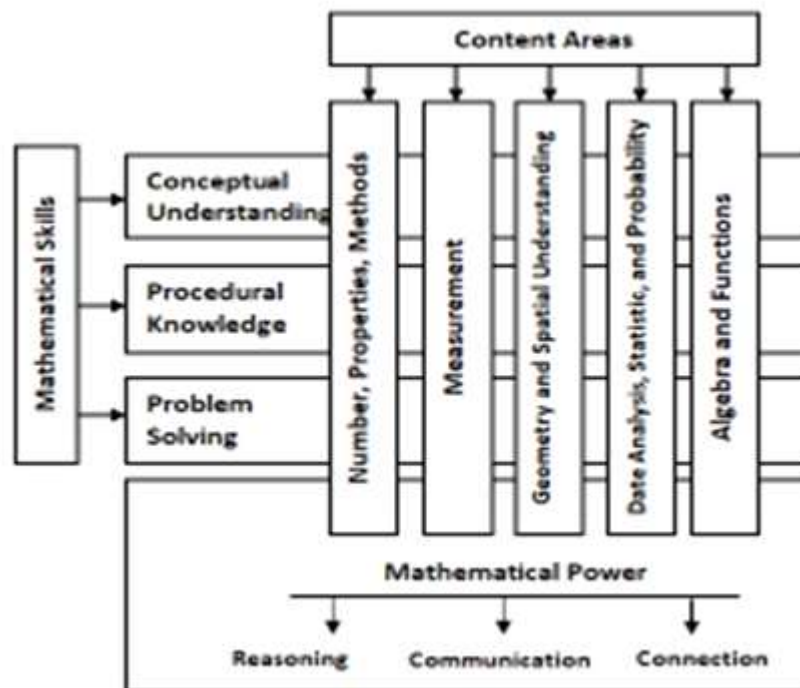
While learning mathematics, some students experience obstacles in their cognitive

development. In fact, not a few students experience mathematical anxiety, lack of self-confidence, and misconceptions, thus hindering their learning achievement (Khasawneh et al., 2021; Kusmaryono et al., 2022). Therefore, every individual who learns mathematics needs mathematical power to reason, connect, communicate, and represent abstract mathematical symbols (Kusmaryono et al., 2019; Lin Li, 2024).

Mathematical power is one of the important goals of mathematics learning in schools (National

Council of Teachers of Mathematics, 2014). After reviewing some related literature (National Council of Teachers of Mathematics, 2014; Pilten, 2010; ahin & Baki, 2010), mathematical power is defined as students' ability to use mathematical knowledge to solve problems through logical reasoning (reasoning), communicate mathematical ideas

(communication); and connections between ideas in mathematics or with other sciences (connection) in order to develop mathematical confidence and disposition. Reasoning, communication, and connection are the main components of mathematical power (Pilten, 2010). Components of mathematical power in learning can be seen in Figure 1 below.



**Figure 1.** Components of mathematical power in learning (NAEP, 2002, Cited in Pilten, 2010)

Figure 1 shows that mathematical power, which includes the ability to reason, communicate in or through mathematics, and relate mathematical ideas to other intellectual activities, is a product of mathematical skills. This mathematical ability is obtained when students apply mathematical knowledge, conceptual understanding, and problem-solving together within the framework of the specified content, according to the desired mathematical skills in the development of mathematical power.

In this paper, the content standard of mathematical power is geometry. Mathematical

ability is focused on conceptual knowledge of parallelograms. Meanwhile, in the standard of the mathematics learning process, researchers will investigate cases of student and teacher misconceptions in reasoning and representation of parallelograms. As stated by previous researchers (Rafianti & Pujiastuti, 2017; Setiawan et al., 2022), every individual (student and teacher) has mathematical power, but the mathematical power possessed by each individual will be different and dynamic.

In certain mathematical content knowledge, an individual's mathematical power can be high or

low depending on the readiness and initial abilities possessed by the individual (Kusmaryono et al., 2019; Nendi et al., 2023). Therefore, students and teachers must continue improving mathematical power and individual cognitive development.

Teachers play a major role in the learning process to achieve successful educational goals (Darling-Hammond et al., 2024; Singh, 2021). Have we ever realized that misconceptions experienced by students start from the teaching of teachers at school? It would be very wise if teachers at school could provide students with the correct understanding of knowledge. If someone experiences a misconception of mathematics in the first lesson and is not immediately corrected, it will impact subsequent mathematics learning. Therefore, teachers must detect and correct learning misconceptions immediately (Turmuzi et al., 2024).

