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Students' Conjectures on Open Classical Analogy Problem: Expanding or Narrowing?

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Abstract: Students' Conjectures on Open Classical Analogy Problem: Expanding or Narrowing?. Objectives: This study aims to describe students' mathematical conjecture in the context of open classical analogy problem. **Methods:** This descriptive research describes students' conjectural profiles using open classical analogies. Data were collected from the responses of 68 students. Data analyzed using constructor example, process, and quality of the conjecture. **Findings:** Results show that the example of constructing the conjecture has not paid attention to all possible cases, while the process of the conjecture used by students was from simple to more expanding or narrowing. Some of the proposed conjecture quality does not include the necessary constraints, and some include unnecessary limitations. Students can add to statements that sharpen the conjecture proposed previously. **Conclusion:** The results of this study indicate that in the construction of conjectures in the context of an open classical analogy, students involve their critical thinking skills.

Keywords: conjecture, example, open classical analogy, process, quality of conjecture.

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INTRODUCTION

Making conjectures or guesses is one of the skills that should be learned in every mathematics learning. In the standard reasoning and proving process, creating, and investigating mathematical conjectures is a skill that every student should proceed with (NCTM, 2000; CCSSM, 2010). According to Mason, Burton, and Stacey (2010), making conjectures is one of the competencies that must be achieved in learning mathematics. In Indonesia, the 2013 curriculum stipulates that the activity of making conjectures is part of the reasoning that will be developed in every mathematics learning (Minister of Education and Culture, 2013).

In fact, not many teachers teach students to make conjectures. Belnap and Parrot (2013) state that conjecture is a neglected aspect of learning mathematics. At the lower level, mathematics learning is oriented towards understanding and applying known theorems, while at the upper level, it is more aimed at developing proof and logic. Furthermore, conjectures are not only accessible to students but also beginners because they are only guesswork (Belnap & Parrot, 2013) and can even be conditioned for elementary school students (Komatsu, 2010).

Conjecture is a statement that is allegedly true but has not been proven (Houston, 2009; Mason et al., 2010; Sutarto et al., 2018). Cañadas & Castro (2007) stated that a conjecture is a statement based on empirical facts but has not been validated. The conjecture is still hypothetical (Astawa et al., 2018). Therefore, conjecture is a statement as a result of logical reasoning that has not been verified. Thus, conjecture can be true or false. If an example of a contradicting is found, the conjecture is wrong, but if it can be proven true, then the conjecture becomes a theorem.

In constructing conjectures, examples play an essential role (Ellis et al., 2012; Lockwood et al., 2016). The completeness of the sample will determine the quality of the proposed conjecture. The existence of a missed example case will provide a considerable chance of possible errors in the proposed conjecture (Balacheff, 1987; Alcock & Inglis, 2008; Sa'dijah et al, 2023).

One type of conjecture is an analogy, and in the process, each type of conjecture involves analogical reasoning (Cañadas et al., 2007). Conjecture formed by analogy rests on things that are already known. Thus a deep understanding of what is learned becomes a prerequisite for constructing conjectures through analogy. In this case, knowledge allows a person to build meaning through interpreting, making examples, concluding, and comparing (Wilson, 2016; Sa'dijah et al, 2021).

There has been much research on the use of analogy in making conjectures. Some of them focus on the use of analogy in mathematics learning to encourage students to make conjectures on open classical analogy problems (Lee & Sriraman, 2011); explore the use of analogies when students construct conjectures (Supratman & Rustina, 2016); conjecture construction uses analogical reasoning in ordinary students and gifted students (Supratman, & Ryane, 2018), the use of examples by mathematicians in the activity of proving conjectures (Lockwood et l., 2016), comparisons between mathematicians and students in using examples to make conjectures and substantiation (Lynch & Lockwood, 2019).

Belnap & Parrott (2013) revealed that the dimensions of characteristics and behavior of making mathematical conjectures include the overall process, the object under investigation, the nature of observations, the quality of written conjectures, and conjectural qualifications. The process dimension is related to the approach used, analysis, and mathematical knowledge used to construct conjectures. The dimensions of the object being observed are related to the examples used or considered to construct the conjectures, while the nature of the observations relates to measurements or observations of precise criteria and properties by considering impossible cases. The dimensions of the quality of written conjectures include the accuracy of conjectural statements that can be tested and the use of mathematical terminology, while the qualification of conjectures is related to the belief and evidence that conjectures apply in various cases.

In terms of the process of making conjectures using an analogy, Cañadas et al. (2007) stated that there are six steps required, namely observing two cases, finding, and determining the similarity of the two cases, formulating conjectures based on similarity, validation, generalization, and justification of the constructed generalizations. The process of constructing a conjecture using an analogy put forward by Cañadas et al. (2007) occurs in the case of classical analogies, namely reasoning in the form of A: B: C: D, where C and D are related in the same way as the relationship between A and B (English, 2004). In this classical analogy, components A, B, and C are known, while component D is unknown. This study focuses on an open classical analogy with only component A given, while components B, C, and D must be formulated (Lee & Sriraman, 2011). Furthermore, Lee & Sriraman (2011) state that further research is needed to make conjectures on open classical analogies outside the topic of triangles.

