25 (2), 2024, 847-863

Jurnal Pendidikan MIPA

e-ISSN: 2685-5488 | p-ISSN: 1411-2531

http://jurnal.fkip.unila.ac.id/index.php/jpmipa/

How is the Learning Evaluation of the Pythagorean Theorem?: A Systematic Literature Review

Usman Aripin[1], Zulkardi[1*], Ratu Ilma Indra Putri[1], & Heris Hendriana[2]

[1]Departement of Doctoral Program in Mathematics Education, Universitas Sriwijaya, Indonesia

[2]Departement of Mathematics and Science Education, IKIP Siliwangi, Indonesia

Abstract: Mathematics learning problems, especially on the Pythagorean theorem material, cannot be studied only from one side, but need to be seen comprehensively from various integrative learning studies. This study aims to provide an overview of the evaluation of learning Pythagorean theorem material from reputable studies. This research uses the Systematic Literature Review (SRL) method. A review was conducted of 13 relevant articles published in various reputable international journals in the period 2019 to 2024. The data analysis used was carried out with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines. Based on the results of the review, it was found that in the Pythagorean theorem material there are still many misconceptions of understanding, with implications for the ability of abstraction that is not optimal. The quality of teaching is influenced by teachers' motivational beliefs in teaching, technology integration can help increase learning motivation, streamline learning and students are happier with teachers who use learning media, and teacher motivation in applying technology has an important role. In the realm of learning innovation, problem-based learning with a realistic mathematics approach is proven to be valid and effective and encourages active students and positive responses to the Pythagorean theorem material. Evaluation instruments must be valid, reliable, well designed to be effective in optimizing the learning process and diagnosing learning difficulties.

Keywords: pythagorean theorem, learning evaluation, mathematics learning.

▪ INTRODUCTION

Students' mathematical thinking skills are the focus of mathematics learning in schools (Hendriana et al., 2019). Mathematics Pythagorean thinking refers to the ability to understand and solve problems involving the Pythagorean theorem in various contexts. This skill involves understanding the basic concepts in the Pythagorean theorem and the ability to apply them in different situations. It requires imagination, creative thinking, and logical analysis to structure relationships and identify possible solutions (Wittmann, 2021).

Pythagorean thinking has significant applications in a variety of fields, including mathematics, physics, engineering, and development. In the field of mathematics, Pythagorean thinking helps students solve problems involving the calculation of side lengths of right triangles. In addition, these skills can also be used in a physics context to calculate distances or vectors in a coordinate system. Understanding and developing Pythagorean thinking skills can be done through practice and application in various problems and situations. Understanding the basic concepts of the Pythagorean theorem, students can improve their ability to solve math problems and apply this theorem in real-world contexts. This can help students become more confident and skillful in facing challenges involving right triangle relationships and the use of the Pythagorean theorem (Güner, 2018).

Zulkardi
*Email: zulkardi@unsri.ac.id

DOI: http://dx.doi.org/10.23960/jpmipa/v25i2.pp847-863
Received: 24 September 2024
Accepted: 03 November 2024
Published: 06 November 2024

In the process of learning the Pythagorean theorem, students often face a number of conditions that can affect their understanding of the concept. One of the common difficulties encountered is the difficulty in understanding the relationship between the side lengths of a right triangle, especially the concept of squaring the side lengths. Students may have difficulty identifying the hypotenuse and legs of a triangle, as well as difficulty understanding the reason why the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs of the triangle (Rudi et al., 2020).

The Pythagorean theorem is the only material that requires students to prove, but the fact that the proof of the Pythagorean theorem is considered to be the most difficult topic for students is not an easy task (Rudi et al., 2022; Zazkis & Zazkis, 2016). Although they understand the concept theoretically, applying it in the context of daily life or practical problems can be challenging. This could be due to a lack of mathematical modelling skills or difficulty in recognizing situations where the Pythagorean theorem can be applied (Yakar & Yilmaz, 2022). These problems need special attention for both teachers and researchers in designing teaching that can reduce students' difficulties in learning. Teaching by teachers is one of the important factors in a meaningful learning process, not only mechanically understanding but also understanding the concept and its application in various problems.