Misconceptions refer to concepts that do not follow scientific understanding. Misconceptions indicate a misunderstanding or difference between preconceptions and scientific conceptions (Soeharto & Csapó, 2022; Suprpto, 2020; Turmuzi et al., 2024). Misconceptions that last a long time and are repeated will be stable and permanent misunderstandings, so they are called ontological misconceptions (Kusmaryono et al., 2020; Suprpto, 2020). Teachers must consider how to reveal and correct misconceptions through knowledge restructuring. In the discussion of this case, knowledge restructuring is referred to as mathematical power restructuring.

The researcher focuses on the parallelogram misconception in mathematics learning in this case study. This case will be studied in depth to reveal the reality behind the phenomenon of parallelogram misconception.

The study's objectives are (a) to investigate cases of parallelogram teaching misconception in mathematics learning and (b) to restructure the mathematical power of teachers and students in understanding the concept of parallelograms.

The results of this study are expected to provide new information and insights for students and teachers about (a) the existence of misconceptions in parallelogram learning, (b) the process of restructuring the mathematical power of teachers and students in understanding the concept of parallelograms, and (c) the importance of students and mathematics teachers understanding the concept of parallelograms correctly and precisely so that they can increase mathematical power.

## ■ METHOD

### Participants

The population of this study was 310 participants from 6 schools with three different levels, namely two elementary schools, two junior high schools, and two senior high schools. This study took a sample of 120 students and ten mathematics teachers. The sample determination used a purposive sampling technique. The criteria for students used as samples were students in the final grade, namely 40 students in grade 6, 40 students in grade 9, and 40 students in grade 12. The criteria for teachers selected were two teachers in each school, requiring teachers to have a teacher certificate and more than five years of experience teaching mathematics.

### Research Design and Procedure

This research is a case study (single-case instrumental) with a qualitative approach. Case studies emphasize a deeper understanding of certain phenomena for individuals (Takahashi & Araujo, 2020). In this case study, the researcher focuses on misconceptions about teaching parallelograms.

The researcher will conduct an in-depth analysis of misconceptions and efforts to restructure knowledge (mathematical power) about parallelograms for students and teachers.

This case study research was conducted within three months by the research team. The

steps for case study research go through five stages (Creswell, 2014), namely: (a) Choosing a theme, topic, and case: Researchers must be able to find cases that are part of the field being studied; (b) Studying Literature and formulating problems: Researchers conduct literature reviews to broaden their insights and sharpen the formulation of problems; (c) Data collection and analysis: Data collection through documentation, observation, and interviews. Data collection is natural and holistic. Data analysis is carried out in-depth by identifying data patterns; (d) Confirmability of findings: Researchers triangulate data sources and forum group discussions; and (e) Conclusions: Researchers synthesize the facts of the findings with the formulation of research problems.

### Instruments

The data collection instruments in this case study were documentation or archive sheets, survey questions, and interview questions. Documentation sheets were used to collect test score data and evidence of student work (test answers). The researcher developed seven survey questions. Respondents answered the survey questions by choosing “Yes or No.” The validity test of the questionnaire used the Kappa statistical technique with a value of  $K = 0.691$ , indicating good agreement (Aithal & Aithal, 2020). The reliability test of the instrument used the Cronbach’s alpha technique with a value of  $\alpha = 0.768$ . This result means that the survey instrument is consistent and has high internal reliability (Tsang et al., 2017).

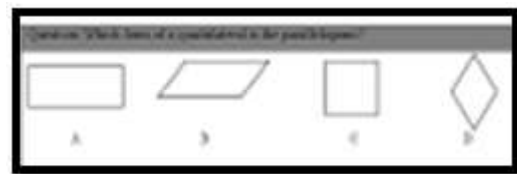
Meanwhile, the interview questions were semi-structured to explore participants’ thoughts, feelings, and beliefs regarding parallelograms.

