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Assessing Pre-service Mathematics Education Teachers Deductive Reasoning via Proof Writing in Basic Geometry: The Power of SOLO Taxonomy

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Abstract: Assessing Pre-service Mathematics Education Teachers Deductive Reasoning via Proof Writing in Basic Geometry: The Power of SOLO Taxonomy. Objectives: This study characterizes the developmental patterns of deductive reasoning via proof-writing of pre-service mathematics education teachers using the Structure of the Observed Learning Outcome (SOLO) taxonomy. **Methods:** One hundred three pre-service teachers were given twelve items involving basic concepts of plane geometry to assess. An in-depth analysis of their proof was done and they were grouped through a two-step clustering technique. **Findings:** Four compatible levels of developmental pattern to the SOLO level were detected. At level 0, students do not know how to establish proof. At level 1, students provided a single or few valid idea/s. Students demonstrating level 2 thinking provided many true ideas. However, the proofs are unclear and illogical. Level 3 students' proof is precisely logical. **Conclusion:** The research concluded that the SOLO taxonomy is a precise framework for conceptual knowledge assessment and that knowledge indeed has structure.

Keywords: assessment, deductive reasoning, proof-writing, SOLO taxonomy.

Abstrak: Penilaian Penalaran Deduktif Calon Guru Pendidikan Matematika melalui Proof Writing Geometri Dasar: Kemampuan Taksonomi SOLO. Tujuan: *Penelitian ini mengkarakterisasi pola perkembangan penalaran deduktif melalui penulisan bukti guru pendidikan matematika prajabatan dengan menggunakan taksonomi Structure of the Observed Learning Outcome (SOLO).* ***Metode:*** *Seratus tiga guru pra-jabatan diberikan dua belas item yang melibatkan konsep dasar geometri bidang untuk dinilai. Analisis mendalam terhadap pembuktian mereka telah dilakukan dan mereka dikelompokkan melalui teknik pengelompokan dua langkah.* ***Temuan:*** *Empat tingkat pola perkembangan yang kompatibel dengan tingkat SOLO terdeteksi. Pada level 0, siswa belum mengetahui cara pembuktian. Pada tingkat 1, siswa memberikan satu atau beberapa ide yang valid. Siswa yang mendemonstrasikan pemikiran tingkat 2 memberikan banyak ide yang benar. Namun, bukti-buktinya tidak jelas dan tidak logis. Pembuktian siswa tingkat 3 sangat logis.* ***Kesimpulan:*** *Penelitian tersebut menyimpulkan bahwa taksonomi SOLO merupakan kerangka kerja yang tepat untuk penilaian pengetahuan konseptual dan bahwa pengetahuan memang memiliki struktur.*

Kata kunci: *asesmen, penalaran deduktif, proof-writing, taksonomi SOLO.*

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INTRODUCTION

In mathematics, knowing the interrelation of concepts is essential in providing proof or an argument, hence it is the fundamental unit of cognition (Wang, 2008). For Conner (2013), deductive reasoning is essential in proof writing in mathematics (as a mathematical argument), which is built upon axioms and the use of theorems and other mathematical concepts in a logical manner. Besides problem-solving (Wong & Bukalov, 2013), its embodiment is central to writing a mathematical proof as a chain of deductive inferences (Carrascal, 2015). The knowledge for assertion is connected into a web of arguments, or translated these into second order symbol system. Carrascal (2015) encourages to include argumentation to improve thought and creativity. This claim indicates the importance of proof writing hence future mathematics teachers must be trained in this thinking activity to develop their deductive reasoning skills.

Proof writing is a communication of thought and allows students to intra-connect and establish relationship between mathematical concepts and the requirement in a mathematical task. This is a written mathematical communication (Sekaryanti, Cholily, Darmayanti, Rahma, & Maryanto, 2022). Sekaryanti et al. (2022) concluded that students with low math ability do not solve completely and described their mathematical communication to be monostructural, an equivalent term for uni-structural level of the Structured of Observed Learning Outcome (SOLO) taxonomy (Biggs & Collis, 1982; Biggs & Tang, 2011). They added that students with moderate math skills reached at polystructural (multi-structural level)-a situation of which students know the correct procedure but is false short in using and processing the problem information. Students with higher math skill know the procedures and connect them well reaching at the relational stage but not the extended abstract level (Sekaryanti et al., 2022).

Writing a proof in mathematics is a specie of argumentation and a form of communication (Carrascal, 2015). While the practice of traditional method (students are given pre-determined statement to prove) of proof is evident in some instances, has done little impact on students understanding of proof writing (Cox, 2004). Sinclair and Robutti (2012) carried out the notion of proving process in discussing about proof and proving. They enumerated two phases; the first one involves the formulation of conjecture and the second one is the provision of proof that follows the rule of logical consequence.

Wong et al. (2013) considered proof as a problem solving activity. Wong et al. (2013) grounded their work from the van Heili levels, and. identified four complexity levels in performing geometry problem solving in which at level four, students solve problems by using deductive reasoning to prove mathematical statements.

The attempt to broaden the understanding on the possible existence of thinking hierarchy within the deductive reasoning level described by Wong et al. (2013) is less explored in the tertiary level. Hence, Nardi, Gonzalez-Martin, Gueudet, Iannone, and Winsløw (2014) noted that at the university level, mathematics education research is not fully mature although educational researches for tertiary level mathematics started to expand in number. At the same time, using the SOLO taxonomy in the assessment of proof writing is scant despite the fact that it is more appropriate since at this stage students are expected to have the ability to establish relationship between concepts (Sekaryanti et al., 2022) with formal language (Stålne, Kjellström, & Utriainen, 2015). Biggs et al. (1982, 1989) argued that formal mode of learning that is the ability of the student to make hypothesis and ask questions about things, reason with principles, go beyond the information, and examine conclusion should be the primary concern at the university level of education.

