25 (3), 2024, 1037-1050
Jurnal Pendidikan MIPA
e-ISSN: 2685-5488 | p-ISSN: 1411-2531
http://jurnal.fkip.unila.ac.id/index.php/jpmipa/

Differences in Mathematical Adaptive Reasoning Abilities and Productive Struggle Achievement Through Problem-Based Learning

Siti Masitoh, Tatang Herman*, & Aan Hasanah
Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

Abstract: The low adaptive reasoning abilities and productive struggle of junior high school students in learning mathematics drive this study. The research aims to determine whether there are differences in adaptive reasoning abilities and the achievement of productive struggle between students using problem-based learning and direct instruction. This research uses a quasi-experimental design with a pre-test-post-test control group design. The population consists of all seventh-grade students from a junior high school in Bandung. The sample includes class VII-B, with 31 students as the control group using the direct instruction model, and class VII-K, with 26 students as the experimental group using the problem-based learning model. The sampling method employe dis purposive sampling. The research instruments include an adaptive reasoning ability test and a productive struggle scale questionnaire. Data is analyzed both descriptively and inferentially. The results show that: (1) There is a significant difference in adaptive reasoning abilities between students using problem-based learning and those using direct learning. (2) There is no difference in the achievement of the productive struggle between students using the different learning models. This research concludes that problem-based learning can have a positive impact on adaptive reasoning abilities but does not directly influence the achievement of productive struggle compared to direct learning.

Keywords: adaptive reasoning, problem-based learning, productive struggle.

▪ INTRODUCTION

Mathematics as the queen of science and service, can develop independently and help advance various fields of knowledge. Mathematics in 21st-century learning can help students develop various skills to face the challenges and needs of the modern world. Mathematics in the learning process helps students build skills for solving a wide range of mathematical problems. This is supported by NCTM (2000), which highlights that mathematics education involves five key standard competencies that from the foundation for mathematical thinking: problem-solving skills, reasoning abilities, connection skills, communication skills, and representation abilities.

Reasoning abilities as one of the main standard competencies is important for students to possess. The reasoning ability that students have as part of the standard mathematics learning competencies helps them solve problems effectively (Agustin, Cahya, & Herman, 2023). Problems that are solved with reasoning ability are non-routine problems, so that mathematical reasoning becomes part of high-level thinking skills (Shodikin, 2017). As Reasoning ability in the 2020 mathematics curriculum competency standards includes the use of reasoning on patterns and properties, mathematical manipulation to make generalizations, proof construction, and explanation of mathematical ideas or statements. Winarni et al., (2024) States that mathematics learning emphasizes the importance of deeper mathematical reasoning, so that students do not merely follow the available steps but are instead guides to understand concepts

Tatang Herman
*Email: tatangherman@upi.edu

DOI: http://dx.doi.org/10.23960/jpmipa/v23i2.pp1037-1050
Received: 11 November 2024
Accepted: 23 November 2024
Published: 28 November 2024

comprehensively. This shows how important reasoning skills are in helping students solve various problems in mathematics learning.

Reasoning is the process of logical thinking used to draw conclusions or generate new statements from previously established ones (Astiati, 2020). Another opinion reveals that reasoning ability is the process of logical thinking to get conclusions that use both induction and deduction techniques (Sari & Darhim, 2020). Conclusions can be drawn deductively or inductively. Deductive reasoning is the process of drawing conclusions based on formal logical rules without the need for experimental validation, while inductive reasoning involves drawing conclusions based on observations of specific examples, such as facts, premises, or cases validated by experiments (Sosa-Moguel & Aparicio-Landa, 2021). The National Research Council introduced adaptive reasoning, which combines both deductive and inductive reasoning, in a publication from 2001.

Adaptive reasoning is one of the five mathematical proficiencies developed by Jeremy Kipatrick, Jane Swafford, and Bradford Findell in their 2001 book, "Adding it Up: Helping Children Learn Mathematics". Kilpatrick et al., (2001) defined adaptive reasoning as the ability to describe, explain, justify, and think logically about the relationships between concepts and situations. This skill involves using logical thinking to reflect, explain, and validate, including making conjectures and providing reasons or evidence for statements by seeking mathematical models or clues to draw conclusions (Darmayanti, Sugianto, Muhammad, & Santiago, 2022). Adaptive reasoning as part of reasoning ability is certainly an important aspect of mathematics learning.