Research reveals that students' conjecture profiles are important to map the weaknesses and strengths of the conjectures they propose. This interest is supported by one of the unanswered questions in Belnap & Parrott's (2013) research, how can constructing a conjecture be nurtured and developed? In the Indonesian context, the mapping will provide direction for many teachers who have not yet practiced making conjecture skills in mathematics learning.

Previous research related to student conjectures, only seen from one aspect, for example, from the part of the example (Lockwood et al., 2016; Lynch & Lockwood, 2019) or the process (Cañadas et al., 2007). In this study, students' conjecture profiles are seen from the dimensions of the constructor examples, the conjecture formulation process, and the quality of the conjecture. With these three dimensions, this study provides a more complete picture of the conjecture constructed by students.

METHOD

Participants

This study's subjects were 68 students in two science classes at a public senior high school in Bojonegoro City, East Java province, Indonesia. The two classes had the best academic ability (especially in mathematics) compared to other science classes. Information related to student intellectual abilities was obtained from one of the mathematics teachers at the school. The two classes were chosen because students had never been trained to make and were not familiar with conjectures in mathematics class. With an excellent academic background in mathematics, the data were expected to be sufficient regarding their conjectural profile. Determination of research subjects was carried out using a purposive approach. Researchers recruit subjects by giving tests.

Research Design and Procedures

This is a descriptive study that portrays the conjecture profile of students on the open classical analog. The research flow was carried out in the following stages. This test question is given to participants. Then the answers to the test questions are grouped based on the variations in the answers submitted by paying attention to the consistency of the form of the answers to the two problems being solved. Then the research subject is determined through answers to test questions based on these variations. One representative was selected for each consistent group to be used as a research subject by paying attention to the communication skills of the prospective subject. The subject's answers on the test are then coded. This coding is carried out based on the indications that are read from the subject's answers.

Instrument

The research instrument was a test and students' narrative. The test questions of test instrument in this research were developed by researchers through focus group discussions of the research team. Then the research test questions were validated by two different experts before being given to students. The test was consisting of two questions. The first question was related to making conjectures in an open classical analogy, and the other asked for a narrative flow of the subject's thinking when making conjectures. The following are the test instrument in this research.

1. What are the properties of three consecutive integers? Then, define a property of n consecutive integers (for n e" 4), which is analogous to the property you specified for three consecutive integers.
2. Describe the line of thought you used to determine these traits.

Data Analysis

From the data obtained, data analysis was carried out through 3 stages, data reduction, data presentation and conclusion drawing. Data reduction means summarizing, selecting the main things, focusing on the important things, looking for themes and patterns and removing unnecessary things. Presenting data means displaying data that has been organized and arranged in certain patterns and relationships. Drawing conclusions is used to answer the problem formulation. Data reduction and presentation activities, as well as drawing conclusions are carried out during and after the data collection process.

The data were then analyzed by coding the conjectures made by the participants. After that, students' narrative conjectures were also analyzed, and the conjecture constructors, along with their conjuncture validation, were cross-checked. The validity of the data was validated using triangulation method techniques. Method triangulation was carried out by paying attention to the match between test results and student narratives. The participants' conjecture profile was considered from three dimensions, namely examples of conjecture builder, process, and quality of the conjecture. The indicators for these three dimensions can be seen in table 1.

Table 1. The dimensions of the conjecture profile in the classical analogy

No.	Dimension	Indicators
1	Example of a conjecture builder	Complete examples of conjecture builder cases
2	Process	• The approach used to construct conjectures • Conjecture validation
3	Conjecture Quality	• Accuracy of conjectural statements, testable and not nontrivial • Conjecture truth

Modification from Lockwood et al. (2016); Lynch & Lockwood (2019), & Cañadas et al.(2007).

RESULT AND DISCUSSION

The conjecture proposed by the students can be categorized into three groups, namely the addition of n consecutive integers, the multiplication of consecutive integers, and others. The complete conjecture submitted by students can be seen in table 2.

Table 1 describes the distribution of students' conjectures on the classical analogy problem. The statements indicating the conjectures in the table have been edited by the researcher. The following demonstrates students' conjectures divided into three-dimensional profiles: conjectures builder examples, process dimension, and conjecture quality dimensions.