This present study attempted describing the level of deductive reasoning through proof writing among the pre-service Bachelor of Secondary Education students specializing mathematics in the basic concepts of plane geometry. Günhan, (2014) claimed that successful geometry teaching, aside from the ability to teach also depends on the geometry knowledge the teacher has. Hence, mathematics teacher preparation must provide activities that promote the habits of the mind that includes but not limited to reasoning and explaining, discourse, and thinking (Caniglia & Meadows, 2018).

There are five levels of cognition that describe the structural quality of knowledge (pre-structural, uni-structural, multi-structural, relational, and extended abstract) that progress in a cyclical fashion (Biggs et al.,1982; Biggs et al., 2011). Each level reflects learners' organization and structural complexity of knowledge in performing an academic task (Biggs et al., 1982; Mosley, Baumeld, Elliott, Gregson, Higgins, Miller, & Newton, 2005; Nor & Idris, 2010; Huey, 2011; Gagani & Misa, 2018). Each succeeding level requires an increasing amount of working memory or attention span. At the upper levels, there are additional variables to consider, relationships between variables to consider, and distinctions between hypothetical and real-world scenarios to be formed (Gagani et al., 2018).

The contribution of this study is of two perspectives. First, it purports the use of the SOLO taxonomy as assessment framework for proof writing as it can assess students' knowledge structure allowing teacher get insights pertaining to the student ability to intra-connect prior learning to new ones, relate mathematical ideas into meaningful and valid argument, and at the same time communicate mathematically. This is expected of pre-service mathematics teachers (Sekaryanti et al. 2022). Second, it gives an insight to teachers that math teacher preparation should include proof writing activity. By having this, the teachers can draw insights from the assessment results of the pre-service math teachers needs in terms of strengthening their conceptual knowledge to have a better proof writing skills and eventually improved their explanatory power when teaching.

Some Investigation on Proof Writing

In the cross-sectional study of Senk (1985) that investigated proof writing in geometry that includes topics in congruent and similar triangles, parallel lines, and quadrilaterals revealed that only 30% of the 2 699 public high school students reached 75% mastery of proof writing. Senk (1985) noted that students whose initial thinking level is at higher van Hiele levels has the higher probability of success. Senk (1989) further noted that proof writing achievement among the two hundred forty-one secondary students significantly vary with the van Hiele levels with either with entering geometry knowledge or in geometry achievement. Senk (1989) notably found out the existence of moderate relationship (r ranges from.5-.6) between proof writing and van Hiele levels and high correlation (r=.7) between entering knowledge and achievement in standard geometry content. However, Senk (1989) noted that the hypothesis that students at higher levels (deduction and rigor) to write a proof consistently was partially supported. This projects that students are not consistent in providing a well-connected proofs.

Crompton, Grant & Shraim (2018) supported Senks' (1989) findings. Crompton et al. (2018) synthesized empirical evidence and reported that the relational level of understanding geometry concepts is is not learned. This implicates that students is less able to connect related and interrelate concepts in their justification. To note, the ability to use and connect the previously learned concepts to novel ones is a demonstration of a higher level thinking (Biggs et al., 1982; Gagani et al., 2018). Wong et al. (2013) claimed that higher order thinking skills is a requirement in geometry and suggested that students cannot proceed at the higher levels without having experience the lower thinking skills.

In the sequential explanatory study participated with 30 prospective mathematics teacher on proof construction (indirect and direct proving), Maarif, Perbowo, Noto, and Harisman (2019) found out obstacles in constructing geometrical proof. The tasks in the study of Maarif et al. (2019) involved proving two sides of a triangle equal (indirect proving) given two equal angles. The second task involved proving if a line passing through the mid-point of a side of a triangle and is parallel to the second side will cut the third side at its mid-point (direct proof).The result accounted only 6.67% of the students who were able to write indirect proving while 13.33% in the case of the direct proving.

Proof Writing as Deductive Reasoning

Writing a proof is a form of deductive reasoning that is essential in the primary level (Harel & Weber, 2020; Sinclair et al., 2012), and in the high school level since it helps them in the tertiary level mathematics most especially in proof based mathematics (Benkhalti, 2017; Sinclair et al., 2012; & Senk, 1985). Conner (2013) affirmed that the essential element in school mathematics is the ability and propensity in reasoning and in providing justification, hence the inclusion of proof writing in geometry curriculum is an important goal (Senk, 1989). Despite its importance, Benkhali (2017) noted that a problem of proof writing is evident in most undergraduate students from an undergraduate inquiry-based transition-to-proof course, and the difficulties maybe due to poor school programs, or the students make detrimental actions, or perhaps this may be because of teaching approaches that only caters surface level of understanding (Crompton, Grant, & Shraim, 2018). Likewise, Conner (2013) recognized the existence of proof writing difficulties in all school levels.

Theoretical framework

Deductive reasoning in this current investigation is the pre-service mathematics teacher to provide written proof to their claim by intra-connecting concepts in geometry in logical manner. The written proof is in a paragraph format rather than the conventional two-column proof or the indirect method of proof writing as usually seen in most plane geometry textbooks.

One essential question in the educative process is on how to assess the student's quality of knowledge structure, like for example, in solving complex problems or the students learning outcomes on a specific subject (Stålne et al., 2015). Stålne, et al. (2015) implied that students' learning could be assessed in their solution to a given problem, or in an answer to an open-ended question. This implies the need of an assessment framework that allows the characterization of the quality of knowledge. Hence, the SOLO taxonomy was used in this investigation as the framework for the assessment of students' proof writing to know their level of deductive reasoning through proof writing in the basic plane geometry concepts since it is an assessment model that can distinguish surface and deep conceptual understandings in a wide variety of school subjects (Dudley et al., 2009). The SOLO model was the appropriate model for evaluating their written justification or argumentation of the pre-service math teachers (Imrie,1995; Luo, Wei, Shi, & Xiao, 2020).