Teaching efforts that are less relevant to everyday life or a lack of contextual approaches in learning can also affect students' understanding of the Pythagorean theorem (Sembiring & Amaliah, 2017). This happens because not all students are able to understand abstract forms, with contextual problems expected to help students to bridge their knowledge from concrete to abstract so that learning is more meaningful. As Bruner's theory in mathematics learning there are three stages, namely (1) enactive where students are directly involved in manipulating concrete objects, (2) iconic where students visualise visually and verbally represent it, (3) symbolic where students are able to use symbols to present concepts (Dwijayanti et al., 2017). Therefore, it is necessary to pay special attention to designing learning that provides practical meaning for students so that they can relate the concepts of the Pythagorean theorem to real situations, gain a deeper understanding, and overcome the difficulties they may face (Güner, 2018).

To achieve deeper understanding, a more holistic and contextualized learning approach is recommended. Educators need to introduce mathematical concepts in a way that is relevant and connected to everyday life (Wittmann, 2021). In addition, interactivity in learning also plays an important role. Students can be invited to participate in activities that encourage them to discover, understand and apply the concepts of the Pythagorean theorem. With a more contextual and interactive approach, it is expected that students can discover mathematical concepts themselves, develop deeper understanding, develop skills in solving mathematical problems, and create a more meaningful and relevant learning experience for them (Adhitama et al., 2018).

The contextual learning approach can be optimized by integrating mathematical concept rediscovery activities through relevant problem contexts. Involving students in exploring and solving real problems can improve their understanding of mathematical concepts, including the concept of the Pythagorean theorem. By providing interesting and contextual challenges, students have the opportunity to identify mathematical patterns and relationships on their own (Putra, 2020). For example, they can apply the Pythagorean theorem in scenarios involving distance measurement in the context of everyday life. This

rediscovery process not only improves their understanding, but also motivates learning as students can see the direct relevance between mathematical concepts and the real world.

The context used in designing mathematical concept rediscovery activities should be relevant to everyday life so that it is familiar to students and easy for students to imagine (Güner, 2018; Wittmann, 2021). For example, when teaching the Pythagorean theorem, educators can design activities that involve measuring distances in the context of real situations, such as room mapping or room layout planning. In this way, mathematical concepts are no longer perceived as separate theories, but as valuable tools in solving everyday problems. Therefore, it is an important concern for teachers and researchers, especially in designing and implementing learning, to look for contexts that students can imagine so that they can overcome various student problems. In addition, various studies that have been conducted by researchers can be a reference in designing innovative learning so it is necessary to systematically collect studies on the Pythagorean theorem and comprehensively evaluate all learning activities.

There have been several previous studies on the topic of Pythagoras theorem in mathematics learning. In general, many qualitative, quantitative and developmental studies have been conducted, but there is no research that comprehensively evaluates the learning process starting from how students learn the Pythagorean theorem, various forms of learning intervention development, and student outcomes in learning Pythagorean theorem material. Therefore, it is necessary to conduct a systematic literature review in order to collect, evaluate, and synthesis research results comprehensively on the topic of Pythagoras theorem.

This research aims to review in depth the evaluation of learning the Pythagorean theorem material. This study aims to answer the following research questions: (1) how are Misconceptions and learning difficulties? (2) how is the quality of teaching (3) what are the Integration of technology? (4) how is Learning Innovation, (5) how is the development of Evaluation Instruments? By conducting this research, it is hoped that it will be an in-depth insight for teachers and researchers to design and develop effective learning strategies to improve student understanding day.

▪ METHOD

Research Design

The method used in this research is systematic literature review. A systematic review of scientific literature in a particular field is important to identify research questions based on the studies that have been conducted (Swartz, 2011; Torres-Carrión et al., 2017).. This method is relevant to the purpose of the research conducted, namely examining the learning situation on the material and learning innovations that are effectively applied to the Pythagorean theorem material based on relevant studies. This research used the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines (Moher et al., 2010).

Study Search Procedure

The article search process in this study was conducted to investigate scientifically published articles focused on the Scopus database using the Publish or Perish 8 application. To select articles, we applied the inclusion criteria: (1) the keyword

"Pythagorean Theorem" from 2019 to 2024 were gathered. The steps used were as follows: We (2) identified the topic and searched for relevant studies; (3) screened documents to identify important studies; (4) reviewed feasibility studies; and (5) loaded the analysis documents, synthesized, and described the results of the study.