Conjecture Builder Example Dimensions

Most of the examples that students use to build conjectures are still partial and minimal. In the conjecture construct for $n = 3$, only one

Table 2. Students' conjectures on the open classical analogy problem and their distribution

Conjecture for $n = 3$	Number of responses	The conjecture for $n \geq 4$	Number of responses
1. Addition			
		1.1.1 The sum of n consecutive integers with n odd, always divisible by the middle number, and the result is n itself.	18
		1.1.2 The sum of n consecutive integers with n odd, always divisible	

1.1 The result of summing three consecutive integers is always evenly divided by the integer in the middle. Note: There is one participant that requires a number in the middle not to be zero.	28	by the middle number. Note: There is one subject that requires the number in the middle to be other than zero.	7
		1.1.3 The sum of 5 consecutive integers is always divisible by the middle number.	1
		1.1.4 The sum of 5 consecutive integers is always divisible by the middle number, and the result is 5 itself.	1
		1.1.5 The sum of n consecutive integers is the product of the many numbers (n) with the middle number.	1
1.2 The result of summing three consecutive integers is odd; if the first number is even.	5	1.2.1 The sum of 5 consecutive integers is always divisible by the middle number, and the result is 5 itself.	1
		1.2.2 The sum of 6 consecutive integers is odd; if the first number is even.	2
		1.2.3 The sum of n (n odd) consecutive integers is odd; if the first number is even.	1
		1.2.4 The sum of the first and last numbers from an even number sequence is odd, and vice versa.	1
1.3 The result of summing three consecutive integers is even; if the first number is odd.	8	1.3.1 The sum of 4 consecutive integers is even; if the first number is odd.	4
		1.3.2 The sum of 7 consecutive integers is even; if the first number is odd.	1
		1.3.3 Not making	2
		1.3.4 The sum of 8 consecutive integers is even; if the first number is odd.	1
2. Multiplication properties of n consecutive integers			
2.1 The product of 3 consecutive integers is divisible by 3.	15	2.1.1 Product of n successive integers for $n \geq 4$ is divisible by 3.	7
		2.1.2 Product of 5 consecutive integers is divisible by 5.	1
		2.1.3 Product of n consecutive integers for $n \geq 4$ is divisible by n.	6
		2.1.4 The product of n consecutive integers is not divided by the prime number, which is greater than the largest number.	1
		2.2.1 The product of n consecutive	

2.2 The product of 3 consecutive integers is even (or is divisible by 2).	9	integers ($n \geq 4$) is divisible by 1, 2, 3,…, n.	1
		2.2.2 The product of n consecutive integers is even.	6
		2.2.3 The product of 5 consecutive integers is even.	2
2.3 The product of 3 consecutive integers is evenly divided by three factorials.	1	2.3.1 Product of n consecutive integers is divisible by n factorials.	1
2.4 Product of three consecutive integers is divisible by the middle number (trivial).	1	2.4.1 The product of five consecutive integers is evenly divisible by 2 and 3. It can also be divided by the middle number.	1
2. Other findings			
3.1 For three consecutive integers, multiply the middle number by the number on the right minus the product of the central number with the number on the left. If divided by the middle number, the result must be 2.	1	3.1.1 For n ($n \geq 4$, and odd) consecutive integers, multiply the middle number by all the numbers to the right minus the middle number's product with all the numbers on the left. The result is evenly divided by the middle number.	1

participant considers all possible models. This participant constructs conjecture 1.1, and the examples used involve positive integers, negative integers, and zeros. In this case, the three consecutive numbers are all negative, all positive, or mixed. The example made by this participant has also looked at the first number in the three consecutive integers he selected. There are examples where the first number is odd, and some are even.

Two more participants involve positive numbers, negative numbers, and zeros, but there is no example of all negative numbers. In constructing the 2.1 conjecture, these two participants have not considered the first number in each model. The first number selected is all odd. As many as 65 participants make all examples of constructors of conjecture by selecting three consecutive positive integers, only 9 of them presented the first number of cases oddly and even.

For the case for $n \geq 4$, the example of constructing a conjecture made by the participant is no different from the case for . They pay less attention to all possible issues, only one complete participant in generating an example of a conjecture builder. This one participant is also a complete participant in presenting examples of constructors for n = 3. The conjecture this participant makes is conjecture 1.1.2.

Nearly all of the examples involved in the participant constructing conjectures for are only positive numbers. Only 4 participants, including one participant whose sample is complete, use negative integers and zero in the example. However, those who make conjectures that do not replace n with a specific number (for example, the conjecture 1.1.1), for the most part, have sample three different values of n and consider odd-even on the first number.

Referring to Ellis et al. (2019), participants tended to choose examples that were easy, random, and limited from the aspect of the sample criteria used. The following narrative passages demonstrate the considerations students used in selecting examples.

"*Initially, I looked at problem no.1, and to determine its properties I chose the numbers that I thought were the easiest.*"
"In making an example, I chose a random number starting from the number 6."

The limitations of the examples of constructing students' constructs can be seen from examples that do not pay attention to all possible cases. This can be seen from the minimal involvement of negative integers and zeros in their examples and paying less attention to the possibility of the first number (which can be even or odd) in the sequence of integers.

This finding reinforces the research results by Lynch & Lockwood (2019), which states that the examples that students choose are easy, but the strategies they use in selecting samples are often not related to the purpose of these examples. This finding needs attention if teachers want to train students to build conjectures through examples, given the critical role of examples in making conjectures (Lockwood et al., 2016; Lynch & Lockwood, 2019).

Process Dimensions

In general, there are two approaches that the participant uses to construct conjectures in the open classical analogy when only component A is given (A: B: C: D); in this case, A given is three consecutive integers. The first approach determines B first (the conjecture of the n consecutive integer property for), followed by deciding D to conjecture the n sequential integer property for). The second approach is the opposite, determining D first followed by selecting B. The first approach is chosen because it departs from simple to develop, while the second approach is chosen on the basis of effectiveness or there are difficulties in implementing the first approach.