There are two frameworks that can be used is assessing geometry knowledge, one is the frequently used van Hiele model (see Jurdak,1991) and the second one is the least used SOLO taxonomy (Biggs & Collis, 1982). According to Jurdak (1991), the van Hiele and the SOLO taxonomy levels are quite comparable, and it was suggested that the latter be utilized as an operational method for describing learning outcomes in geometrical problems.

Two decades ago, Chick (1998) enumerated that the SOLO taxonomy have been useful for classifying the quality of responses so far in the areas of polynomials, basic algebra and function notation, basic algebra operations and relationships, and complicated arithmetical progression. Recently, as assessment framework for assessing the depth of knowledge gain through formal schooling has begun to surface at the tertiary level mathematics and statistics. The SOLO model serves as an assessment framework in some researches in mathematics. The focus of interest were problem solving in statistics (Mulbar, Rahman, & Ahmar, 2017), measures of centrality (Groth & Bergner 2006), inductive reasoning level in geometry (Gagani et al., 2017), solving composition of function with multiple representations (Afriyani, Sa'dijah, & Muksa, 2018), one-question problem posing in making cones task (Caniglia & Meadows, 2018), students reasoning about variation and variability (Chaphalkar & Wu, 2020), inferential reasoning in statistics (Huey, 2011), algorithmic thinking (Niemela, Mikkolainen, & Vuorinen, 2018), and spatial orientation skills in geometry (Özdemir & Yildiz,2015).

Consequently, three decades ago, Jurdak (1991) noted that many studies regarding the development of geometry knowledge have utilized the van Hiele model to label students with the van Hiele levels based on their achievement in geometry. Jurdak (1991) added that another means of knowing achievement in geometry is by classifying achievement rather than on classifying individuals. This is where the SOLO model fits in this study because it is for assessing the quality of learning at the tertiary level (Biggs et al., 1982, 1989; Biggs & Tang, 2011).

The van Hiele models are intended to describe geometric thinking following a sequence of cognitive abilities that characterizes development stages in a sequential order. It does not provide the framework for describing the learned outcomes in a cyclical manner of which the increasing structural complexities from uni-structural to extended abstract can be observed in a cyclical fashion at each mode of cognitive functioning, from iconic mode to post-formal mode (Biggs et al., 1982; Chick, 1998).

The validity of the cyclical levels and the usefulness of the SOLO taxonomy in classifying students into interpretable groups are well established (Jurdak, 1991). On the other hand, the rigor stage of the van Hiele model has not been formally examined in the pre-university level students and is scant for to those concentrating in mathematics (Jurdak 1991). The SOLO model is a research-based model that could precisely quantify the quality of learning gain (Imrie ,1995; Biggs et al., 2011). It can also be used to evaluate the organization of thoughts in an article, responses to technical questions, medical diagnoses, or student reports, according to Imrie (1995).

Despite its usability in assessing the quality of knowledge, Chan, Tsui, Chan and Hong (2002), noted conceptual ambiguity of the SOLO taxonomy however affirmed that it can be used as measurement framework for various learning ends. Radmehr and Drake (2019) acknowledged it as framework for designing tasks for assessment and as model for assessing, and classifying responses to an assessment task but have demarcated its limitation in the areas wherein metacognitive and affective element of learning in mathematics is sought.

The five levels of cognition that describe the structural quality of knowledge progress in a cyclical fashion in each mode of cognitive functioning, from iconic to post-formal mode (Biggs et al., 1982). Each level reflects learners' organization and structural complexity of knowledge in performing an academic task (Biggs et al., 1982; Mosley et al.,, 2005; Nor et al., 2010; Huey, 2011; Gagani et al., 2017). With each level up demands more working memory and knowledge. At the upper levels, there are many factors to take into account, more connections between components to look at, and more distinctions to be made between real-world and fictitious scenarios (Gagani et al., 2017). Putnam, Lampert, and Peterson (1990) claimed that a focus on knowledge structure is important because it makes explicit the implicit knowledge for mathematics competence

Since this study aimed of describing the quality of deductive reasoning skill of the pre-service teacher, an open-ended type of item where given to give way the categorization of responses into the thinking levels of the SOLO taxonomy (Groth et al., 2006). Stålne et al. (2015) noted that in higher education, working within the formal mode is usually evident. This allows the SOLO model as the appropriate model for evaluating their written justification or argumentation (Imrie,1995; Luo, Wei, Shi, & Xiao, 2020).

METHODS

Research Design

This is a descriptive survey with complete enumeration sampling design. The participants were from a state university in Cebu City Philippines consisting of one hundred and three out of the one hundred and eighteen pre-service mathematics teachers in their practice teaching year in the school year 2018-2019.

Before the study commences, a letter of permission was sent to the school supervisor and was approved. An informed consent, information about the study, and a questionnaire for understanding were given to the participants prior to the collection. These three instruments were tailored fit from the ethics review committee of the same university. The information about the study explains that purpose of the study, provides confidentiality and anonymity statement that their identity will be protected and that they can voluntarily participate. It also articulated that there is no monetary compensation for their participation and that they can withdraw anytime even at the course of answering the test questions. They are given one week to understand what is articulated in the information about the study. They are instructed that if they are willing to participate, then they will answer the questionnaire for understanding and the informed consent. Once they have decided, they are required to bring the questionnaire for understanding and the informed consent form on the day of the test. The participant voluntarily participated in the study.

Participants

A total of one hundred thirty pre-service mathematics teachers participated in the study of which twenty seven (10 males and 17 females) participated in the pilot testing of the deductive reasoning task (mean age = 19.25; S.D. = 1.25). These twenty-seven pre-service mathematics teachers are from three universities in Cebu City, Philippines. The one hundred and three were from another university. This university is the leading producer of math teachers in terms of number per year in Cebu City, Philippines.