The importance of adaptive reasoning skills does not align with the situation on the ground. Several studies indicate that students' adaptive reasoning abilities in mathematics learning are still below optimal levels. Study focusing on the reasoning abilities of junior high school students regarding the Pythagorean theorem showed even lower results, with only 20.63% (Ramdan & Roesdiana, 2022). Furthermore, the results of the study qualitatively found that the majority of students were unable to solve adaptive reasoning problems (Fitri, Prabawanto, & Mulyaning, 2024). Based on several research findings, it shows that students' adaptive reasoning abilities in mathematics learning are categorized as low.

The low adaptive reasoning ability requires support from certain factors to improve it. Adaptive reasoning ability, as a high-level skill, requires strong motivation for students to solve mathematical problems. One important factor is productive struggle, productive struggle is a skill of person who endures when experiencing difficulties and persists to get the desired goal (Stoltz, 1997). Productive struggle is essential to helping students solve math problems that require adaptive reasoning. As the results of the study show that the productive struggle to understand mathematics is an important component of learning mathematics with understanding (Hiebert & Grouws, 2007) However, the results of the study show that the productive struggle of students in solving problems is still low, so that in the end students give up and cannot solve the given problems (Amidon, Monroe, Rock, & Cook, 2020; Roble, 2017).

The low mathematical reasoning ability and productive struggle of students are influenced by both internal and external factors. Internal factors stem from within the students themselves, while external factors include elements such as teaching models, methods, and learning strategies (Wijaya, Fahinu, & Ruslan, 2017). Moreover, students' problems in adaptive reasoning are caused by their lack of practice in solving reasoning problems, leading to a lack of deep understanding, teacher-centered learning, mathematical concepts being delivered informatively, and teachers focusing too much on mechanistic and procedural aspects (Herman, 2007). The role of educators in the learning process is crucial for fostering and enhancing students' productive struggle in learning (Mayasari & Pagiling, 2020). This suggests that teachers play a key role in developing students' adaptive reasoning abilities and productive struggle. To address the issue, a solution is needed to help improve students' adaptive reasoning abilities and productive struggle.

One of the solutions that can be implemented is using problem-based learning. The problem-based learning model is a learning activity that focuses on providing real-world problems at the beginning of learning and resulting in a process of working towards understanding the problem (Chen, 2013). Through this model, students are trained to recognize various types of questions and become accustomed to solving problems by finding patterns and drawing accurate conclusions. Therefore, implementing problem-based learning is crucial for improving students' adaptive reasoning abilities and productive struggle.

Previous research on problem-based learning has shown good results in enhancing students' adaptive reasoning abilities and productive struggle. The research by Darwani, Zubainur, and Saminan (2020), suggests that implementing problem-based learning helps develop students' adaptive reasoning skills through non-routine problems so that students are used to solving problems. This finding is supported by Dorimana et al (2022), whose research indicates that problem-based learning can improve students' reasoning skills in mathematics education. Additionally, Salazar (2022) found a significant difference in student test scores that fosters productive struggle during the learning process. Consequently, this study focuses on the impact of problem-based learning on students' adaptive reasoning abilities and productive struggle in mathematics education.

Building on the problem background described above, the research questions are formulated as follows: (1) Is there a difference in the adaptive reasoning abilities of students who use problem-based learning compared to those who use direct instruction? (2) Is there a difference in the achievement of students' productive struggle between those utilizing problem-based learning and those engaged in direct learning? Accordingly, the aim of this research is to assess whether there are differences in adaptive reasoning abilities and in the achievement of productive struggle among students using problem-based learning and direct instruction.

▪ METHOD

Research Design and Procedures

This research is a quasi-experimental study, meaning that the researcher does not randomly assign subjects to groups but instead uses pre-existing groups (Fraenkel, Wallen, & Hyun, 2012). The design employed is a non-equivalent control group design with pre-tests and post-tests, where both the control and experimental groups are administered the same instruments for the pre-tests and post-test, but only the experimental group receives the treatment (Creswell & Creswell, 2018). In this study, two research groups were selected as samples: the experimental group utilizing a problem-based learning model and the control group employing a direct learning model. The research design on adaptive reasoning ability begins with the provision of a pre-test for each group, then the treatment is given only to the experimental group in the form of problem-based learning and ends with a post-test for each group. The research design for productive struggle is only given at the end of learning to see the achievement of student's productive struggle in each research group.