One of the participants using the first approach is a participant with conjectures 2.2 and 2.2.1. The participant's explanation regarding the approach he uses is as follows.

"At the beginning, I chose the numbers that I thought were the easiest. And among the three consecutive numbers, there must be an even number (divisible by 2). So from that, I chose the property. The product of 3 consecutive numbers is divisible by 2. Then, I tried the property for , and I could. I tried several variations of n, and the product of the consecutive number is divisible by 2. I tried to divide the product of these numbers by 3 and 4, and it also worked, so I concluded that if there are consecutive numbers, then the effect of the consecutive numbers is divisible by y if . "

Participants using the second approach are participants with conjectures 1.1 and 1.1.2. By adding conditions to the conjecture, the number in the middle cannot be equal to 0. The participant's explanation regarding the approach he uses is as follows.

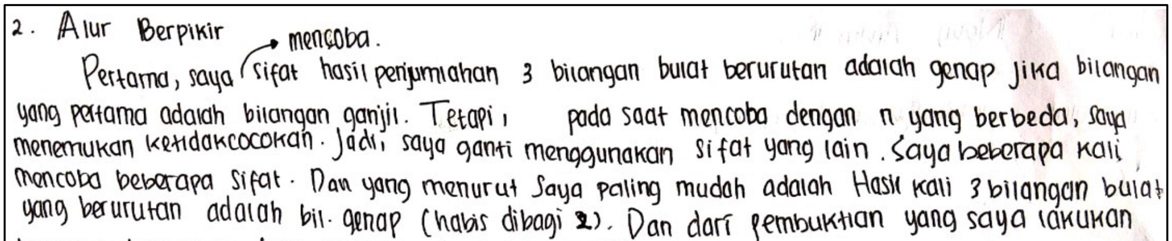
"When working on this problem, I tried to make the n-number property in sequence for first and tried to prove whether the property was appropriate or not for the sake of effectiveness in working on the problem. Once I feel it fits, I choose one of n consecutive numbers. I also try for n other consecutive numbers to prove whether the properties really apply to n different. "

There are also participants (conjectures 1.1 and 1.1.3) who actually intend to use the first approach but have difficulty. Finally, use the second approach. Here's the explanation.

"At first, I tried to determine the properties of three consecutive integers first, but I was confused in determining the properties when

, because I cannot find all numbers in that property. So I tried to determine the properties for , then I experimented. I started with n = 5 because it is not much different from , which is both odd, and I found the property, the sum of the five numbers, then divided by the middle number, then the quotient is 5. "

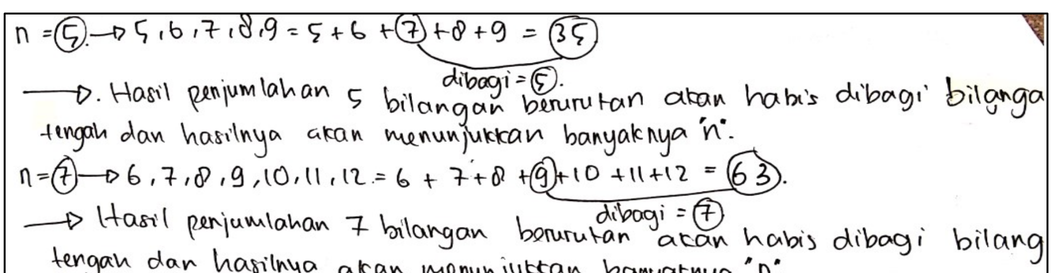
In the process, for the first approach, something happened not in one cycle. A participant tries to determine the property of 3 consecutive integers and tries it for () consecutive integers but fails. The participant has successfully looked for another alternative trait for , and finds its match for (). The following is the narrative of one of the participants (Figure 1).



Translation:
First, I tried the property of the sum of three consecutive numbers is even if the first number is odd. However, while trying with different *n*, I found a mismatch. So, I replace it by using another trait. I tried nature several times. In my opinion, the easiest property is the property that the product of three consecutive integers is even (divisible by two). This turned out to be true, I tried with whatever the value of *n*, the result is an even number.

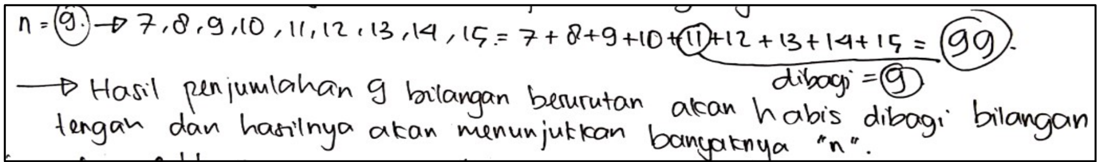
Figure 1. The narrative of one of the participants

From the validation aspect, most of the participants used other examples to validate their conjectures. For conjecture 1.1 and "derivatives" (conjecture 1.1.1-1.1.5), all participants (28 people) validated by adding a different example, then showing that the conjecture is still valid. For example, one of the participants made the 1.1.1 conjecture, constructing the conjecture made examples for $n = 5$ and $n = 7$ (Figure 2), and validated using $n = 9$ (Figure 3).