The total number of pre-service teachers of which the one hundred and three (28 males and 75 females; mean age = 19.50; S.D.=.75) were taken is one hundred thirty eight (32 males and 106 females; mean age=20.25; S.D.=.5). All one hundred thirty eight pre-service mathematics teachers were invited.

Instrument

The instrument contained twelve tasks (refer to supplementary file) designed to know the knowledge structure of the participants. The tasks included three basic contents as follows: pair of angles; polygons, and the angles and sides relationships in a triangle. The test has a perfect content validity rating (CVR=1, Lawshe,1975) from five mathematics teachers who hold masters degree in mathematics education. Lawshes' (1975) analytical method allowed the raters to give an item three points if it can elicit deductive inferences, two points when it is but not relevant, and one if is irrelevant. For a panel of size five, the method requires a hundred percent agreement to an item to be acceptable. After this stage, the test items was pilot tested to twenty seven pre-service math teachers from three universities. Originally, thirty were invited but only the twenty seven came. The scoring was done with three math teachers with the use of the scoring guide (see data analysis section). The raters decided the appropriate score for a certain justification.

After recording the data it was subjected into an internal consistency estimation. Two measures of reliability measures was obtained with SPSS 24, the omega value and the alpha value. To get these measures, a maximum likelihood extraction method of factor analysis were employed. The omega value utilized the item factor loading whereas the alpha was obtained with the use of the covariance (Padilla & Divers, 2016). Due to the small sample, an iterations was implemented and one factor solution was extracted in five iteration. The test is reliable with an omega value of 0.859 and 0.755 for n=27 and n=103, respectively; and alpha value of .864 and .747 for n= 27 and n=103, respectively (Goodboy & Martin, 2020; Savalei & Reise, 2018). The correlations of the scores for each item and the total scores ranges from 0.40-0.65.

With partial credit scoring, and with the classical approach of identifying item difficulty level or p-value (Aiken, 1979; Hingorjo & Jaleel, 2012), the difficulty of the test items ranges from easy to difficult as shown by the indexes in table 1. An index, which is greater than 0.70 puts the item at the too easy scale, between 0.3-0.7, is average, and less than 0.3 is very difficult (Hingorjo et al., 2012).

The pair of angles requires the use of knowledge about the Vertical Angles Theorem and the Linear Pair Postulate. For the polygons, the consistency of understanding about the properties of square, rhombus, rectangle, and equilateral and isosceles triangles are essential. The angles and sides relationship in a triangle requires the concepts of the Triangle Angles Sum Theorem, the Linear Pair Postulate, and the Remote-Exterior Angles Theorem in a proof. A 5-item SOLO-based scoring guide that was examined by Biggs, one of the authors of SOLO taxonomy was also used. The scoring guide was made to reflect the SOLO levels, from uni-structural to extended abract level (Bigg & Collis, 1982; Biggs and Tang, 2011).

As mentioned, proof writing is the same as mathematical deductive reasoning (Harel & Weber, 2020). Following Harel and Webers' argument, a justification is considered mathematical deductive reasoning if it fits to any of the following theoretical perspectives. First is when the deductive inference follows a decontextualized logical rule for example the use of De Morgans Law or modus ponens in a two-column proof. Second is when their justification fits transformational reasoning (TR) (see Simon, 1996 for further explanation of TR). A third criterion puts mathematical reasoning as socially sanctioned rules of reasoning (Harel & Weber) which was not considered in this study.

Data Analysis

To fit the SOLO-model as the assessment framework, justifications are gauge according to the following scoring guide.

If there is a claim to a task or provide justification but are meaningless, it is interpreted as a display of deductive reasoning at the pre-structural stage and the work is given a score of zero, likewise no attempt to answer receives zero mark ;

If a correct claim is provided and an attempt to justify are apparent but the use of theorems, or geometry concepts necessary for the proof are illogical is assigned with the uni-structural level and is given a score of one;

If there is a correct claim and some theorems or geometry concepts present in the justification but some are irrelevant and other are appropriate but connections are missing is assigned with multi-structural level and is given a score of two. If there is a correct claim and the use of theorems or geometry concepts are meaningfully connected but is still lacking in enforcing a convincing and logical proof.

If there is a correct claim and the use of theorems or geometry concepts and other relevant mathematical principles, rules, or properties are meaningfully utilized in proving the claim is assigned with the relational level and is given a score of three. At this level, students are able to intra-connect previous learning.

If students provided a proof and utilized other learning's beyond the concept of geometry, four points were given.

The deductive reasoning test was given to the participants with no time limit to avoid time pressure. The participants were required to show all possible justifications or proof.

There were three assessors of the proof for each respective item. Zero point corresponded to unconnected, not related, and disorganized proof, or not engaged. This was given zero point. One point corresponded to a single of few correct concepts but connections between them are illogical. Two points corresponded to proof with some connections between existing ideas are apparent, but there are missing meta-connections. Three points was given to proof with element/ ideas in the situation concerning the general rule/ principles behind the situation are present, proof wherein the connections between and among ideas are established. When students provided a proof and utilized other learning beyond the concept of geometry, four points were given. After the scoring, the data was subjected to two-steps clustering. The sum of the scores received from each item was used in the clustering. Proof was analyzed using a SOLO-based rubric to describe their deductive reasoning level.

All items were made so that a task requires the pre-service mathematics teachers to utilize their mathematics learning but not limited to geometry knowledge to intra-connect them in their proof. This strategy allows the utilization of other learning in other field of mathematics providing them the opportunity to display deductive reasoning skill at the extended abstract level (Biggs & Collis, 1982).