The research was conducted during five meetings. Before conducting the research, the researcher conducted a test of adaptive reasoning ability test instruments to check validity, reliability, difficulty level, and differentiation. The adaptive reasoning ability test instrument used is to meet the category of valid, relable, moderate and difficult difficulty, and have good and excellent differentiation. The test of the productive struggle scale instrument was carried out to determine the validity and reliability, the statement used was the one that met the valid and reliable category. In addition, for observation sheets and interviews, only logical validity is carried out by a team of experts (lecturers and teachers). After the instrument test was carried out, the research at the first meeting was carried out by providing a pre-test for each research group. Continued learning for 3 meetings and ended with a post-test meeting and the provision of a productive struggle force questionnaire. Then an interview was conducted according to the needs of the research.

Participants

The research population consists of all seventh-grade students at one of the schools in the city of Bandung. The sample selection in this study uses the purposive sampling technique, which means the samples are used as they are, in the sense that the researcher does not group the samples randomly (Babbie, 2007). So, two research groups were chosen: class VII-B as the control group and class VII-K as the experimental group.

Instruments

The research instruments used consist of tests and non-tests. The test instrument is a problem-solving ability test consisting of 4 essay questions that cover 5 indicators of adaptive reasoning ability. The non-test instrument consists of a productive struggle scale questionnaire with 30 statements, including both positive and negative statements based on 4 indicators of productive struggle. All instruments have been tested for validity by experts in their respective fields. In addition, the test instrument underwent empirical testing in the form of validity, reliability, difficulty level, and discrimination power tests. The productive struggle scale underwent validity and reliability tests. Pre-test, post-test, and productive struggle scale data were collected at the conclusion of the learning process as part of the data collection method used in this study.

The research variables are: (1) the independent variable, which is the problem-based learning model, and (2) the dependent variable, which are adaptive reasoning ability and productive struggle. The independent variable, students' adaptive reasoning ability, is assessed through specific indicators, including (1) making conjectures or hypotheses, (2) providing reasons for the given answers, (3) drawing conclusions from a statement, (4) examining the validity of an argument, and (5) finding patterns in a mathematical problem (Permana, Setiani, & Nurcahyono, 2020). As for the dependent variable of productive struggle, it is measured based on the indicators used, namely question, encourage, give time, and acknowledge (Warshauer, 2015). Here is an explanation regarding the indicator of productive resilience.

Table 1. Indicators adaptive reasoning

Indicators	Explanation
Making conjectures or hypotheses	The ability to formulate hypotheses refers to the student's skill in generating various possibilities based on their existing knowledge.
Providing reasons for the given answers	A character statement in this question places emphasis on how students articulate their reasons for the validity of a statement.
Drawing conclusions from a statement	The ability to draw conclusions from statements involves a cognitive process that utilizes knowledge to generate a coherent thought or understanding.
Examining the validity of an argument	The ability to examine an argument is a skill that involves students critically evaluating the validity of a given statement to determine its truth.
Finding patterns in a mathematical problem	The ability to recognize patterns from mathematical phenomena and make generalizations refers to a student's skill in identifying trends or methods within given statements and developing them into mathematical expressions.

Table 2. Indicators productive struggle

Indicators	Student Indicators of a Productive Struggle
Question	Students inquire to pinpoint the source of their difficulties, jot down their thoughts, discuss and clarify their ideas with others, and explore alternative strategies or presentations to overcome their challenges.
Encourage	Students are encouraged to put in effort to solve problems and understand their work, rather than just being satisfied with getting the correct answer or labelling themselves as smart or not.
Give Time	Students utilize their time to refine and implement their strategies, assess their progress, and recognize what they have accomplished and what tasks still need to be completed.
Acknowledge	Students remain dedicated to understanding and solving their problems, not easily giving up or becoming discouraged.

Data Analysis

The data analysis methods used in this study consist of both descriptive and inferential techniques. Descriptive analysis techniques are utilized to summarize the results of the pre-test and post-test regarding adaptive reasoning ability and the productive struggle scale. In contrast, Inferential data analysis is applied to statistically test the research hypothesis and address the formulated research questions. The hypotheses in this study are: (1) There is a difference in adaptive reasoning ability between students who experience problem-based learning and those who receive direct instruction; (2) There is a difference in the achievement of productive struggle between students who undergo problem-based learning and those who receive direct instruction.