### Data Analysis

The analysis of survey data results was carried out descriptively and qualitatively. Qualitative data analysis through thematic Analysis with the following steps: (a) recognizing data patterns and making a list of data; (b) combining and developing data into themes by involving coding and transcripts; (c) recognizing themes and sub-themes, noting recurring patterns, and grouping different data results, (d) synthesizing sub-themes into themes and concluding analysis units; (e) interpreting Literature, and (f) formulating results (Naeem et al., 2023; Sovacool et al., 2023). In the final data analysis, researchers confirmed through group discussion forums and source triangulation to ensure the credibility of the findings and conclusions (Noble, 2019).

## ■ RESULT AND DISCUSSION

The main data for this research was obtained through a survey. Survey questions were designed to determine respondents’ understanding of the parallelogram concept. The parallelogram shapes are presented in Figure 2 below.



**Figure 2.** Parallelogram shapes

Based on Figure 2, the researcher developed seven survey questions. The survey form has been distributed and filled out by 140 respondents. Respondents answered the survey questions by selecting “Yes or No” answers. The survey results are presented in Table 1 below.

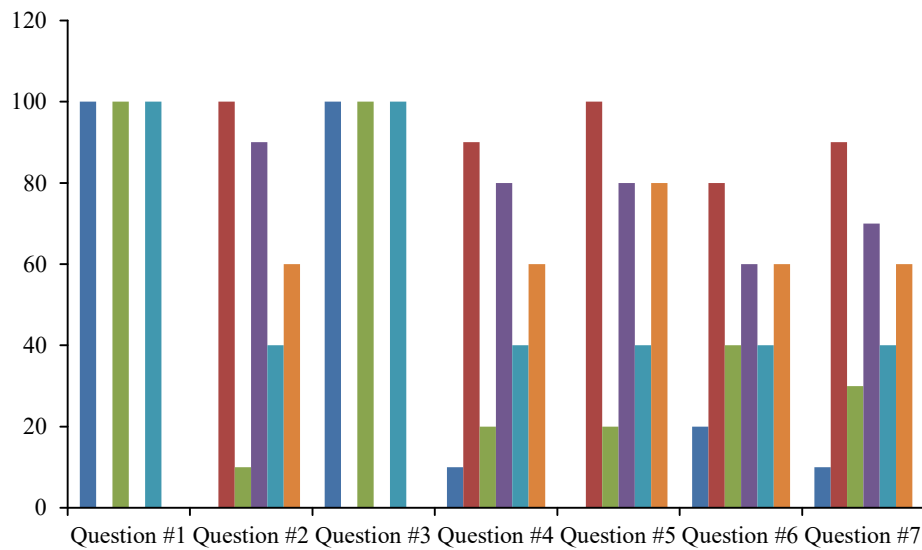
**Table 1.** Survey results with student respondents

No.	Statement Survey	Percentage of Student Answers (%)		
		A	B	C
1.	Pictures A, B, C, and D are the shapes of a rectangular plane	Yes: 100 No: 0	Yes: 100 No: 0	Yes: 100 No: 0

2.	Picture A is the shape of a parallelogram plane.	Yes: 0 No: 100	Yes: 10 No: 90	Yes: 40 No: 60
3.	Picture B is the shape of a parallelogram plane.	Yes: 100 No: 0	Yes: 100 No: 0	Yes: 100 No: 0
4.	Picture C is the shape of a parallelogram plane.	Yes: 10 No: 90	Yes: 20 No: 80	Yes: 40 No: 60
5.	Picture D is the shape of a parallelogram plane.	Yes: 0 No: 100	Yes: 10 No: 90	Yes: 20 No: 80
6.	Pictures A, B, C, and D are the shapes of parallelogram planes.	Yes: 20 No: 80	Yes: 40 No: 60	Yes: 40 No: 60
7.	Do you understand the definition (concept) of a parallelogram?	Yes: 10 No: 90	Yes: 30 No: 70	Yes: 40 No: 60
* Description: A = elementary school students; B = junior high school students C = senior high school students				

The survey data (Table 1) shows that the percentage of “Yes” answers to questions 2, 4, 5, and 6 is still below 50%, meaning that respondents do not understand the concept of parallelograms. While in questions 1 and 3, all respondents can answer 100% “Yes”. This result means that

respondents can only understand the shape of a quadrilateral in general, and their understanding of parallelograms is limited to one picture. The graph of the survey results of students’ understanding of parallelograms is shown in Figure 3 below.