RESULTS AND DISCUSSION

The Pre-service mathematics teachers' Level of Deductive Reasoning in Proof writing

The examination of the developmental trend followed the procedures namely: First, the difficulty levels of each item were examined base on correct proof (refer to table 1). Second, the students' groupings were identified through two-steps clustering. Lastly is the characterization of the trends of cognition level in deductive reasoning. Table 1 presented the task difficulty level based on Hingorjo et al. (2012).

Table 1. Task difficulty level

	Deductive Reasoning Task Items			
	1	**2**	**3**	**4**
Difficulty	0.23	0.17	0.37	0.50
Correlation	0.62	0.40	0.53	0.58
	5	6	7	8
Difficulty	0.38	0.37	0.56	0.03
Correlation	0.42	0.65	0.51	0.34

	9	10	11	12
Difficulty	0.15	0.30	0.30	0.33
Correlation	0.48	0.61	0.52	0.46

** Through two-steps clustering of the data sets grouped the students into class 1 (n=23), class 2 (n=66), and class 3 (n=14).*

Table 2 presents students in each class who provided correct proof. The data included in table 2 suggested a pattern in students' deductive reasoning proof writing because successful justification of a task by the students' in a class was also evident with the peers successes in each upper classes with the exception of successes in item 9 hence no class 3 students is successful for item nine. Some students in the next class provided strong justification to another task/s that students in the other class fail to deliver.

Table 2. Developmental pattern of students' deductive reasoning in proof writing

Class 1	**Class 2**	**Class 3**
n=23	**n=66**	**n=14**
	(3.03%,1)	(21.43%; 14.28%^{ext}, 1)
(4.35%, 2)	(1.52%,2)	(21.43%; 7.14%^{ext}, 2)
	(9.09%,3)	(42.86%,3)
(13.04%,4)	(33.33%; 1.52%^{ext},4)	(50%; 28.57%^{ext}, 4)
	(9.09%,5)	(28.57%, 5)
	(18.18%,6)	(57.14%; 21.43%^{ext}, 6)
(17.39%, 7)	(54.55%, 7)	(57.14%; 21.43%^{ext}, 7)
		(7.14%^{ext},8)
	(3.03%, 9)	
	(9.09%, 10)	(21.43%; 14.29^{ext},10)
		(21.43%, 11)
	(12.12%, 12)	(21.43%;7.14%^{ext} 12)

Note: The number with the percentage symbol means percentage of successful proof, the percentage with the superscript ext means proof at the extended abstract level, the number after the percentage/s is the item whom the successfully provided a proof.

The finding seems to suggest that there are levels that characterize students' cognition in deductive reasoning. To note, only a few members in class 1 completed task 2,4,6, and 7, respectively. Similarly, although no class 2 member was able to provide complete proof for item 2, but there were few who provided reasonable proof for items 4,6, and 7 and there were members who provided a proof for items 5, 7, 10 and 12, that no class 1 members has provided. Meanwhile, class 3 has responded with more successes on the items 4,6, and 7 compared to groups 1 and 2. Additionally, there were members from class 3 who succeeded tasks 1,3,5,10,12 which no students from groups 1 and 2 provided proof. It suggested that class 3 students were more better in providing proofs compared to classes 1 and 2.

Students in class 4 showed better deductive reasoning compare to the three groups for more than 60% of them were able to complete the proofs for items 4,6, and 7. Furthermore, there were those who provide proofs for items 1, 8, and 9 which are difficult items (Hingorjo et al., 2012). No student in the other groups was successful in these items.

It is very interesting that the items (4,6,&7) are common to the successes of the four classes. These items requires the possible utilization of the following concepts: for item 4, linear pair postulate, vertical angles theorem, substitution, addition property of equality and multiplication property of equality; for item 6, linear pair postulate, interior angles sum theorem, transitivity and addition property of equality; for item 7, interior angle sum theorem, linear pair postulate, and addition property of equality. The fact that each succeeding classes provided with more success suggest a developmental trend of cognition (Biggs et al. 1982). The students who performed at a higher level of cognition were able to utilize their knowledge in their written justification of their claims. Another interesting observation from the respondents answer is that at least 60% of the pre-service teachers in classes 3 and 4 were successful in utilizing the concepts needed for items 4, 6, and 7 but were not able to transfer these concepts in items 1, 2, and 9. Items 1, 2, and 9 require the same concepts in items 4, 6, or 7. Interestingly, items 4, 6, and 7 have figures in it, while items 1, 2, and 9 do not have. This suggests that students' cognition was facilitated by the use of figures and were visual. This is what Biggs et al. (1982) called operating at the concrete mode.

The data in table 2 presented a scenario of students reaching a higher level of cognition in deductive reasoning. Consequently, students, who reached at higher level cognition may have remembered well the principle and theories needed to justify their claim.

The fact that classes 1, 2, and 3 were unable to provide proof to difficult tasks such as in items 1, 8, and 9 suggested that the information in table 2 appear to produce a structure of cognition. The researcher claimed that the structure of cognition in deductive reasoning in proof writing correspond to the thinking levels identified by Biggs et al. (1982). The inclusion of a level 0 thinking for the fact that there are unsuccessful attempts to prove an item. The formulation of the description of the thinking levels was from their written justification.

Entry level: Students at this level do not know how to establish a proof. The doer of the proof may have background ideas but the construction of the evidence is questionable, or the provision of a single correct approach to a task is impossible. For example, students at entry level seems do not familiar with the vertical angles theorem. The knowledge of vertical angles theorem is vital for the proof in items 1, 4, and 10. For the justification of item 2, they also fail to claim that two angles that are supplements to congruent angles are also congruent. These suggested their inability to use already known principles in justifying a geometrical claim. Figure 1 illustrates this kind of thinking.