Inclusion and Exclusion Criteria

The stages carried out in obtaining articles, in order to obtain new findings that are more comprehensive about learning Pythagoras theorem material. Technically, articles that are included in the inclusion criteria are (1) Scopus indexed articles, (2) Articles describing learning Pythagorean theorem material, (3) Articles published between 2020-2024 and (4) open access articles. As for the exclusion criteria, namely (1) articles not written in English, (2) close access and (3) not Original Research.

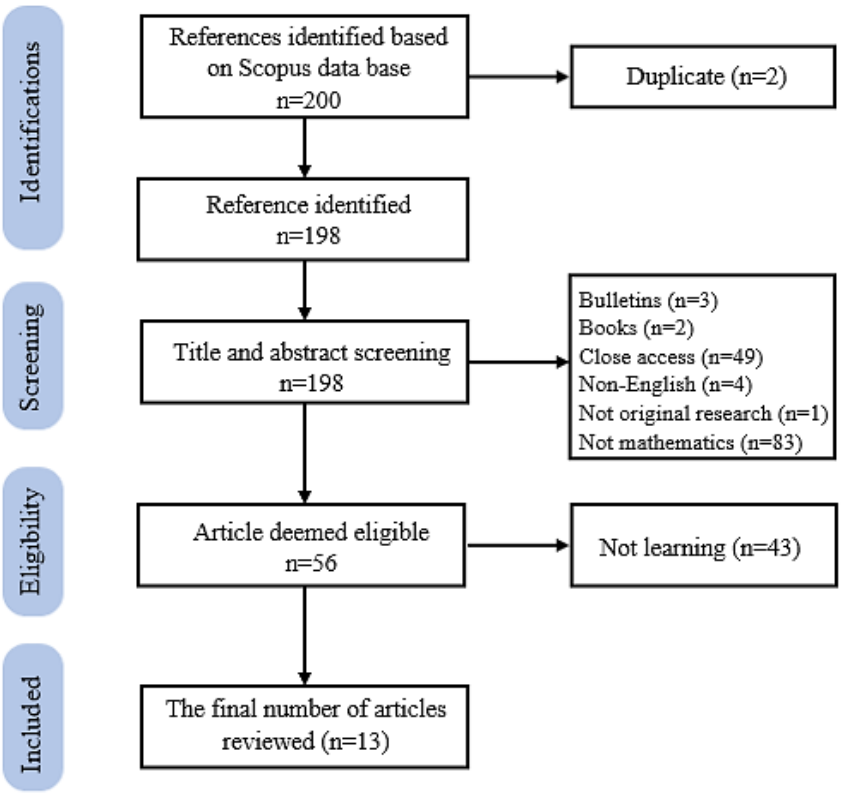

Figure 1. Stages of systematic literature review

Data Analysis

From the data extraction results, the researcher conducted an in-depth review of the learning situation and various teaching innovations related to the Pythagorean Theorem material. The learning situation includes various findings from the articles, ranging from students' difficulties, misconceptions, and their thinking processes in solving problems related to the Pythagorean Theorem. The review of various studies on effective teaching innovations regarding the Pythagorean Theorem material includes the development of learning media, learning trajectories, teaching models, and the development of relevant

assessment tools. The results of the analysis from various research findings became a comprehensive synthesis of new knowledge on the Pythagorean Theorem material. This synthesis serves as new knowledge and a strategic step toward developing more relevant research for teaching the Pythagorean Theorem material.

▪ RESULT AND DISSCUSSION

The research results presented focus on examining the evaluation results on learning the Pythagorean theorem material. In this section, it will be divided into two parts, each of which will be discussed based on the findings in the field which are associated with relevant theories or research results. The first part presents the learning situation on Pythagoras material and the second part presents effective learning innovations to be applied. In general, the final references are eleven articles that will be analyzed for their research results presented in the following table.