**Figure 3.** Graph of conceptual understanding of parallelograms

Figure 3 shows an upward graph on (answer “Yes”) understanding of questions 2, 4, 5, and 6. Elementary school students have a very

low understanding of parallelograms. Junior high school students better understand parallelograms than elementary school students. Senior high

school students better understand parallelograms than junior high school students. Unfortunately, students' understanding of parallelograms is still below 50%. These findings can be interpreted as follows: the higher the students' education level, the more understanding and reasoning they have about the concept of parallelograms. However, overall, the results of this survey point to evidence that students do not understand the concept of parallelograms (see Table 1, question 7). Therefore, the researcher conducted further investigation by interviewing student representatives at each level of education. Excerpts from the interview results are presented below.

Question no 1: From Whom do you understand parallelogram only in picture B?

**S-72:** From my teacher in elementary school

**S-81:** From my teacher in elementary school until now (high school)

Question no 2: How do you store conceptual knowledge about parallelograms in your memory?

**S-09:** I understand parallelograms only based on visualizations presented by the teacher, such as in Figure 2-B.

**S-25:** A parallelogram is formed from a pair of parallel lines cut by two oblique lines.

Question no 3: How many references have you read about parallelograms?

**S-72:** I only study math books at school.

**S-81:** I read more than one book, but there is only one parallelogram like in picture B

Question no. 4: Do Pictures A, C, and D have two pairs of parallel sides, and each pair of sides is the same length?

**S-72:** Yes, all sides are parallel and the same length. Is this a parallelogram?

**S-81:** I still doubt if Picture D is a parallelogram.

Question no. 5: Do Pictures A, C, and D have opposite angles of equal size?

**All students:** Yes, all Pictures A, C, and D fulfill the elements of a parallelogram

Question no. 6: Do you now understand the definition of a parallelogram?

**S-72:** I understand the parallelogram shape.

**S-81:** Wow. It turns out that a parallelogram is a quadrilateral that has two pairs of parallel sides and two pairs of opposite angles that are the same size.

Question no 7: Why is a trapezoid, not a parallelogram?

**S-09:** A trapezoid only has one pair of parallel sides.

**S-81:** Because the two parallel sides are not the same length

**S-25:** Because, in a trapezoid, the opposite angles are not the same size

Question no 8: Are you now sure that rectangles, squares, and rhombuses are parallelograms?

**S-72:** I am sure there are other forms of parallelograms.

**S-25 and S-81:** Based on the definition of a parallelogram, I am sure and without a doubt that rectangles, squares, and rhombuses are parallelograms.

We examined the data in Table 1, which shows that most students did not understand the concept of a parallelogram. Thus, it can be said that students' mathematical power is low, especially regarding reasoning ability. When interviewing students, the researcher found misconceptions about the parallelogram concept.

Considering the case of student misconceptions in the problem of Figure 2, it was stated that students faced preconceptions, namely, being unable to distinguish between parallelograms and non-parallelograms (interview question no. 2 answered by S-09 and S-25). Preconception is an initial error before someone understands the concept properly (Schwichow et al., 2022). In addition, students were also identified as experiencing modeling errors when students only imitated examples of parallelogram images from the teacher (interview question no. 1 answered by S-72 and S-81) (Stemele & Jina



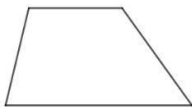

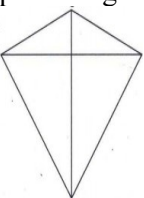
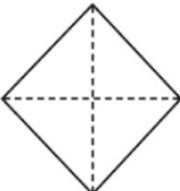
Asvat, 2024). For students, the concept may be abstract, counterintuitive, or complex.

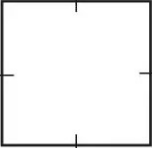
Therefore, students' understanding of the concept of a parallelogram is wrong. Therefore, changing the teacher's framework is the key to improving misconceptions in mathematics teaching.

Misconceptions tend to be very resistant to teaching because learning requires radical replacement or reorganization of students' knowledge. Misconceptions can be replaced or eliminated by changing or restructuring students' knowledge and thinking frameworks (Makhrus & Busyairi, 2022). The understanding of new

concepts obtained by students supports the reconstruction of knowledge, but sometimes there is a cognitive conflict that conflicts with previous conceptual understanding (interview question no. 4 answered by S-72 and S-81) (Lestary et al., 2022; Mufit et al., 2023). Misconceptions experienced by students can be caused by inappropriate teaching factors (knowledge) of teachers. We conducted a survey and interviews with mathematics teachers based on student errors. The survey was conducted to determine teachers' understanding of parallelograms. The following are the results of teachers' answers in the survey (Table 2).