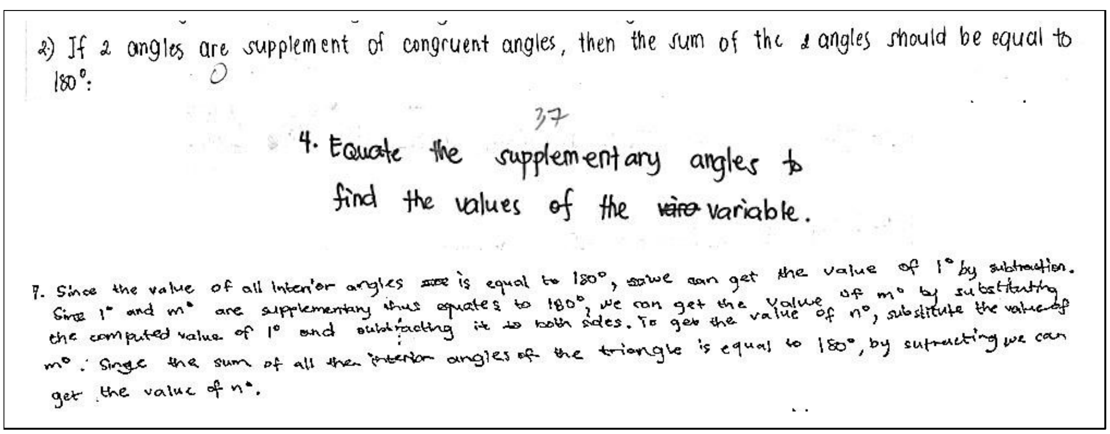
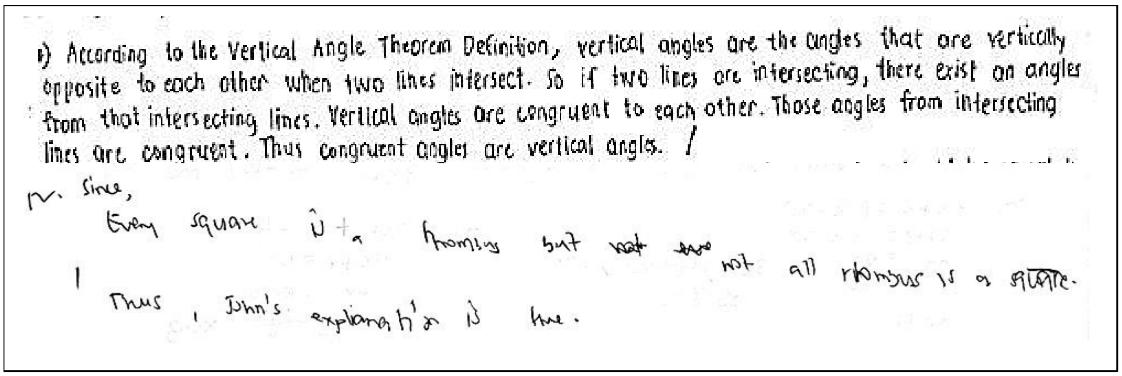


Figure 1. Collated students justification in items at entry level

In figure 1, a student claim that if two angles are supplement of congruent angles, then their sum of the angles should be equal to 180°. This claim is not true. In here, the pre-service teacher did not realize that the two angles are also congruent since they are supplementary to two congruent angles or to the same angle. Another student reasoning at the entry level was observed in justifying how to get the values of x and y in the diagram in item four (refer to the appendix) where in the explanation has no sense. The student here was not able to use the concept of linear pair postulated and the vertical angles theorem. The student here fails to connect the concept of the sum of the interior angles in a triangle, the sum of the measure of two linear pair of angles, and the relationship between the measure of the exterior angles and the sum of the measures of two remote interior angles in a triangle.

The entry-level cognition is characterizes by students no, or slight understanding of the task and geometric thinking are often unconnected. They have tried solving and engaging the tasks but failed to utilize relevant and the known concepts (Sekaryanti., 2022; Gagani et al., 2018).

First level: At this level, students provided a single or few valid ideas. They still fail to prove their claim. For instance, in item 1 as shown in figures 2, students at this level mentioned that vertical angles are congruent but did not substantiate this claim. Students at this level also manifest learned concepts on a task; however, they were not able to connect these with sense.


1) According to the Vertical Angle Theorem Definition, vertical angles are the angles that are vertically opposite to each other when two lines intersect. So if two lines are intersecting, there exist an angles from that intersecting lines. Vertical angles are congruent to each other. Those angles from intersecting lines are congruent. Thus congruent angles are vertical angles.

12. Since,
Every square is a rhombus but not ~~two~~ not all rhombus is a square.
Thus, John's explanation is true.

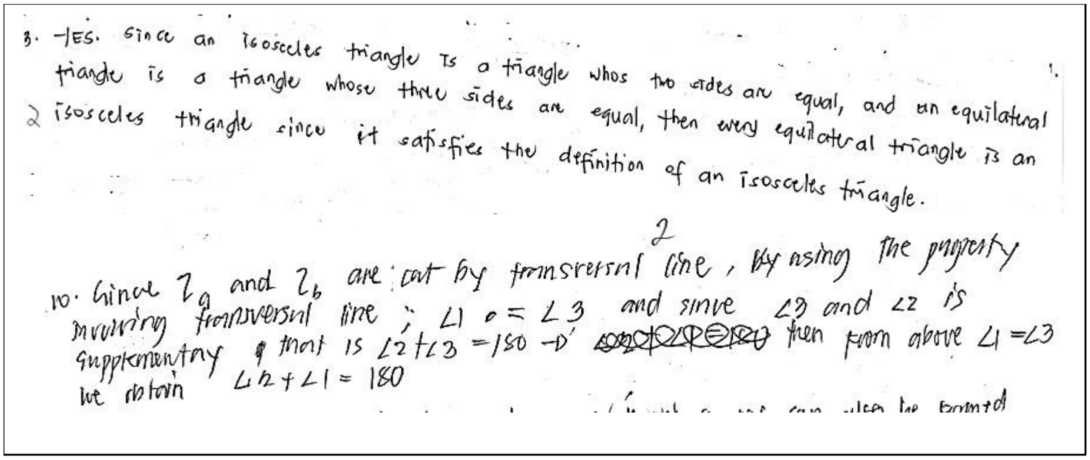
Figure 2. Collated students justification in the first level

The typical justification for item 12 at first level provides a claim that a square is a rhombus but not all rhombi are squares is valid but not explained.

