Vol. 14, No. 01, pp. 704-712, 2024 DOI: 10.23960/jpp.v14.i2.202451

Jurnal Pendidikan Progresif

e-ISSN: 2550-1313 | p-ISSN: 2087-9849
http://jurnal.fkip.unila.ac.id/index.php/jpp/

Profile of Pre-Service Math Teachers' Conception about the Definition of Limit Functions Based on Mathematical Ability

Usman*, Muhammad Hasbi, & Asnawi Muslem
Department Mathematics Education, Universitas Syiah Kuala, Indonesia

*Corresponding email: usmangani@usk.ac.id

Received: 16 May 2024 *Accepted: 20 June 2024* *Published: 15 July 2024*

Abstract: Profile of Pre-Service Math Teachers' Conception about the Definition of Limit Functions Based on Mathematical Ability. Objective: The study aims to explore the profile of pre-service math teachers' Conception about the definition of limit functions based on mathematical ability between high and low ability and to find out their perceptions on the topic. **Methods**: The methods used in this study were mixed-methods in nature; quantitative and qualitative. The participants of the study are 64 pre-service math teachers who are studying in the fifth-semester academic year 2022, Faculty of Teacher Training and Education, Syiah Kuala University. The reason of selecting this participant is because they have gained experience studying calculus and objective analysis. The data collection for this study is obtained through math ability tests on limit conception tests and interviews. The data are analysed by using descriptive quantitative and qualitative. The data from the interview are analysed by reducing data, exposing, interpreting, and concluding. **Findings:** This study reported that high pre-service math teachers group outperformed the low-ability group in terms of math tests. In addition, The pre-service math teachers' perceptions on expressing ideas, and explaining formal definitions of the limit function and mental image were relatively good. **Conclusion:** The study concluded that the profile of high ability of the pre-service math teachers performed better than those low in math test competence and their understanding of the definition of limit function, mental image and the concepts of limit function and limit function relation were relatively good. Of course, this study has a limitation in terms of comprehensive information related to understanding math concepts and practice.

Keywords: profile, pre-service teacher, conception, limit function, mathematical ability.

To cite this article:

Usman, Hasbi, M., & Muslem, A. (2024). Profile of Pre-Service Math Teachers' Conception about the Definition of Limit Functions Based on Mathematical Ability. *Jurnal Pendidikan Progresif*, *14*(1), 704-712. doi: 10.23960/jpp.v14.i2.202451.

INTRODUCTION

The limit function is a fundamental concept in calculus and mathematical analysis. Cetin (2009: 323) asserted that "Limit is one the most fundamental and important concepts of calculus since it established a framework necessary to completely acquire the basic concepts of calculus such as continuity, differentiation integral. Ervynck also confirms (1981) that the limit concept has long been considered fundamental to understanding calculus and real analysis. Davis and Vinner (1986) explain why the concept of "limit" is fundamental to calculus. Students' failure to express the meaningful idea of the limit concepts' role in calculus may largely be used to inappropriate and weak mental links between Limit knowledge and knowledge of other calculus concepts such as continuity, derivative, and integral. The formal definition of Limit at a point" $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - a| < \delta \rightarrow |f(x) - L| < \varepsilon$" is foundational as students proceed to more formal, rigorous mathematics, continuity, derivatives, integrals, and Taylor series approximations are just a few of topics in an analysis course for which the standard definition of Limit serves as an indispensable component Swinyard and Larsen (2012). Referring to some of these expositions, the concept of function limits has an essential role in academic considerations and as an object of research in mathematics education.

Conception research on limit function is one of the essential topics in mathematics education research (Szydlik, 2000; Roh, 2008; Sebsibe & Feza, 2019). Bezuidenhout (2001) explained that "the student with mature conceptions of the mentioned concepts should be able to relate those concepts in a meaningful way. Notions about mathematical concepts are constructed in the mind of students with a strong concept then will be able to link between one concept with another concept in a meaningful way. Students will be able to connect the concept of function limit to the concept of distance, interval, absolute value, and various forms of representation (Bezuidenhout, 2001; Denbel, 2014). The research findings explained that a "Well-constructed mental model of the network of relationships among calculus concepts is essential for a thorough understanding of the conceptual underpinning of the calculus, which includes the fundamental role of the limit concepts (Bingölbali & Coºkun, 2016). Such a network of mental creations underlies an individual's problem-solving ability in calculus.