**Table 2.** Restructuring of mathematical power about quadrilaterals

Statement	Evaluation	Teacher Answers
<p><b>Trapezium</b> is a parallelogram</p> 	<p><b>Incorrect</b></p> <hr/> <p><b>Correct</b></p>	<p>A trapezoid is a quadrilateral with one pair of parallel sides.</p> <hr/> <p>A trapezoid only has one pair of parallel sides. A trapezoid is not a parallelogram. Because, a parallelogram must have two pairs of parallel sides. Whatever the definition of a trapezoid is used, a trapezoid is not a parallelogram.</p>
<p>The rectangle is a parallelogram.</p> 	<p><b>Incorrect</b></p> <hr/> <p><b>Correct</b></p>	<p>A rectangle is a quadrilateral formed by two pairs of parallel sides of equal length.</p> <hr/> <p>A rectangle is a quadrilateral formed by two pairs of equal-length parallel sides with four right angles. A rectangle is a parallelogram that has four right angles.</p>
<p>A kite is a parallelogram.</p> 	<p><b>Incorrect</b></p> <hr/> <p><b>Correct</b></p>	<p>A kite is a quadrilateral with two pairs of sides of equal length but not parallel.</p> <hr/> <p>A kite is not a parallelogram. A kite does not have a pair of parallel sides.</p>
Statement	Evaluation	Teacher Answers
<p>A rhombus is a parallelogram.</p> 	<p><b>Incorrect</b></p> <hr/> <p><b>Correct</b></p>	<p>A rhombus is a plane figure formed by four right triangles, each of which is the same size as the opposite angle.</p> <hr/> <p>A rhombus is a quadrilateral whose sides are the same length and opposite angles are the same size. A rhombus is a parallelogram whose four sides are the same length.</p>

A square is a parallelogram. 	<b>Incorrect</b>	A square is a quadrilateral with four corners and sides of the same length.
	<b>Correct</b>	A square is a quadrilateral with four sides of the same length and four angles of 90 degrees. A square is a parallelogram whose sides are the same length and four angles of 90 degrees.

Based on the research data that we have collected (Tables 1 and 2), we analyze and compare mathematical power between teachers and students. The comparative analysis of mathematical power includes three indicators: mathematical reasoning, mathematical communication, and mathematical connections (NAEP, 2002, Cited in Pilten, 2010). The analysis result can be seen in Table 3 below.

The comparison of mathematical power between students and teachers (Table 3) shows a significant difference between the mathematical power of students and teachers. Students have low mathematical power, and teachers have relatively high mathematical power. Students' abilities in the three indicators of mathematical power, namely reasoning, communication, and mathematical connections, still need to improve.

**Table 3.** Comparison of mathematical power between teachers and students

Indicator of Mathematical Power	Description of Mathematical Power	
	Teachers	Students
Mathematical Reasoning	Teachers can define parallelograms correctly (verbal understanding), but teachers fail to interpret the definition of parallelograms into pictorial form.	Students can only show one picture of a parallelogram Students can define parallelograms but not accurately Students do not understand the concept of parallelograms
Mathematical Communication	Some teachers can communicate abstract ideas about the concept of parallelograms quite well.	Students fail to communicate abstract ideas about the concept of parallelograms.
Mathematical Connection	Some teachers have been unable to perfectly link the concepts of parallelograms and polygons.	Students have not been able to link the concepts of parallelograms, namely rectangles, rhombuses, and squares.

Meanwhile, the mathematical power of teachers is relatively high. Teachers can communicate abstract ideas about the concept of parallelograms quite well. Teachers understand the definition, but they need to interpret the meaning of parallelograms into pictures. Teachers have not been able to perfectly link the concepts of parallelograms and polygons.

The results of the survey on teachers in Tables 1 and 2 were very surprising to the researcher. The

researcher assumed that the teachers did not yet have a strong conceptual foundation in learning polygonal planes. Most of the teachers' answers were incorrect in explaining the definition of quadrilaterals (trapezoids, rectangles, kites, rhombuses, and squares).