Translation:
$n = 5$ → 5, 6, 7, 8, 9 = 5 + 6 + 7 + 8 +9 = 35
The sum of five consecutive integers is divisible by the number at the very middle and the sum will show the number of "*n*"
$n = 7$ → 6, 7, 8, 9, 10, 11, 12 = 6 + 7 + 8 + 9 + 10 + 11 +12 = 63
The sum of seven consecutive integers is divisible by the number at the very middle and the sum will show the number of "*n*"

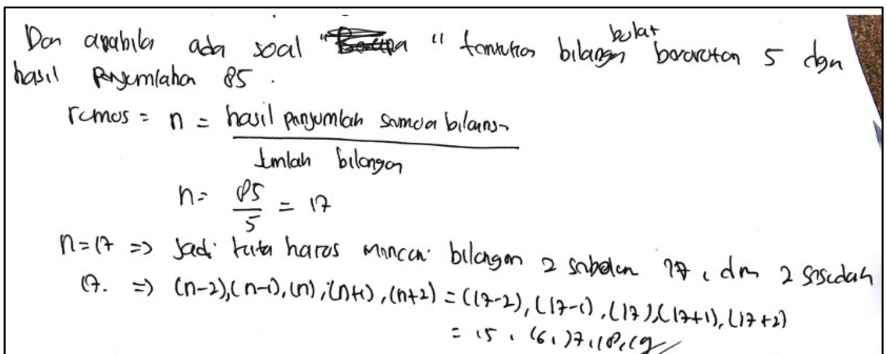
Figure 2. Constructing the conjecture made examples for $n = 5$ and



Translation:
$n = 9$ → 7, 8, 9, 10, 11, 12, 13, 14, 15 = 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 99
The sum of nine consecutive integers is divisible by the number at the very middle and the sum will show the number of "n"

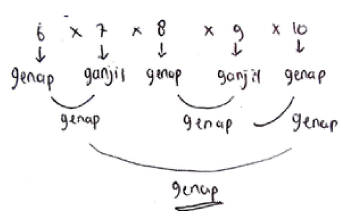
Figure 3. Constructing the conjecture validated using

In fact, there is one participant, with conjectures 1.1 and 1.1.5, trying to use the general form representation of 5 consecutive integers, but it is not used to validate his conjecture. He uses it to answer self-made problems that are still related to the nature of 5 consecutive integers, as shown in Figure 4.



Translation:
If there is a question, "find five consecutive integers whose sum is 85.", then n can be found by the following formula:
$n = \frac{the\ sum\ of\ all\ integers}{number\ \ of\ integers}$
$n = \frac{85}{5}$=17
Then, we should find two integers before 17 and two integers after 17, $(n-2), (n-1), (n), (n+1), (n+2)$. Thus, we obtain 15, 16, 17, 18, 19.

Figure 4. Constructing the nature of 5 consecutive integers



Translation: *genap* is even number, *ganjil* is odd number

Figure 5. An example of what the participant does when validating conjectures 2.2.2.

There are (no more than 5 participants) participants who validate the conjecture not by other examples but by analyzing the examples that have been made. Figure 5 is an example of what the participant does when validating conjectures 2.2.2.

From the process of compiling conjectures, the approaches used by students can be categorized as expanding or narrowing. The approach extends (Figures 7 and 8) when students determine the properties of 3 consecutive numbers first, followed by n consecutive integers (ne”4), while the narrowing approach (Figures 9 and 10) is made the opposite. The open-minded nature of the classical analogy proposed facilitates this to happen. These results confirm that the submission of conjectures in the context of an open classical analogy provides a space for creativity for students (Lee & Sriraman, 2011). Both of these approaches can produce the correct conjecture, so in learning that aims to develop the ability to construct conjectures, it is best to give students space to choose their own approaches.

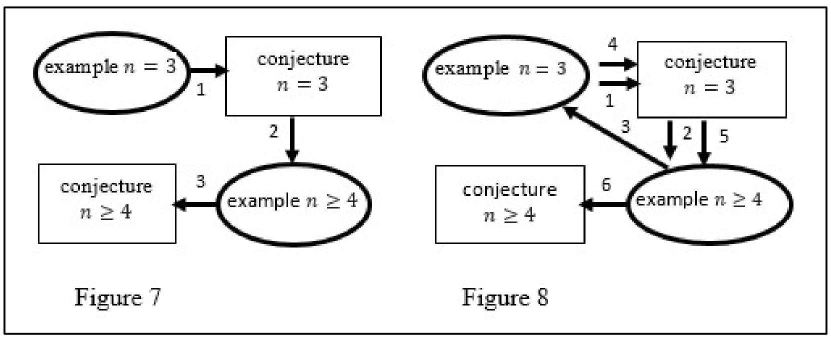


Figure 7 and 8. Students determine the properties of 3 consecutive numbers first, followed by n consecutive integers ($n \geq 4$)