This research explored student conceptions about function limit, which consists of understanding and mental image (Hiebert & Carpenter, 1992; Tall & Vinner, 1981; Ndagijimana, 2022). The theoretical framework that constructs the notion is the understanding described by Skemp (1976), According to Skemp (1976), relational understanding involves knowing both what to do and why it works, while instrumental knowledge consists in knowing what to do, the rule, not the reason why the role works. So, relational understanding comes from an experience of a deeper relationship among the concepts and processes associated with a particular concept. Instrumental performance refers to an algorithm's understanding of a concept or process. While Hiebert and Carpenter (1992) explained that" a mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, mathematics is understood if its mental representations are part of network representations. This theory considers that a student understands the concept, fact, or procedure of mathematics to build a network of internal understanding in his mind. The student understands the concept of Limit if the representation of the definition of the limit function built into the student's mind is part of the representative network of the overall concept of function limits. Suppose the student could construct various representations of the definition of limit function in mind, which is part of the representation network of the concept of limit function. In that case, it is said that the student understood the limit function.

Conception can also be a mental image of Liyad (1998). Tall dan Vinner (1981) explained the mental image in mathematics. The term concept image describes the whole cognitive structure associated with the concepts, including all the mental picture and associated properties and processes." In this study, mental images are a collection of cognitive designs in a person's mind associated with the concept of a function limit consisting of images in the mind and associated with the nature of the concepts and processes of the function limit. The mental image of the function limit in the form of words graphs or illustrations of functions has limits or no limits at a point, associated with the properties of any values approaching a number, the value of a part coming to a deal. And the process of relationship between the value of the independent variable and the dependent variable. Viirman, Attorps, & Tossavainen, (2010) reported that the character of the definitions given in the textbooks used by the students affect their concept images.

Researchers have done some research on the concept of function limits. For example, Duru (2010) has undertaken research on conceptions to explore the understanding of prospective student teachers about limit functions, continuous and differential functions, and their relationships. Bezuidenhout (2001) explores the student's conception of semester two concerning limit functions and continuity in calculus. The theoretical framework developed by Bezuidenhout refers to constructivist theory and concept imagery. Therefore, research on student conception of the Limit of Function is essential to be explored so that the lecturer can get a picture of the student's notion of the concept of the Limit of Function. Another related research has been conducted by Sulastri, Suryadi, Prabawanto, Siagian, & Tamur, (2021). They found that students understood the concept of limit function in the same way that limit is an unreachable function. Shaldehi, Shaldehi, & Hedayatpanah, (2022) reported that functions limit education is considered by combining mental and sensory imagery through allegory or simile with six puppet images. The experiences in this research can be used by teachers of teacher training centres, teachers and high schools and universities and designers of educational materials. A previous study showed that most of students were still at the level of action conception. They could calculate and use procedure precisely to the mathematics objects that was given, but could not reach the higher conception yet (Afgani, Suryadi, & Dahlan, 2017).

Mathematics education students were one group of individuals prepared to become math teachers. Students had different characteristics when viewed from the ability of mathematics—some research results on differences in mathematical ability and gender of men and women. For example, Baton et al.'s (1999) study concluded that boys tend to obtain high scores from women on space representation, measurement, and complex problems. In contrast, women tend to score higher than men in computing, Simple problems, and graph reading. The research results reinforce by Artila et al. (2011) that the ability of men and women lies in the capacity of verbal, spatial, and arithmetic. Women generally excel in verbal skills, while men excel in spatial and arithmetic abilities. This study showed differences in the capacity of mathematical components of oral, spatial, computing, reading graphs, and solving complex problems in men and women.