Based on the triangulation analysis of sources (Tables 1, 2, and 3), we describe that misconceptions occur when students' reasoning about mathematical concepts does not match the



actual concept. On the other hand, if students' mathematical communication and connection abilities are low, it will affect their understanding of concepts related to subsequent material.

On the teacher's side, it is documented that teachers can communicate abstract ideas about parallelograms quite well. Teachers naturally form ideas from everyday experiences, but not all ideas developed are correct concerning evidence in the given discipline (Darling-Hammond et al., 2024). If teachers do not have high mathematical power, it will hinder understanding and mastery of learning materials, including misconceptions.

These results show that the teachers' mathematical reasoning, connection, and communication abilities are still low. In other words, teachers have weak mathematical power. Therefore, the researcher conducted further investigation by interviewing teachers at each level of education. A summary of the interview results is presented below.

Question no 1: Since when did you understand that a parallelogram is only picture B?

All Teachers: From elementary school until now

Question no 2: How many references have you read about parallelograms?

Teacher-02: I have read four math learning resource books.

Teacher-03: I have read 4 to 5 books.

Question no 3: How do you understand the conceptual knowledge of parallelograms?

Teacher-01: A parallelogram is a flat quadrilateral that has two, and the angles are the same size.

Teacher-02: A parallelogram is a flat quadrilateral that has two pairs of parallel sides.

Teacher-03: A parallelogram is a quadrilateral with two pairs of parallel sides, obtuse angles, and an acute angle.

Question no 4: Do you agree with the statement: "A parallelogram is a quadrilateral

that has two pairs of parallel sides (each pair of sides is the same length) and the opposite angles are the same size"?

Teacher-02: Strongly agree. The statement is quite clear, and I can understand it.

Teacher-03: I agree, although I was shocked to get the surprise that pictures A, C, and D are parallelograms.

Question no 5: Look again at Figure 2. Do Figure 2: A, B, C, and D fulfill the elements as a parallelogram?

All Teachers: Yes, all four images (A, B, C, and D) fulfill the elements of the sides and angles as a parallelogram.

Question no 6: After you understand the definition of a parallelogram, do you believe and are sure there has been a misconception in teaching parallelograms?

Teacher-01: I realize that there has been a misconception about parallelograms.

Teacher-03: I feel guilty towards the students because I have made a fatal mistake.

Based on the data obtained through the survey (Table 1) and the results of interviews with students (Table 2), the researcher considers the data to have reached the point of saturation. The indicators for achieving data saturation are (a) all respondents have participated in the survey, (b) all survey questions have been answered by respondents, (c) the data obtained is adequate and fully represents the research model construct, and (d) there is no new data information that contributes (Hennink & Kaiser, 2022; Saunders et al., 2018).

Researchers have identified misconceptions in teaching parallelograms based on the survey results (Table 2) and data reduction from interviews. In general, teachers can only present visual images without properly understanding the

concept of parallelograms. The results of document checks on mathematics textbooks in schools have correctly described the definition of parallelograms. However, the book only provides one model of a parallelogram image, as shown in Figure 2-B. Researchers argue that the book's author does not properly visualize the definition of a parallelogram. Mathematics books in schools are the main reference for teachers and students when learning.

The results of interviews with students showed that they have a weak understanding of the concept of parallelograms. They obtain knowledge received from teachers only through pictures (Figure 2-B) without recognizing the elements of parallelograms. Students certainly experience cognitive conflict in their thinking framework. After they understood that a parallelogram's shape is not only one (Figure 2) (Makhrus & Busyairi, 2022), they were surprised that pictures A, C, and D are parallelograms.