After the data on adaptive reasoning abilitu and productive struggle are collected, then data processing is carried out with descriptive analysis and inferential analysis. Data processing with descriptive analysis is carried out to obtain data in the form of data amount, average, standard deviation, maximum value, and minimum value. Then an inferential analysis was carried out by conducting a hypothesis test, but preciously a normality and homogeneity condition test was carried out. Hypothesis tests are carried out using the t-test if the data is normal and homogeneous, if the data is not homogeneous, the t' test is used, and if the data is not normal, the Mann Whitney test is carried out. Data processing is carried out using the help of SPSS software and Statistical testing was conducted at a significance level of α = 0.05.

▪ RESULT AND DISSCUSSION

There are two findings and discussions from the hypothesis testing conducted by the researcher: the differences in adaptive reasoning abilities and the achievement of productive struggle between the experimental and control groups. Below are the results and explanations from the hypothesis testing in the study.

The Adaptive Reasoning Ability

The analysis of students' adaptive mathematical reasoning abilities was carried out using data collected from pre-tests and post-tests of both the experimental group and the control groups. Below are the pre-test and post-test results of the experimental group utilizing the problem-based learning model and the control group employing the direct learning model:

Table 3. Results pre-test and post-test adaptive reasoning ability

Group	Assessment	Results				
		N	Average	Standard Deviation	Maximum Score	Minimum Score
Experiment	Pre-test	26	28.43	19.07	64.29	0
	Post-test		68.68	27.21	96.43	10.71
Control	Pre-test	31	48.50	25.76	92.86	17.86
	Post-test		43.43	26.70	92.86	7.14

According to Table 3, the average pre-test score for adaptive reasoning ability in the experimental group was lower than that of the control group. However, the average post-test score for adaptive reasoning ability was higher in the experimental group compared to the control group. Based on this, it can be seen that there is a difference in adaptive reasoning ability between the experimental group and the control group. The following is a graph of the differences in the results of the pre-test and post-test adaptive reasoning abilities of the experimental group and the control group:

Subsequently, a normality assumption test was performed on data for adaptive reasoning ability. Based on the results of the shapiro-wilk normality rest, pre-test and post-test of the experimental group were obtained a significance value of 0.002 > 0.05, leading to the rejection of $H0$, which indicates that the data is not normally distributed. Similarly, the normality test results for the control group yield a significance value 0.041 < 0.000, also resulting in the rejection of $H0$, indicating that this data is not normal either. Following this, a homogeneity test was conducted to verify the homogeneity requirement.

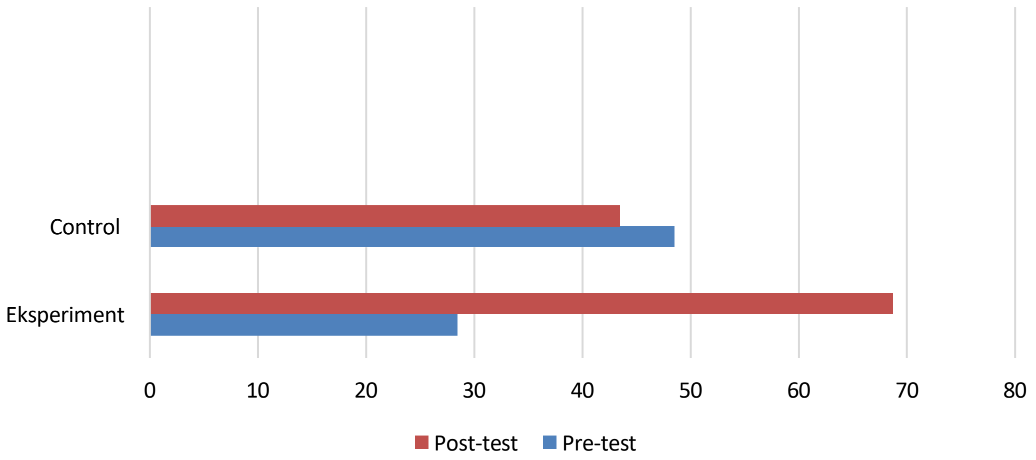

Figure 1. Pre-test and post-test adaptive reasoning ability

Based on the results of the levene statistical test for homogeneity obtained a significance value of 0.405 > 0.05, meaning $H0$ is accepted, which suggests that the variance of pre-test and post-test scores for adaptive reasoning ability between the groups is homogeneous. However, based on the analysis of normality and homogeneity assumptions, the data shows that the pre-test and post-test scores are neither normally distributed nor homogeneous. Therefore, a Mann-Whitney test was performed to assess the difference. Below are the results of the test comparing the average adaptive reasoning ability of students:

Table 4. Results of testing differences in adaptive reasoning ability

Test	Results Test
Mann-Whitney U	205.000
Wilcoxon W	701.000
Z	-3.180
Asymp. Sig. (2-tailed)	0.001

From the table above, the Asymp. Sig. (2-tailed) value is 0.001 < 0.05, leading to the rejection of $H0$. This indicates that there is a significant difference in the average adaptive reasoning ability between the problem-based learning group and the direct learning group. Based on the results of the data analysis, it was found that the hypothesis test results for adaptive reasoning ability showed a significant difference between the problem-based learning group and the direct learning group. This shows that the learning model influences students' adaptive reasoning abilities in mathematics learning. Adaptive reasoning ability is part of the student's thinking process. According Jatisunda & Nahdi (2020), problem-based learning positively influences the development of students' thinking processes. In this study, problem-based learning, used as the treatment for the experimental group, is a teaching model that introduces problems at the beginning of the learning process. The learning process follows the steps of problem-based learning, which involve introducing students to the problem, organizing their tasks, guiding them through individual or group investigations, assisting them in developing and presenting their findings, and analyzing and evaluating the problem-solving process.

The steps of problem-based learning start by introducing problems to the students. The problem is given to students in student's worksheets. Next, the students were organized into heterogeneous groups. The formation of groups consists of 3-4 students in each group. Grouping students is done so that they actively participate in the learning process, both in groups and by interacting with friends and teachers. In the problem investigation step, students in the experimental group solve problems found in student's worksheets. In each session, students are presented with problems related to the material the teacher intendes to conver. During this third step, the teacher takes on the role of a facilitator, guiding students through the investigation process, either individually or in groups. At this stage, students exchange ideas and interact with both their peers and the teacher to solve the assigned problems. Acting as a fasilitator, the teacher supports students in exploring solutions independently or collaboratively. The following step involves presenting the solutions for collective analysis and evaluation, aiming to achieve the intended learning objectives.

Based on the learning model utilized, there are notable differences in adaptive reasoning abilities between the experimental and control groups. The introduction of problems at the start of the lesson, coupled with group organization, fosters a more interactive learning environment and boosts student enthusiasm for problem-solving. As noted by Naziroh et al. (2022), problem-based learning necessitates active student participation in the learning process, which optimizes their reasoning abilities throughout various stages of the of the problem-based learning experience. Supporting this, research by Dorimana et al. (2022) indicated that students prefer problem-based learning because its structured phases aid in knowledge application, thus improving their reasoning, creativity, and critical thinking skills. So that the problem-based learning model used in the control group helps strengthen student's adaptive reasoning skills.

Efforts to improve students' adaptive reasoning skills in problem-based learning are incorporated into student's worksheets. These worksheets are structured around problem-solving steps aimed at developing students' adaptive reasoning. This approach contrasts with teacher-centered learning, where observations indicate that students are often passive participants. In traditional lessons, the teacher introduces the material using the lecture method, and students are not exposed to diverse problems that could stimulate their adaptive reasoning abilities.

The learning model applied to students in the control group is the one typically employed by teachers during mathematics instruction. In this context, the teacher in the control group utilized a direct learning. The learning process is based on learning steps, including explaining objectives and preparing students, demonstrating knowledge or skills, guiding practice, reviewing understanding and providing feedback, and offering opportunities for further practice and application (Arends, 2012; Joyce & Weil, 2003). Direct instruction refers to teaching patterns where the teacher explains concepts or skills more to the students (Hastari, Zuhroh, Purwanto, & Susiana, 2020). In the control group mathematics learning activities, the teacher provides an explanation of the material at the beginning of the lesson, so the students are merely listeners and do not engage or participate much. The learning process indicates that the teacher assumes a central role by delivering a significant amount of information or content at the beginning of the lesson.