Conception is a cognitive structure as a formula of one's construction results to a mathematical concept based on interaction with the environment and experience. A person's learning experience in constructing mathematical concepts, the relationship between concepts, and problem-solving affect the person's conceptions of advanced mathematical concepts. Mathematical skills influenced the conception of the definition of limits constructed by students. The mathematical abilities in this study are performance obtained through tests on knowledge and skills resulting from interaction with the environment and mathematics learning experience. The knowledge and skills tested consist of (1) pre-calculus: Function, absolute value, limit function, (2) calculus: derivative and integral, (3) algebra: polynomial, sequence and series and liner program, (4) space geometry. The results of the mathematics ability test of teacher candidates were grouped into the high and low groups. The study aims to explore the profile of pre-service math teachers' Conception about the definition of limit functions based on mathematical ability between high and low ability and to find out their perceptions on the topic.

METHOD

Participant

The total population of this study was 83 pre-service math teachers academic year 2022 from the Mathematics Education Study Program, Faculty of Teacher Training and Education, Syiah Kuala University, Darussalam Banda Aceh, Indonesia. The real participants of this study were 64 pre-service math teachers. All the participants involved based on the presence of the initial test. Thus, all the existing pre-service math teachers who took the initial test became the participants.

Research Design and Procedures

This study employed mixed methods in nature; quantitative and qualitative. After collecting all participants in a class, the authors gave the test to them. The test was conducted individually for 60 minutes. The test consisted of 10 essay questions related to the limit function test. After that, the authors interviewed two pre-service math teachers randomly selected in high and low-ability groups. Each group selected one participant to be interviewed. The test was given to 64 pre-service math teachers for the two groups; high and low ability. Whereas, the interview was conducted with only two participants who had been selected randomly from high and low-ability groups. After the participants responded to each question, proceed with an interview to explore the understanding and mental image of the definition of limit functions. Aspects of learning and mental shadow

Instrument

The instrument used in this study was test and interviews. The tests related to math ability test and limit function test (see appendix 1&2). The tests focused on pre-calculus include absolute value, function, and limit function, calculus that includes integral derivatives, algebra that includes polynomials, sequence and series, linear programming, and geometry; space geometry. Meanwhile, the interview questions were related to expressing the notion of limit function, left-hand limit, right-hand limit, definition of the limit function, left-hand limit, and right-hand limit with words, graphs, tables, and symbols.

Data Analysis

This study used mixed methods in analysing the data. The data from the math test ability were analysed by using descriptive statistics. The math tests were presented between scores of high and low-ability groups. Then, the mean scores of the two groups were presented. Whereas, the data from the interview were analysed qualitatively.

RESULT AND DISCUSSION

The results of pre-service math teachers' tests between high and low-ability groups are presented in Table 1.

Table 1 shows the scores of math test between high and low groups' abilities. It can be seen that the lowest score from the high group is 83 and the highest score is 94. Whereas the score in the low group ability shows the lowest score is 48 and the highest score is 65. The high and low groups' scores do not overlap. The high group's lowest score (83) exceeds the low group's highest score (65). This wide divergence shows that the two groups have significantly different abilities. High Group: Scores in this group reflect a higher level of competence, with a relatively narrow range of 11 points. This could imply that the upper group has more homogeneous abilities. Low Group: Scores in this group indicate a lesser level of ability, with a larger range of 17 points. This broader range shows that the low group's abilities vary more. Based on the results between the two groups, it can be implied that the distinct non-overlapping score ranges could indicate that the criteria for categorizing the high and low groups

Table 1. Pre-service math teachers' test scores between high and low-ability groups

No	Participants (64)	Scores	
		High Group	Low Group
1		84	65
2		84	60
3		85	64
4		86	64
5		86	63
6		86	59
7		87	62
8		87	60
9		88	59
10		89	62
11		90	65
12		92	49
13		90	50
14		90	62
15		83	56
16		85	57
17		83	59
18		89	48
19		90	57
20		94	57
21		88	55
22		88	50
23		92	53
24		92	54
25		84	56
26		85	52
27		89	49
28		90	60
29		90	64
30		89	62
31		87	61
32		85	58

are effective in distinguishing between different levels of ability. The narrower range in the high group might suggest that higher ability students tend to perform more consistently, whereas the wider range in the low group suggests more variability in performance among lower-ability students. The following table 2 displays the mean score of the two groups.