Table 1. Reference paper on learning evaluation of pythagorean theorem

No	Author	Year	Method	Learning Evaluation
1	Pauli et.al	2024	Quasi-Experimental	• Characteristics of teaching quality • Content of Pythagoras' theorem • Selection of learning context
2	Hartmann et.al	2023	Qualitative Study	• Problem posing that is based on real-world situations • Self-modelling problem posing can encourage modelling activities
3	Deshpande et.al	2021	Qualitative Descriptive	• Students with learning disabilities • Spend more time making metacognitive verbalizations • Analyse students' answers
4	Li et.al	2021	Research and Development	• Learning media development with Hawgent • Validitas, Validity, positive teacher and student responses
5	Casanova et.al	2021	Descriptive Research	• Van Hiele levels • Misconceptions in triangle • Geometry teaching methods need to be evaluated
6	Rewah et.al	2021	Research and Development	• Development of valid, practical and effective learning tools • PBL model using RME approach on Pythagorean Theorem material
7	Rahayu et.al	2021	Didactical Design Research	• Didactic and epistemological obstacles • Understanding of procedures using the Pythagorean theorem and algebraic concepts
8	Backfisch et.al	2020	Correlation Study	• Teachers' confidence and motivation in applying technology • Effective technology

9	Alghadari & Noor	2020	Case Study	• Application Pythagorean theorem for triangle property • Abstraction thinking problems
10	Rudi et., al	2020	Qualitative Descriptive	• Teacher knowledge in teaching • Knowledge of how to teach cognitive, epistemic, ecological, and interactional aspects. • Student responses to teacher teaching
11	Zulfah et.al	2019	Development Studies: (Design Research)	• Open-ended, contextual, and non-routine problem development • Problems developed have clear pictures and instructions
12	Puspitarani & Retnawati	2019	Qualitative Descriptive	• Difficulty in finding problem solutions • Factors causing learning difficulties
13	Towe & Julie	2019	Design Research	• The developed context helps to understand the concept of the Pythagorean theorem • Hypothetical learning trajectory (HLT) use RME

Thirteen (13) articles were selected based on Scopus-indexed journals and proceedings. The studies that describe the overall learning of Pythagorean theorem material are shown in Table 1, with a variety of research methods so as to see comprehensively in conducting a thorough evaluation. This evaluation process was analyses with a thematic analysis approach into five themes. The resulting themes are (1) Misconceptions and learning difficulties, (2) teaching quality, (3) technology integration, (4) learning innovation, (5) evaluation instruments. The themes collected to evaluate the learning of Pythagorean theorem material are explained as follows:

Theme 1. Misconceptions and Learning Difficulties

There are 5 articles describing how Pythagorean theorem material is learned by students. The results of the research in the article generally discuss how significant correlation of teaching quality on one learning topic, expert judgement and student perception, students achieve, diagnose student abilities, misconceptions, student errors in answering questions and descriptions of people with disabilities in learning mathematics, especially the Pythagorean theorem material. Furthermore, the various research findings were analysed to describe the situational learning of Pythagorean theorem material.

The findings of Casanova et al. (2021) show that there are student misconceptions about triangles, especially considering right triangles and obtuse triangles, where students do not use correct terminology, write incorrect notation, have difficulty identifying properties and are unable to connect previous concepts. This has an impact on students' initial understanding of the principles of the Pythagorean theorem and its application in solving problems. This is in line with the research results of Sari et al. (2019) the low initial knowledge of students has implications for improving problem solving skills. A deep initial understanding of mathematical concepts is the basis for building mathematical concept structures (Cai & Ding, 2017; Chi & VanLehn, 2012; Richland &

Begolli, 2016). Factors due to the influence of metacognitive knowledge, the student's focus is directed at his tendency to apply procedures to find the properties of the shapes needed to solve geometry problems (Casanova et al., 2021). Generally, problems in geometry problem solving based on students' conceptual knowledge have relevance to the given information, the concepts used, and the problems overcome.

Further analysis showed that students had difficulties in several aspects, including presenting concepts in various mathematical representations, understanding the concept of orthogonal projection, and constructing spatial buildings (Rahayu et al., 2021). Deshpande et al. (2021) research results show that, compared to general education students, students with learning disabilities make shorter and fewer verbalisations to achieve accurate problem solutions. Other findings also show that students' understanding of the Pythagorean theorem tends to arise from abstract thinking in the design of a particular problem, even though it exists in their conceptual system, which may hinder the achievement of optimal abstraction (Alghadari & Noor, 2020). From the results of these studies, it shows that there are still difficulties in learning Pythagorean theorem material, especially in presenting concepts, representing, verbalising, and difficulty reaching the abstract level.