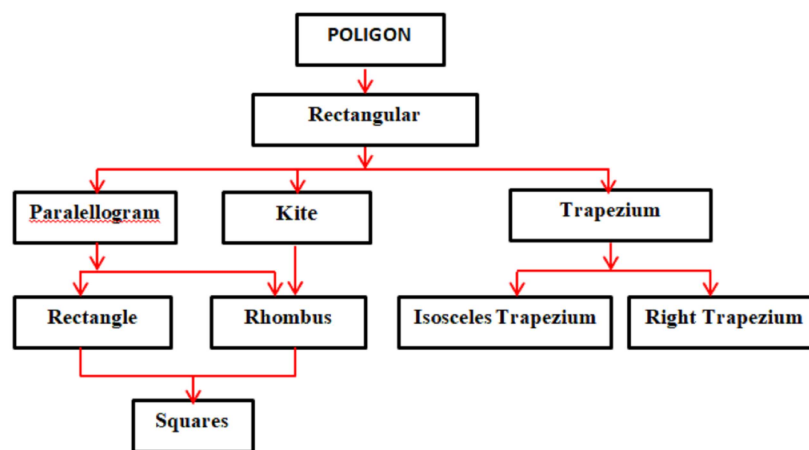
On the other hand, teachers understand the concept of parallelograms, but teachers cannot visualize the concept of parallelograms correctly. The teachers only focus on one picture, namely Figure 2-B. This result shows that students and teachers have weak mathematical power,

especially in reasoning about parallelograms and connections between parallelogram families.

During the interview with the teacher (see question 4), the teacher experienced cognitive conflict in his thinking process (Makhrus & Busyairi, 2022). Teacher 03 seemed hesitant and tried to reason and re-understand the concept of a parallelogram. This cognitive conflict triggered the teacher to achieve equilibrium in his thinking process (Maarroof & Thujil, 2023).

So, in the end, the teacher believed that images A, C, and D fulfilled the elements of a parallelogram. After a fairly serious discussion and explanation of the elements, students and teachers could restructure their thinking to become real. The teachers realized there had been a misconception about teaching parallelograms (see teacher interview in question no. 5).

During the interview, we and the teachers had a very serious discussion. We provided scaffolding assistance to the teacher. We guided them to be able to compile a parallelogram and trapezium relationship scheme. Finally, they were able to explain the parallelogram family relationship. The parallelogram family scheme is shown in Figure 4 below.



**Figure 4.** Family relationships in a parallelogram

Based on Figure 4, the researcher would like to reiterate that a parallelogram is a quadrilateral plane that has two pairs of parallel sides (each pair of sides is the same length) and opposite angles are equal (Biber et al., 2019; Yavuzsoy-Köse et al., 2019). The definition of a plane, including a parallelogram, has two requirements that must be met: mentioning the elements of the sides and angles (Yavuzsoy-Köse et al., 2019; Zembat & Gürhan, 2023).

The errors or misconceptions in teaching planes are more in mentioning incomplete elements, resulting in a wrong perception. Figure 4 is the result of the teacher's work with the help of scaffolding from the researcher. After going through the cognitive conflict and scaffolding process, the teacher carried out cognitive equilibrium in his thinking process to connect conceptual knowledge and the visual form of the parallelogram (Maarof & Thujil, 2023). So that the quadrilateral family scheme was realized (see Figure 4). Based on Figure 4, it can be stated that the teacher has restructured mathematical power, especially in the ability to reason about parallelograms, connections between parallelogram families, and the ability to communicate parallelogram knowledge. Correctly restructured mathematical power about the concept of parallelograms through the process of cognitive conflict, scaffolding, and cognitive equilibrium. The researcher concluded that in this case study, the weakened mathematical power of teachers and students could be strengthened again after going through the process of restructuring the parallelogram concept correctly. In other words, mathematical power has been restructured at the level of cognitive abilities of students and teachers.

## ■ CONCLUSION

In mathematics learning from elementary school to high school level, there has been an ontological misconception about understanding

the concept of parallelograms. Students and teachers experience two types of misconceptions: preconception and modeling error. However, in the end, students and teachers managed to correct misconceptions through knowledge or mathematical power restructuring so that students and teachers could understand the concept of parallelograms. Mathematical power restructuring is characterized by cognitive conflict, scaffolding, and cognitive equilibrium in their thinking processes (teachers and students).

The results of this study have implications for mathematics learning in schools, especially the material on plane shapes. When teaching plane shapes, teachers must ensure that conceptual knowledge about parallelograms is instilled correctly. Teachers can apply problem-based or project-based learning strategies or inquiry strategies so that students can construct their own knowledge, ultimately building students' mathematical power.

The limitations of this study are the small number of teacher samples (minimum) and the fact that it only represents three schools. This study has not revealed the factors that influence the occurrence of misconceptions experienced by students and teachers. In the future, research can be conducted on the factors that influence the occurrence of misconceptions by involving a larger number of teacher samples.

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