Table 2. The mean scores between high and low-ability groups

No	Participants (64)	Mean Scores	
		High Group	Low Group
		87.71	57.87

Table 2 shows the mean score between high and low-ability groups of pre-service math teachers. It can be seen that the mean score of the high group is 87.71 and the mean score of the low group is 57.87. It can be indicated that, on average, the mean scores in the high group are 87.71. This mean is closer to the lower end of the high group score range (83 to 94), suggesting that most scores cluster around this average. In the low group indicates that, on average, the scores in the low group are 57.87. This mean is also somewhat closer to the middle of the low group score range (48 to 65), suggesting a spread around this central value. The mean score provides a central value around which the scores are distributed. For the high group, 87.71 is the central point of their scores, and for the low group, 57.87 is the central point. The significant difference between the mean scores of the high group (87.71) and the low group (57.87) highlights a clear disparity in performance levels between the two groups. This suggests that the high group consistently performs better than the low group. The mean scores effectively summarize the average performance levels of each group, with the high group performing significantly better on average than the low group. This central tendency measure, combined with further statistical analysis, can provide a comprehensive understanding of the performance dynamics within and between the groups.

The following presents the findings from interview data. The data from interviews were analysed qualitatively. Two pre-service math teachers who involve in the interview part.

Question 1: What is your view on function limits? What is the meaning of the definition of function limit that you wrote? They responded that "*I think the (subject) limit of a function is the real number L as x approaches c, towards, near, the limit. The meaning of the definition of limit is that ã(x) approaches L when x approaches c or x approaches c, limit is near, limit".*

Quesntion 2: Explain the meaning of the definition of function limit that you stated. What is the meaning of the symbols contained in the limit definition? They said that "*The meaning of the limit definition in the form of this symbol is that the limit value of the function ã(x) is L where x approaches c. then ã(x) approaches L. The meaning of the definition of the symbol is that given any epsilon where the epsilon of the real member is positive there is a positive delta where the absolute value of the distance difference ã(x) to L is smaller than the epsilon, where the absolute value of the distance difference x to c is smaller than positive delta".*

Question 3: Explain the definition of limit that you stated in the form of a graphical representation. What does the graphic representation have to do with this verbal definition? Regarding to this item, they responded that "*I choose any c on the select x on the left, if I am on the right, what I select can be if I select the limit ã(x) is equal to L. The closer L is in the picture. then this means that the limit x goes to c, ã(x) goes to L*".

Question 4: How to determine the limit value of a function? Is there any other way? What is the meaning of the limit value you determined? They said that "*In my opinion, the way to determine the limit of this function is by factoring first, then crossing out the algebraic form in the denominator and the numerator, then simplifying, and finally using the substitution method to get the limit value. The meaning of the function limit there is that the function limit ã(x) is equal to L when x approaches the number c".*

Question 5: What is your view on proving limit problems? How do you prove the limit of this function? They pointed out that "*In my opinion, proving the function limit problem is to take* *any positive real number epsilon and then look for a positive delta number so that the absolute value of the difference between this function and this number is less than epsilon when x approaches c. To prove that by taking an arbitrary epsilon in the first step, the second step determines the positive delta number so that it satisfies this inequality*". This process illustrates how to use the õõ-ää definition to rigorously prove that a function approaches a certain limit as xx approaches a specific point.

Question 6: Explain the relationship between the left and right limits. Explain the necessary and sufficient conditions for a function to have a limit at one point. Lastly, they responded that "*The limit of h(x) is equal to L There are two cases here, there is this if and only if so, if this is the x approximation of a, while in this case, L is the limit function. The point is that they exist and are the same, so approach A is completely certain".*

CONCLUSION

The study concluded Pre-service math teachers with high ability performed better in the math test, demonstrating a stronger understanding of the definition of limit functions, mental imagery, and concepts related to limit functions. Those with lower math test competence did not perform as well, indicating gaps in their understanding and application of limit function concepts. High-ability students showed a relatively good grasp of the formal definition, including the õ\epsilonõ-ä\deltaä definition. They also had a better mental image of what a limit function represents, likely understanding it both graphically and conceptually. Their overall conceptual understanding of limit functions and the relationships between various limit concepts was stronger.