Overall, Puspitarani and Retnawati (2020) mentioned that the factors that cause students of SMP Muhammadiyah 9 Todanan and SMP 1 Todanan to experience difficulties in solving math diagnostic test questions on Pythagorean Theorem material are as follows: Students cannot solve root form problems, students often forget the formula and how to solve the problems given, students are less careful in solving problems, students are too hasty in solving problems, students give up quickly when trying to solve problems, students feel the time given to solve problems is lacking, students' readiness in learning in learning is still lacking.

Puspitarani and Retnawati (2020) identifies the factors that cause students‘ difficulties in learning through diagnostic tests of Pythagorean theorem material, the results obtained are students unable to solve problems, students often forget the formulas and ways of solving the problems given, students are less careful in solving problems, students are too hasty in solving problems, students give up quickly when trying to solve problems, students feel the time given to solve the problem is lacking, students’ readiness to learn in learning is still lacking and difficulty checking back answers. These findings provide a clear picture of students' errors, misconceptions and difficulties in learning Pythagorean theorem material, so for teachers and researchers it is important to know the readiness of learning before teaching is carried out. Successful learning can be achieved if there has been an evaluation with critical thinking in learning mathematics (Pratama & Retnawati, 2018; Sachdeva & Eggen, 2021).

Theme 2. Teaching Quality

Rudi, Backfich, pauli. The findings in Figure 2 show a significant correlation at the classroom level between subject-based teaching quality as assessed by experts and students' perceptions and understanding of learning, as well as assessing the general understanding orientation of teachers (Pauli et al., 2024). This shows that the role of the teacher has an important role in presenting learning, especially on pythgaoras teroema material, based on expert assessment. Students perceptions of the material affect student learning outcomes, this shows the role of the teacher not only in improving learning

outcomes but influencing students' perceptions of learning which has implications for student learning outcomes. The quality of mathematics teaching is a factor considered in relation to student learning outcomes so that teaching practices with good teaching quality affect mathematics achievement and affective learning outcomes (Llinares, 2021; Yang & Kaiser, 2022).

The quality of teaching carried out by teachers is not only the ability to present learning materials, but the ability to identify student problems is no less important. Research by Rudi et al. (2020) shows that mathematics teachers lack awareness in identifying student errors and have limitations in providing advice for students who are experiencing difficulties. The importance of the ability to identify student problems will have implications for the learning design that is applied. The material designed will be right on target so that the learning process becomes optimal and reduces student learning difficulties. This analysis shows the importance of the teacher's role in improving student learning outcomes, especially knowledge in analyzing student difficulties which has implications for the ease of teachers in designing learning materials (Hill et al., 2005; Rudi et al., 2020).

Backfisch et al. (2020) in their research revealed that motivational beliefs are a further element of teacher professional competence, where this is an important aspect of classroom learning and teacher motivational beliefs should be more strongly considered in teacher education. This shows that the quality of teaching is influenced by teachers' motivational beliefs in teaching. Other findings show there is a significant positive correlation between teachers' motivational beliefs and learning engagement, making it an important factor in the learning process (Durksen et al., 2017; Zhang & Liu, 2019). This should be an important concern by making one of the instruments that determine the professional level of teachers in teaching so that this motivational belief is expected to be a reference for improving teacher competence.

Motivational beliefs are indirectly reflected in how the attitude of the teacher teaches in class, of course, students will indirectly feel how enthusiastic teachers are with teachers who lack motivation in teaching by looking at student responses. Teachers are the main actors who shape the learning environment and motivate students to learn (Eccles & Roeser, 2011; Hornstra et al., 2015; Lomos et al., 2021). The role of the teacher in this case has an important role and is one of the aspects considered in terms of classroom management. Thus, the quality of teaching becomes more optimal with the delivery, presentation and encouragement carried out by the teacher which will be one of the important factors in optimizing the learning process.