The students at first level are in the state of cognitive development with an immature and disjointed effort in supporting claims. However, unlike the entry level, the thinking resembles the uni-structural level cognition (Biggs et al., (1982) or the monostructural level (Sekaryanti et al., 2022), because only one or few concepts that are needed for the completion of the task is/are provided. Additionally, the connections are not clear, and thought are disorganized, and frequently reasoning are incoherent and based on limited ideas (Sekaryanti et al., 2022; Gagani et al., 2018).

Second Level: In comparison to the entry and the first levels, students at the second level provided many true ideas. The characteristic of deduction is enriched wherein provision of ideas or concepts as requirements for a proof are reasonable; however, meta-connections among and between them are not clearly and logically connected. Properties of geometric figures are for comparison; however, elaboration of explanation into how they are related is not clear and still disorganized. For example, the students' reasoning in answering item three presented in figure three suggested knowledge of the properties of isosceles and equilateral triangles; however the proof projected a superficial understanding of the features. Hence, they provided some reasonable ideas but still have a misconception.

Figure 3 represents level two thinking. The students show some knowledge about isosceles and equilateral triangle in justifying item three. They used the ideas for comparison. Instead of stating that isosceles triangle has at least two congruent sides, the students claimed that an isosceles triangle has two equal sides. This claim now does


3. Yes. Since an isosceles triangle is a triangle whos two sides are equal, and an equilateral triangle is a triangle whose three sides are equal, then every equilateral triangle is an isosceles triangle since it satisfies the definition of an isosceles triangle.

10. Since l_a and l_b are cut by transversal line, by using the property involving transversal line; $\angle 1 = \angle 3$ and since $\angle 3$ and $\angle 2$ is supplementary & that is $\angle 2 + \angle 3 = 180 \rightarrow$ then from above $\angle 1 = \angle 3$ we obtain $\angle 2 + \angle 1 = 180$

Figure 3: Collated students justification in the second level

not support the claim that every equilateral triangle is an isosceles.

Students in this kind of thinking seemed to display the characteristics of the multi-structural level (Biggs et al.,1982; Sekaryanti et al.,2022; Gagani et al., 2018) for some correct ideas are present, but meta-connections between them are not clear. At this level, students gave few correct ideas but fail to use properties, proven theorem, postulates, and definitions to support their thinking. This level is comparable to the polystructural defined by Sekaryanti et al. (2022). Polystructural is a situation in which students know the correct procedure to solve a problem but is false short in utilizing and processing the provided information (Sekaryanti et al., 2022).

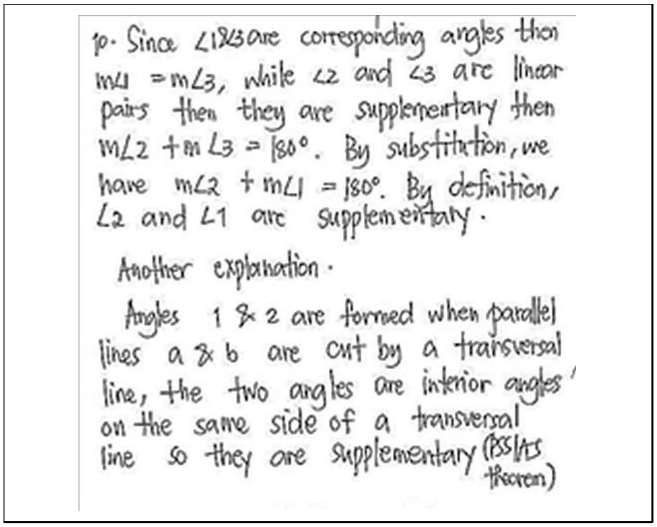
A proof for item 10 shown in figure 3 further manifested this thinking. The task presented a piece of information that a transversal cuts two parallel lines a and b. The participants' needs to justify that angles 1 and 2 are supplementary.

In the justification presented in figure 3, it was claimed that $\angle 1 = \angle 3$. Similarly it is claimed that $\angle 3$ and $\angle 2$ are supplementary without supports. Then, concluding that + =180. The claims was true but meta-connections and the use of postulates, properties, and another mathematical ideas related to establish a logical proof was not clear. Additionally, the proof was incomplete. An acceptable proof is presented below.

" since they are corresponding angles (the corresponding angles of parallel lines cut by a transversal are congruent). So, (congruent angles are equal). are linear pair. By Linear pair postulate, . By substitution, . Thus, are supplementary (definition of supplementary angles."

Third level: Students at this level of cognition displayed precise connections between ideas and were able to used and connected previously learned knowledge in their justification. Students' way of justifying a claim indicates this level. The fact that students at the third level provided a situation to the extent that they used and transferred knowledge in another field of mathematics such as in algebraic manipulation indicates that their thinking is consistent. At this level, the characteristic of cognition is somewhat a blend of relational and extended abstract levels of the SOLO taxonomy.

At this level, the pre-service teachers are capable of deducing the ideas that are contextually based on the situation; provide well-constructed connection among and between them; generate abstractions logically for the mathematical relationships and is skilled of connecting these to



p. Since ∠1&3 are corresponding angles then m∠1 = m∠3, while ∠2 and ∠3 are linear pairs then they are supplementary then m∠2 + m∠3 = 180°. By substitution, we have m∠2 + m∠1 = 180°. By definition, ∠2 and ∠1 are supplementary.