The learning process of the control group carried out in this study is based on the results of observation, starting from the teacher explaining the material with the lecture method with students paying attention to the teacher's explanation. After delivering the material, the teacher provided several sample questions and exercises for students to to. The questions given by the teacher tend to be routine questions found in the book or the same questions as the sample questions given. That way students find it easy to solve, so that students are not given challenging questions and hone their adaptive reasoning skills. After completing the exercise, students are given feedback on what the teacher has done. The difference in adaptive reasoning ability in the experimental group and the control group is in line with the results of research conducted by Darwani, Zubainur, and Saminan (2020), that the use of problem-based learning models is able to develop and improve the adaptive reasoning skills of high school students by training students to solve non-routine problems. This opinion is supported by the results of the research Fery, Wahyudin, and Tatang (2017), mentioned that problem-based learning significantly improved the adaptive reasoning aspect compared to direct learning, where there was a higher average increase in students who used problem-based learning compare to the direct learning model. Based on this research, it supports the difference in adaptive reasoning ability un this study.

The Productive Struggle

The assessment of students' productive struggle in both the experimental and control groups was based on the results of the productive struggle questionnaire administered at the conclusion of the learning process. The following is presented the average results of students questionnaire responses regarding productive struggle in the control group and the experimental group based on the indicators:

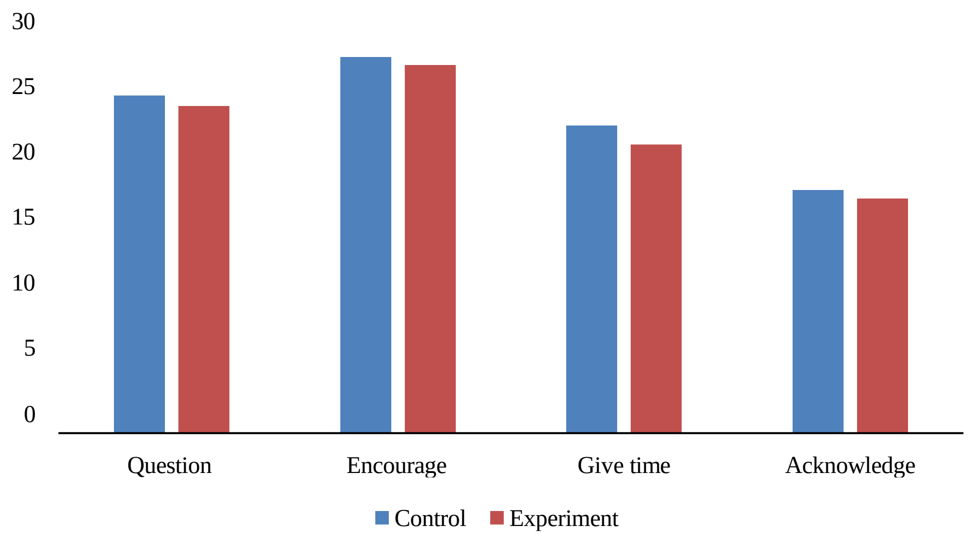

Figure 2. Results of the productive struggle questionnaire

Furthermore, hypothesis tests related to the achievement of productive struggle in the control group and experimental group were carried out. Prior to testing the hypothesis, a normality test was conducted on the productive struggle scale data. Based on the Shapiro-wilk statistical test, the results of the normality test of the productive struggle force questionnaire in the control group ware obtained a significance value of 0.381 > 0.05, meaning $H0$ is accepted, confirming normal distribution. Similarly the normality test for the productive struggle scale in the experimental group show a significance value of 0.563 > 0.05, leading to the acceptance of $H0$, indicating that the data is normally distributed.

Furthermore, a prerequisite test for students' productive struggle scale was carried out. The homogeneity test shows a significance value of 0.665 > 0.05, leading to the acceptance of $H0$, indicating that the variance of the productive struggle scale between the groups is homogeneous. Since both the normality and homogeneity assumptions are satisfied, the next step is to perform a hypothesis test using the t-test. Below are the results of the hypothesis test on the achievement of productive struggle:

Table 5. Results of the productive struggle achievement test

T-test for Equality of Means						
		t	**Df**	**Sig. (2-tailed)**	**Mean Difference**	**Mean Difference**
Productive Struggle Scale	Equal variances assumed	-1.450	55	0.153	-3.23284	2.22892

As shown in Table 11, the hypothesis test results for productive struggle reveal a significance value (2-tailed) of 0.153 > 0.05, which leads to the acceptance of $H0$. This indicates that there is no significant difference in the achievement of productive struggle between the experimental group and the control group.