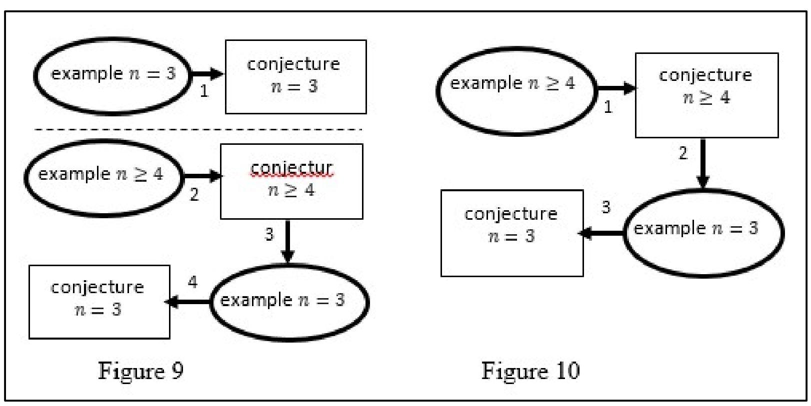


Figure 9 and 10. The reverse refinement approach

Conjecture Quality Dimensions

In general, the conjectural statements the participant makes are testable. All conjectures that are made can be expressed in the form of implications if P then Q. However, one nontrivial conjecture (2.4 conjecture) and three conjectures do not require sufficient conditions (conjecture 1.2.2, conjecture 1.3.1, and conjecture 1.3. 4) because it applies to all cases.

Conjecture statement 2.4 made by the participant, “the nature of multiplying three consecutive numbers regardless of the exact value the result is divisible by the middle value." The participant builds this conjecture by making five examples of 3 consecutive integers, and all of them are positive but have paid attention to the first number. The participant uses two examples, the first number is even, and the other three are odd. This conjecture is a trivial conjecture because the proof is already contained in the conjecture statement. That is, if a, b, c are three consecutive numbers, then $a \cdot b \cdot c$ must be divisible by b.

Here are three examples of a conjecture made by the participant, which really does not require necessary conditions.

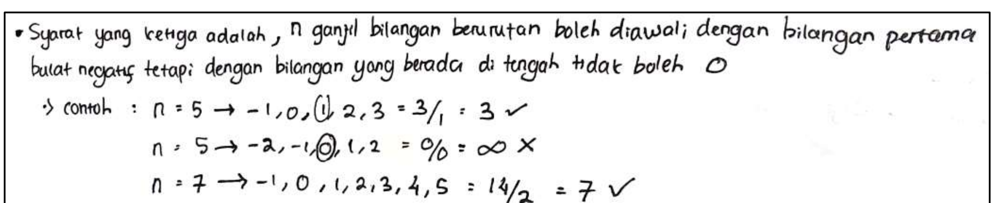
Conjecture 1.2.2 "The sum of 6 consecutive integers is an odd number if the first number is an even number."
Conjecture 1.3.1 "The sum of 4 consecutive integers is even if the first number is an odd number."
Conjecture 1.3.4 "The sum of consecutive integers of 8 numbers is an even number if the first number of the sum is an odd number."

The three conjectures (1.2.2, 1.3.1, and 1.3.4) do not require sufficient conditions. The three conjectures are valid for the first number, even or odd. If traced, this condition's emergence is analogous to the condition that the participant makes in the conjecture for $n = 3$. They do not realize that, for certain n, this condition is not necessary.

In terms of structure, the participant's conjecture for $n \geq 4$ has the same structure as the conjecture for , except for one participant. This participant constructs the 1.2 conjecture for and the 1.2.1 conjecture for . The participant's structural changes were because they did not find the properties in the 1.2 conjecture for . The following is a complete descripti of the participant.

"For , I choose the property of 3 integers if the add up is odd if the first number is even. However, for , I did not find the similarities in the properties I found for . So, I tried another property and found the property for , that if the sum is divided by the middle number, it will all be the same (5). "

In terms of truth, conjecture 1.1, and its derivatives, the truth still requires a condition that the number in the middle should not equal 0. Only one out of 28 participants make conjecture 1.1 and its derivatives, which write these conditions. This one participant makes the conjectures 1.1 and 1.1.2 and is aware of the universe of the conjecture. This one participant did a complete analysis of the conjectures he made using examples. Before deciding to conjecture 1.1.2, this participant began to analyze for and , taking the example that all numbers are positive, and the first number is odd. Next, he tries, with the first number being even. Finally, this participant makes three examples in figure 6 so that he can add his conjectural conditions.