Theme 3. Technology Integration

Currently, technology has an important role in education, especially in learning mathematics in the classroom. However, applying it is not an easy thing, it needs facilities, skills, and last but not least the motivation of teachers in applying it. Backfisch et al. (2020) showed that teachers' perceived motivational beliefs towards educational technology play an important role in integrating technology into teaching. Other findings show a relationship between teachers' TPACK self-efficacy and technology use (Farjon et al., 2019; Knezek & Christensen, 2016). Based on previous findings, the use of technology is inseparable from teachers' motivational beliefs in applying it in mathematics learning.

The integration of technology in learning is very important, because it can help increase learning motivation, student understanding, streamline the learning process, streamline time, energy and costs and optimize the learning process to achieve learning goals. The important role of technology in optimizing and streamlining the learning process of mathematics that can be accessed anytime and anywhere and designed in an attractive and interactive way (Arnellis et al., 2023; O'Keefe et al., 2020; Singh et al., 2022). Juniati & Budayasa (2023) findings that technology in learning can help students realize mistakes, help find solutions more easily, and also make it easier to master the concepts learned and implement problem-solving plans. Technology is always needed by teachers and students, which means that teachers must continue to develop technological pedagogy knowledge and implement it when teaching in the classroom (Li et al., 2021)

Technology becomes an effective learning media in its use. Learning media is a tool that bridges the needs of teachers in achieving learning goals. Several researchers mentioned that the media helps teachers achieve learning objectives, motivates students in learning, learning will be more effective and efficient, and can bridge teachers during the learning process in the classroom, supporting students' higher-order thinking skills (Borzekowski et al., 2019; Maharani et al., 2024; Meryansumayeka et al., 2022). This shows the great need to integrate technology in mathematics learning.

The use of technology-based learning media is very relevant to the current demands, where technology has integrated various human life activities including in the field of education. Technology-integrated learning has many benefits both for teachers as teachers, students as students, of course, train digital skills and furthermore for the achievement of learning objectives. This is in line with Li et al. (2021) findings that students prefer teachers who use learning media in class, especially on Pythagorean theorem material. However, as an implication for teaching, it is necessary to earnestly design learning media.

In its implementation, the use of technology cannot always help the learning process. In line Jung et al. (2019) In classes where teachers use more technology in teaching mathematics, students actually show lower levels of mathematics achievement. Factors hindering the use of technology include lack of basic math concepts, poor study skills, internet connectivity, eye strain, technology not used for educational purposes (Changala & Mbozi, 2017; Letchumanan et al., 2019). In addition, there are conditions where students become preoccupied with using the media and do not focus on the learning material, the implication is that the learning process is not optimal and the use of technology becomes an obstacle. In addition, the creation of technology-based media can also hinder the thinking process because the technology used is not constructing knowledge but providing direct answers, so that students feel enough to get answers.

The Pythagorean theorem material actually includes a branch of geometry, so in developing the right learning media needs the activity of describing geometric shapes to be able to help understand the Pythagorean theorem. In addition, the development of science in the learning process of the Pythagorean theorem is often expressed in the form of algebraic equations. Some of the results of research on learning media on Pythagorean theorem material using Powtoon in the valid and good category to use (Lidya & Istikomah.Endang, 2021), besides using mobile learning which is flexible and provides convenience for learning and learning at any time (Nurwita et al., 2023). Some previous media development has not explicitly how students understand the concept of Pythagoras.

Thus it is necessary to develop media that can accommodate the needs of students to learn, not only geometry skills but algebraic skills are important in understanding the Pythagorean theorem.

Based on the previous description, it is important for teachers or researchers who will design, design technology-integrated media to comprehensively study the objectives, benefits, technical use, so that it can be used properly. The learning objectives of the media should help construct mathematical concepts, not help students find answers so that the benefits of its use will be felt by both teachers and students. In the technical use it is important to determine when the media is used, under what conditions the media will be felt useful by students in order to achieve learning objectives.