Another explanation.

Angles 1 & 2 are formed when parallel lines a & b are cut by a transversal line, the two angles are interior angles on the same side of a transversal line so they are supplementary (SSIATS theorem)

Figure 4: A Student justification in the third level

other mathematical entities not found in the given task into meaningful connections of ideas. Figure 4 exhibits the students' answer at this level in proving the claim of item 10. The case reflects better quality of learning. The use of previously learned concepts into the existing tasks has indicated better cognition processes (Ascraft, 1994; Sonnabend 1997, Gagani et al., 2018).

The present study supported other claims that knowledge has structural complexities and levels (Biggs et al., 1982; Putnam, et al., 1996; De Jong et al.,1996; Moseley et al.,2005; Baturo, et al., 2008; Nor et al., 2010; Huey, 2011; Gagani et al., 2017; Sekaryanti et al. 2022;). As found out, only a small percentage of classes 1 pre-service math teachers were able to provide proof to three average items (4.35%, 2; 13.04%, 4; 17.39%,7). On the other hand, similar to class 1 students, a small percentages of class 2 students were successful for average items (1,2,3,5,6, 9,& 12). However, a couple of them were correct with two items (34.85%,4; 54.55%,7). With this fact, it can be inferred that majority of class 1 and 2 students performed at a lower levels of cognition (Biggs et al.,1982; Biggs et al., 2011). This difficulty is also noted by Maarif, et al., (2019), Benkhali (2017), and Conner (2013). By this account, students are learning at the superficial levels (pre-structural to multi-structural, in this study entry to second levels).

Even three decades ago, this persistent problem was reported by Senk (1989) and was affirmed by Crompton et al. (2018). Crompton et al. (2018) noted that students are not learning at relational level of understanding concepts of geometry. It has been argued that students have difficulty performing at the advance level of thinking without experiencing the lower once (Wong et al., 2013). In this context, it means that a student who is learning at superficial level find it hard to use deductive reasoning skills in proof writing. It can be gleaned in table 2 that there is no easy item among the tasks. The items were categorized as average or very difficult (Aiken, 1979; Hingorjo et al., 2012) may be demanding for the classes 1 and 2 students than their cognitive facility would provide. For Sekaryanti et al. (2022), low math ability students cannot solve completely and will perform to what he termed monostructural, an equivalent term for uni-structural level. In this study, it pertains to the first level of deductive reasoning in proof writing. On the hand, students with moderate math skills can reach polystructural (multi-structural level) wherein they can have correct procedures but false short in using and processing the problem information. In this study, it pertains to the second level.

Meanwhile class 3 students responded with more items (35.71%, 1; 28.57%, 2; 42.86%, 3; 78.57%, 4; 28.57%, 5; 78.57%, 6; 78.57%,7; 7.14%, 8; 35.71%, 10; 21.42%,11; 28.57%, 12). More than 78.57% of them responded correctly on items 4, 6, and 7. It is worth to note that on the common three items that class members provided successful proofs, more than 22.71% of this class justified item 2 correctly than classes 1 and 2 combined. This class also responded 43.72% more than the class 2 students on item 4. Additionally, more than 24.03% of class 3 students were successful than class 2 students on item 7. Despite the fact that no student were able to provide a complete proof for item 9 which is among the very difficult items, some of the members of this group provided proofs to other very difficult items like item 1 (35.71%), item 2 (28.57%), and item 8 (7.14%).

These scenarios project students' cognition reaching at relational level (Biggs et al., 1982; De Jong et al., 1996; Moseley et al., 2005; Baturo, et al., 2008; Nor & Idris, 2010; Huey, 2011; Gagani et al., 2018; Sekaryanti et al. 2022). As purported that, working within the formal mode is usually evident in higher education (Stålne et al., 2015) and that they are expected to establish connections between the basic concepts of plane geometry (Sekaryanti et al. 2022). However, Gagani et al. (2017) found out that even honor students barely reached at the relational level in their inductive reasoning test. Nevertheless, this class is more capable than the other two classes. According to Sekaryanti et al. (2022) students with higher math skill know well be reaching at the relational stage but not the extended abstract level.

The findings of this research demonstrated the power of the SOLO taxonomy as assessment framework. This research purports the use of the taxonomy as it supported other findings in assessing the quality of knowledge. For Dudley et al., (2009), the SOLO taxonomy is able to distinguish surface and deep conceptual understandings in various disciplines, which is evident through the findings of this investigation. Proof writing activity should be done regularly in any mathematics-learning scenario as it a good avenue in checking the quality of knowledge. As Stålne, et al. (2015) puts it, students' learning can be reflected in solution to an answer to an open-ended questions and the SOLO is an appropriate framework since it describes performance quality rather than characterizing the students (Stålne et al., 2015). Additionally, it is an appropriate model for evaluating their written justification or argumentation (Imrie, 1995; Luo et al., 2020).

■ CONCLUSIONS

This current investigation endorses four levels of deductive reasoning in proof writing (entry to third levels) that are compatible with the SOLO Taxonomy levels of cognition. The levels of deductive reasoning in proof writing in this investigation imposes that the knowledge gained from school has structure. With the evidence that majority of the preserve teachers performance at the entry, first , or second levels while others were successful in difficult tasks, suggests a trend in their ability to tap their deductive reasoning skills and the quality of geometry concepts learned. Those who were unable to provide the proof has manifested surface level geometry knowledge. The regular proof writing activity is recommended as a regular activity as it provides teachers the opportunities of checking the quality of knowledge. The author endorse the use of the SOLO taxonomy as assessment framework in gauging the quality of students' knowledge as it provides precise means of checking students' quality of knowledge. This further implies the potential use of the taxonomy in the assessment of knowledge to other areas beyond geometry context.

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