Limitations of the Study

Depth of Understanding: The study might not have fully captured the depth of understanding of math concepts and practices across all participants. Scope of Assessment: The methods used to assess understanding may not cover all relevant aspects of mathematical competence or the full range of skills necessary for teaching math effectively. The findings might be specific to the sample studied and may not be generalizable to all pre-service math teachers or different educational contexts. The study may have focused more on theoretical understanding rather than practical application and teaching practice, which is crucial for future math educators.

REFERENCES

Afgani, M. W., Suryadi, D., & Dahlan, J. A. (2017). Analysis of undergraduate students' mathematical understanding ability of the limit of function based on APOS theory perspective. In Journal of Physics: Conference Series (Vol. 895, No. 1, p. 012056). IOP Publishing.

Artila, A., Rosselli,M., & Inozemtseva, O. (2011). Gender differences in cognitive development. *Development Psychology*. 47(4), Juli 2011.984-990

Beaton, A.E., Mullis, I.V.S., Martin, M., Gonzales, E. J., Kelly, D. L., & Smith, T. A. (1999). Mathematics achievement in the middle school years: iea's third international mathematics and science study (TIMSS). Boston College, USA

Bezuidenhout, J. (2001). Limit and continuity: some conceptions of first-year students. *In International Journal of Mathematical Education in Science and Technology*. 32(4), 487-500.

Bingölbali, E., & Coºkun, M. (2016). A proposed conceptual framework for enhancing the use of making connections skill in mathematics teaching. Egitim ve Bilim, 41(183).

Bulden, S, D, Harries, T, V, & Newton, D, P. (2010). Pre-service Primary teachers' conception of creativity in mathematics.

Journal Mathematical Teacher Education. 13. 325-343. Do1: 101007/ s 10649.009.9207.z

Cetin, N. (2009). The performance of undergraduate students in the limit concept. *International Journal of Mathematical Education in Science and Technology*..40(3), 323-330.

Cornu, B. (1991). Limits, In A, J. Bishop, Mathematics Education Library. 11, 153-166.

Cottrill, J., Dubinsky,E., Nochols, D., Schwingerdoerf,K., Thomas,K., & Vidakovic,D. (1996), Understanding the limit concepts: Beginning with a collaborative process sckeme. *Jourrnal of mathematical Behavior*, 15.167-192.

Davis, R., & Vinner, S. (1986). The notion of limit: some seemingly unavoidable misconception stages. *The journal of mathematical behavior*, 5(3), 281-303.

Denbel, D. G. (2014). Students' misconceptions of the limit concept in a first calculus course. Journal of Education and Practice, 5(34), 24-40.

Duru, A., Koklu, O., & Jakubowski, E. (2010). Pre-service mathematics teachers' conceptions about the relationship between continuity and differentiability of a function. *Scientific Research and Essay*. 5(12). 1519- 1529.

Evangelidou, A., Spyrou, P., Ellia, L., & Gagatsis (2004). University students' conceptions of function. Paper of proceeding of the 28 conference of the international group for the psychology of mathematics education. *Volume 2*, *page: 351-358.*

Ervynck, G. (1981). Conceptual difficulties for first-year university students in acquiring the Limit of a function. In Equipe de Recherche Pedagogie (Ed.), *Proceedings of the Fifth Conference of the International Group for the psychology of mathematics education. Page: 330-333*.

Fernandez, E. (2004). The students take on the epsilon-delta definition of a limit, *Primus*, 14(1), 43-54.