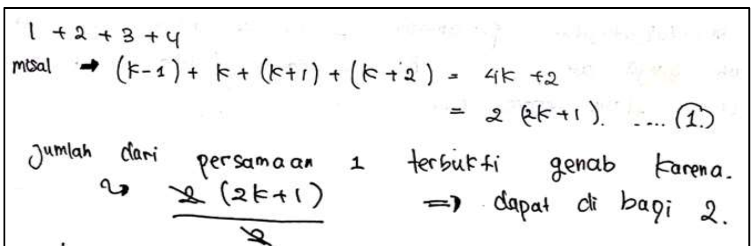
Translation:
The third property is if the number of integers, ??, and then it can be started with any negative integer but the integer in the very middle should not be zero.
Example: n = 5 → -1, 0, 1, 2, 3 = 3/1 = 3
n = 5 → -2, -1, 0, 1, 2 = 0/0 = 8
n = 7 → -1, 0, 1, 2, 3, 4, 5 = 14/2 = 7

Figure 6. Participant makes three examples and able to add his conjectural conditions

In terms of the proposed conjecture quality, most students still focus on the similarity of the surface when making analogies when compiling the conjecture and its development. For example, when constructing conjecture 1.1 and its "derivatives," the student is still unaware of the need for the conjecture to apply. Another evidence that students focus on surface similarity can be seen in students constructing Conjecture 1.3 and Conjecture 1.3.1. They still attach the conditions to the 1.3.1 conjecture just like the conditions they put in the 1.3 conjecture, even though it is not needed. These results are consistent with the research carried out by Lee & Sriraman (2011) and English (2004).

One interesting thing from some of the conjectures that students submitted was adding additional statements to sharpen the conjectures they previously proposed (see conjunctions 1.1 and 1.1.1; conjunctions 1.1 and 1.1.4; conjectures 2.2 and 2.2.1; conjectures 2.3 and 2.3. 1). This shows that the task of making conjectures in an open classical analogy makes students engage their critical thinking skills.

From the aspect of testing validity, a symbolic representation for consecutive numbers (for example, n, $n+1$, and $n+2$) has not been an option for students. In fact, none of the students used the general notation for odd ($2n+1$ or $2n-1$) and even ($2n$) numbers. Only two students try to use the general notation of consecutive numbers, one of which is shown in Figure 4. One other student uses it to validate the 1.3.1 conjecture, as illustrated in Figure 11.



Translation:
1 + 2 + 3 + 4
Let → (??- 1) + ?? + (??+ 1) + (??+ 2) = 4?? + 2
= 2 (2?? + 1)
The total of equation 1 is proved old number because
➔ 2 (2??+ 1)/2 is divided by 2

Figure 11. Student uses it to validate the 1.3.1 conjecture

Figure 11 shows that although the participant tries to use the general notation for consecutive numbers in the validation process, he forgets that the conjecture's first number is odd. The k-1 representation without any explanation is not sufficient if what is meant is an odd number.

The research finding indicates that the students paid less attention to the requirements of the conjecture in the context of the open classical analogy meant that. In the expands approach, the conditions that exist in the initial conjecture are still used in the developed conjecture, even though it is not needed. This requires reflective thinking in proposing conjectures in the context of an open classical analogy (Kholid et al, 2020; Sa'dijah et al, 2021). Students are also not familiar with using symbolic representation for consecutive integers, odd numbers, or even numbers. In this case, students do not have a multi-structural understanding of integers' concepts (Afriyani et al, 2018; Sa'dijah et al, 2018). On the positive side, the process of making conjectures in the context of an open classical analogy allows students to think flexibly, expanding, or narrowing.

CONCLUSION

Even though students have never learned to make conjectures, this study's results indicate that they are "able" to do it. Students' conjecture profile in the dimensions of the conjecture builder example still does not cover all possible cases. In the process dimension, especially the validation is only limited to using other examples, while in the aspect of the approach, some expand and narrow. In the dimensions of the conjecture's quality, they tend to pay less attention to the requirements. There are some drawbacks to the conjecture they propose, but this is a positive indication for teachers who plan to develop conjecture skills in their students. The open classical analogy can be chosen as the context. The reason is that the open classical analogy allows one the freedom to select the appropriate properties or characteristics of the object while at the same time considering its possible application in different analogous cases. The results of this study also indicate that in the construction of conjectures in the context of an open classical analogy, students involve their critical thinking skills.

The limitations of the conjecture builder examples found in this study need to be followed up on the cause and why high school students, who should be formal, tend not to use generic forms to validate their conjectures. The follow-up can be done by looking at the learning process that students have received or in the form of designing learning to overcome this. Regarding the approach during the construction of the conjecture that can expand or narrow in the context of the open classical analogy found in this study, it is necessary to study further what types of conjecture are suitable for both approaches. The finding of further study may answer the possibility of an equally good conjecture when approached using both approaches.

REFERENCES

Alcock, L. J., & Inglis, M. (2008). Doctoral students' use of examples in evaluating and proving conjectures. Educational Studies in Mathematics, 69, 111–129.

Afriyani, D., & Sa'dijah, C., Subanji, S., Muksar, M. (2018). Characteristics of students' mathematical understanding in solving multiple representation task based on solo taxonomy. *International Electronic Journal of Mathematics Education*, *13*(3), 281-287.

Astawa, I., Budayasa, I. K., & Juniati, D. (2018). The process of student cognition in constructing mathematical conjecture. *Journal on Mathematics Education*, *9*(1), 15-26.