The hypothesis test results indicate that there is no significant difference in the productive struggle achievement between students who use the problem-based learning model and those who engage in the direct learning model. According to the observations of the problem-based learning group, the teacher encourages the productive struggle attitude of students through both direct and indirect communication in student's worksheets. The student worksheets is equipped with motivational quotes, and the teacher conveys words and actions that help students develop a productive struggle attitude. The teacher observes the students' productive struggle during the learning process. In addition, learning is conducted in heterogeneous groups to foster interaction among students, thereby motivating each other. Grouping in problem-based learning fosters collaboration that can motivate students to engage in solving complex problems through investigation and joint dialogue (Marsitin & Sesanti, 2023). Furthermore, the research findings of Salazar (2022) indicate that learning that encourages students' perseverance has a positive impact on students' mathematical skills. So that in this case, the stages in problem-based learning support students' productive struggle.

Whereas in direct learning, based on students' opinions, students are motivated by peers who have higher abilities, which makes them eager to complete the tasks assigned by the teacher. Based on the results of observations made by researchers in direct learning, it shows that teachers before learning always provide motivation in learning as an indicator of encouragement to students. As for the direct learning step, there is a phase of independent training which is a means for teachers to give time for students to solve problems idependently. In addition, when in the process of delivering the material, the teacher provides simple questions that are related to daily life so that students are more enthusiastic and feel easy to solve the problems given. With encouragement from teacher, friends, and the environment created by teachers, teachers train students to have a productive struggle in mathematics learning.

From the explanation related to the efforts of each group given to students towards the attitude of productive struggle of students, there is no significant difference. So that this supports the resultas of statistical tests, in addition to that it can be seen in figure 2 showing that for each indicator in productive struggle there is no significant difference and looks relatively the same. The absence of differences in productive struggle as an affective aspect is certainly a concern, this is certainly related to the difficulty of changing the attitude of productive struggle which in only learned during three meetings. Of course, it is an evaluation material for future research to be careful in assessing the attitude of productive struggle in order to consider the time needed for students to show better productive struggle.

Based on the results and discussion of research on adaptive reasoning ability, it shows that the use of problem-based learning models has an impact on adaptibe reasoning ability. The use of problem-based learning model is needed in adaptive reasoning skills, especially in geometry and algebra materials, as in this study using Pythagorean theorem material which is part of geometric elements. The structured problem-based learning approach, which incorporates problems at each step, facilitates the development of students' adaptive reasoning abilities. As the use of the problem-based learning model is carried out effectively for each learning step.

The effectiveness of problem-based learning starts from giving problems at the beginning of learning, the interaction between students in groups, the process of collecting data and investigating that can hone students' adaptive reasoning skills because it requires the ability to choose the right strategy to connect the concept and the wrong problem situation so that the right solution to the problem is found. In addition, the last two steps are in the form of presenting results and analyzing and evaluating which allows students to reflect on what has been done in solving problems assisted by teachers as facilitators who provide justification for problem solving. All stages of the peoblem-based learning model are carried out step by step so that the adaptive reasoning ability in the experimental group is different from that of the control group.

As for productive struggle, it is recommended in the experimental group that uses a problem-based learning model in order to provide more challenging tasks so that students can show a more profound attitude of productive struggle in accordance with Warshauer's theory (2015). In addition, in this theory, teachers have an important role in helping students to develop a productive struggle. Therefore, it is important to pay attention to the right problems and can trigger students to have a productive struggle in the mathematics learning process.

▪ CONCLUSION

The conclusion of the research indicates a significant difference in adaptive reasoning abilities between students who participated in problem-based learning and those who received direct instruction. The difference is evident from the pre-test and post-test results of the experimental and control groups. However, the results of the productive struggle test show no significant difference in the level of productive struggle achieved by students in the problem-based learning group compared to those who underwent direct instruction. The achievement that shows no difference certainly makes the learning process provided to both groups support the students' productive struggle attitudes in different ways. The research that has been conducted does not specifically discuss the differences in adaptive reasoning abilities and how the productive struggle of students varies for each group. In addition, the researchers did not measure the students' productive struggle at the beginning of the learning process, so they did not investigate the extent of the increase in students' productive struggle before receiving the instruction. Therefore, the researchers suggest conducting further studies on more specific aspects regarding the causes or differences in greater depth, necessitating qualitative research for a more in-depth examination.

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