Hiebert, J. & Carpenter, T.P. (1992). Learning and Teaching with understanding. In D, Grows, (Ed), Handbook of Research on Mathematics Teaching and Learning. 65-97. New York: MacMillan

Johnson, L. J., Bluew, G. W., Shimizu, J. K., Graysay, D, & Konnova, S. (2014). A teacher's conceptions of definition and examples when doing and teaching mathematics. *Journal Mathematical Thinking and Learning*. 16(4). 285-311. Doi: 10.1080110986065.2014.953018.

Lioyd, G, M. (1998). Supporting innovations: the impact of a teacher's conceptions of function on implementing a reform curriculum. *Journal for Research in Mathematics Education*. 29(3). 248-274.

Ndagijimana, T. H. (2022). Conceptual understanding of limits and continuity of functions: senior four rwandan secondary schools. Educational Studies in Mathematics, 111(3-2022).

Roh, K, H. (2008). Students' images and their understanding of the limit of a sequence definitions. *Educ Stud Math.* 69. 217-233.

Sebsibe, A. S., & Feza, N. N. (2019). Assessment of students' conceptual knowledge in limit of functions. International Electronic Journal of Mathematics Education, 15(2), em0574.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics,* 22.1-36.

Shaldehi, A. H., Shaldehi, M. H., & Hedayatpanah, B. (2022). A model for combining allegorical mental imagery with intuitive thinking in understanding the limit of a function. Indian Journal of Advanced Mathematics (IJAM), 2(2), 1-7.

Skemp, R. (1986). The psychology of learning mathematics. Expand American Edition. New Jersey: Lawrence Associated Publishers.

Star, J. R & Hoffmann, A. J. (2002). Assessing students' conceptions of reform mathematics. In Mewborn, D., Sztajn, P., White, D., Wiegel, H., Bryant, R., & Nooney, K.(Eds), *Proceeding of the twenty-fourth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education*. 1729-1732.

Stylianou, D, A. (2010). Teachers' conception of representation in middle school mathematics. *Journal Mathematical Teacher Education.* 13. 325-343. Do1: 101007/s 10857:0109143.y

Sulastri, R., Suryadi, D., Prabawanto, S., Cahya, E., Siagian, M. D., & Tamur, M. (2021, May). Prospective mathematics teachers' concept image on the limit of a function. In Journal of Physics: Conference Series (Vol. 1882, No. 1, p. 012068). IOP Publishing.

Swinyard, C & Larser, S, (2012), Coming to understand the Formal Definition of Limit Insights Gained From Engaging Students in Reinvention. *Journal for Research in Mathematics Education.* 43(4), 465-493.

Szydlik, J. E. (2000). Mathematical beliefs and conceptual understanding of the limit of a function. Journal for Research in Mathematics Education, 31(3), 258-276.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Published in Educational Studies in Mathematics*, 12 (2), 151-169.

Turgut, M & Yulmaz, S.(2012). Relationship among preservative primary teachers' gender, academic success and spatial ability. *International Journal of Instruction.* 5(2). 5-20.

Unal, H., Elizabeth Jakubowski & Darryl Corey.(2009). Differences in learning geometry among high and low spatial ability pre-service mathematics teachers. *International Journal of Mathematics Education in Science and Technology*. 40(8), 997-1012.

Viirman, O., Attorps, I., & Tossavainen, T. (2010). Different views–some Swedish mathematics students' concept images of the function concept. Nordic studies in Mathematics education, 15(4), 5-24.

Williams, S, R. (1991). Models of limit held by college calculus students. *Journal for Research in Mathematics Education*. 22(3), 219-236

Yang, J. C, & Chen, S.Y. (2010). Effects of gender differences and spatial abilities within a digital pentominoes game. *Computers & Education*. Vol.55. pp.1220-1233.DOI:10.1016/j:compedu.2010.05.019

Zaslavsky, O & Shir, K. (2005). Students' conceptions of a mathematical definition. Journal for Research in Mathematics Education. 36(4), 317-346.

Zhu, Cheng. (2007). Gender differences in mathematical problem-solving patterns: A review of the literature. *International Educational Journal.* Vol.8.No.2.pp 187-203.http;//iej.com.au