Balacheff, N. (1987). Processus de prevue et situations de validation. *Educational Studies in Mathematics, 18*(2), 147–176

Belnap, J., & Parrott, A. (2013, February). Understanding mathematical conjecturing. In *Proceedings of the 16th SIGMAA on Research in Undergraduate Mathematics Education Conference* (Vol. 2, pp. 25-31).

Cañadas, M. C. & Castro, E. (2007). A proposal of categorisation for analysing inductive reasoning. *PNA, 1*(2), 67-78.

Cañadas, M. C., Deulofeu, J., Figueiras, L., Reid, D., & Yevdokimov, O. (2007). The conjecturing process: Perspectives in theory and implications in practice. *Journal of Teaching and Learning*, *5*(1).

Council of Chief State School Officers. (2010). Common core state standards for mathematics. *Washington, DC: Council of Chief State School Officers*.

Ellis, A. B., Lockwood, E., Williams, C. C., Dogan, M. F., & Knuth, E. (2012). Middle School Students' Example Use in Conjecture Exploration and Justification. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Ellis, A. B., Ozgur, Z., Vinsonhaler, R., Dogan, M. F., Carolan, T., Lockwood, E., ... & Zaslavsky, O. (2019). Student thinking with examples: The criteria-affordances-purposes-strategies framework. *The Journal of Mathematical Behavior*, *53*, 263-283.

English, L. D. (Ed.). (2004). *Mathematical and analogical reasoning of young learners*. Routledge.

Houston, K. (2009). How to think like a mathematician: A companion to undergraduate mathematics. Cambridge University Press.

Kholid, M. N., Sa'dijah, C., Hidayanto, E., & Permadi, H. (2020). How are students' reflective thinking for problem solving?. *Journal for the Education of Gifted Young Scientists*, *8*(3), 1135-1146.

Komatsu, K. (2010). Counter-examples for refinement of conjectures and proofs in primary school mathematics. *The Journal of Mathematical Behavior*, *29*(1), 1-10.

Lee, K. H., & Sriraman, B. (2011). Conjecturing via reconceived classical analogy. *Educational studies in mathematics*, *76*(2), 123-140.

Lockwood, E., Ellis, A. B., & Lynch, A. G. (2016). Mathematicians' example-related activity when exploring and proving conjectures. *International Journal of Research in Undergraduate Mathematics Education*, *2*(2), 165-196.

Lynch, A. G., & Lockwood, E. (2019). A comparison between mathematicians' and students' use of examples for conjecturing and proving. *The Journal of Mathematical Behavior*, *53*, 323-338.

Mason J, Burton L and Stacey K. (2010). *Thingking mathematically second edition,* England: Pearson Education Limited

Minister of Education and Culture. (2013). *Modul pelatihan implementasi kurikulum 2013 sma matematika*. [Training module for implementation of the 2013 high school mathematics curriculum.]. *Jakarta: Badan Pengembangan Sumber Daya Manusia Pendidikan dan Kebudayaan dan Penjaminan Mutu Pendidikan.*

Nasional Council of Teacher of Mathematics. (2000). *Principles and standards for school mathematics Reston*, VA: NCTM

Sa'dijah, C., Afriyani, D., Subanji, Muksar, M., & Anwar, L. (2018). Assessing students' pseudo-mathematical translation using translation-verification model. *AIP Conference Proceedings, 2014*. https://doi.org/10.1063/1.5054548

Sa'dijah, C., Murtafiah, W., Anwar, L., Nurhakiki, R., & Cahyowati, E. T. D. (2021). Teaching higher-order thinking skills in mathematics classrooms: Gender differences. *Journal on Mathematics Education, 12(1), 159–179.* https://doi.org/10.22342/jme.12.1.13087.159-180

Sa'dijah, C., Rahayuningsih, S., Sukoriyanto, S., Qohar, A., & Pujarama, W. (2021). Concept understanding layers of seventh graders based on communication ability in solving fraction problems. *AIP Conference Proceedings, 2330.* https://doi.org/10.1063/5.004372

Sa'dijah, C., Purnomo, H., Abdullah, A. H., Permadi, H., Anwar, L., Cahyowati, E. T. D., & Sa'diyah, M. (2023). Students' numeracy skills in solving numeracy tasks: analysis of students of junior high schools. *AIP Conference Proceedings, 2569.* https://doi.org/10.1063/5.0113664

Supratman, R. S., & Rustina, R. (2016). Conjecturing via analogical reasoning in developing scientific approach in junior high school students. In *J. Phys. Conf. Ser. 693* (Vol. 12017).

Supratman, N. R., & Ryane, S. (2018). Conjecturing via analogical reasoning constructs ordinary students into like gifted student. In *J. Phys. Conf. Ser* (Vol. 943, p. 012025).

Sutarto, Nusantara, T., Subanji, Hastuti, D. I, Dafik, (2018). Global conjecturing process in pattern generalization problem. *JPhCS*, *1008*(1), 012060.

Wilson, L. O. (2016). Anderson and Krathwohl–Bloom's taxonomy revised. *Understanding the New Version of Bloom's Taxonomy*.