Theme 4. Learning Innovation

Research on learning innovation is very important, in developing a learning environment that continues to change according to the needs of the times. The purpose of learning innovation is to develop various alternative solutions in overcoming various learning problems. The results of learning innovations developed by Rewah et al. (2021), namely problem-based learning models with a realistic mathematics approach developed, obtained valid, practical and effective results applied in Pythagorean theorem material. The findings of this study are based on the results of expert validation in assessing the learning tools used, practicality obtained from the assessment results during the learning process and effectiveness based on the results of descriptive analysis of pretest and posttest with the achievement of learning completeness (Rewah et al., 2021). The problem-based learning model presents problems with a realistic approach that students can imagine. This makes learning more meaningful because students are educated to solve problems with teacher guidance and gain a comprehensive concept understanding of the Pythagorean theorem material. This is in line with Husna et al. (2024) the realistic approach allows students to link mathematics with everyday life and the surrounding reality, so that they accept mathematics as an activity that is integrated with personal experience. Zulkardi et al. (2020) mathematics learning should be inspiring, meaningful, taught at every level, and help students build practical knowledge to deal with quantitative situations in everyday life.

Another study innovated the development of the ADDIE model by involving 30 grade VII students, 56 teachers, and ten academics, the results of which showed that the learning materials produced could be used effectively. The findings also illustrate students' preferences towards the use of learning media in the classroom by teachers. The Hypothetical Learning Trajectory (HLT) was designed as an approach to assist students in understanding the concept of the Pythagorean Theorem through three stages, namely calculating the length of the painter's ladder, finding the shortest path from point C to point A, and calculating the area of the tablecloth (Towe & Julie, 2020). According to the cognitive-constructivist perspective, cognitive schemas are products of building knowledge and tools that can be used to build new knowledge (Arends, 2015; Van de Walle et al., 2019). Hartmann et al. (2023) findings explain that prospective teachers should solve problems they create themselves through problem posing, so that all modeling activities are known when learners solve the problem. The results of this study can be a form of innovation in learning Pythagorean theorem material that will be

developed with effective results and help students understand the concept of the Pythagorean theorem.

In developing a learning model on Pythagorean theorem material, it is important to integrate appropriate technology. The use of technology in learning is basically to help, streamline and streamline the learning process that takes place. For example, in the process of concept construction, the presentation of geometric shapes will attract students' attention and facilitate visualization, so that it can help students understand abstract things. This is in line with Li et al. (2021) findings that students prefer teachers who use learning media in class. As a result of Backfisch et al. (2020) research the role of the teacher, in this case confidence and motivation, is very important to use technology effectively in learning mathematics, especially the Pythagorean theorem. The importance of learning innovation by integrating technology has a significant impact on the effectiveness of the learning process. With this it is clear the need to develop various learning innovations, especially on the Pythagorean theorem material that can facilitate meaningful learning, can overcome various student problems, so that learning objectives are optimally achieved.

Theme 5. Evaluation Instrument

Evaluation instruments in learning are one of the important tools to be developed in order to measure the achievement of learning objectives. A good instrument must have clear validity and reliability so that it can be used properly. The results of research by Zulfah et al. (2019) on the development of open-ended, contextual and non-routine problem-based questions on Pythagorean theorem and SPLDV material obtained valid, practical and effective results. In developing questions, it is necessary to consider and determine the validity of content, construction and language aspects assessed by expert validators and presented with clear images and instructions.

Learning evaluation is an activity to measure teaching effectiveness (Dodeen, 2013; Mart, 2017). Evaluation instruments are developed to assess the characteristics of effective teaching, improving classroom teaching, lesson preparation and organization, interaction with students, clarity of teaching, effectiveness of communication, effective use of technology, teacher enthusiasm, frequency and quality of feedback, fairness of assessment and teacher availability outside the classroom (Alok, 2011; Henderson et al., 2014; Pan et al., 2021; Sadrina et al., 2018). Evaluation instruments are also a consideration for further teaching, so that each learning cycle can be evaluated to improve a more optimal learning process.

One of the important evaluation instruments used in learning is diagnostic instruments. Diagnostic instruments function to identify / diagnose students' strengths or weaknesses in learning. Diagnostic instruments can identify factors of student difficulties in solving problems on Pythagorean Theorem material not only seen from students' ability to solve mathematical problems, but also seen from the implementation of the mathematics learning process which is one of the sources of learning difficulties proposed by (Kennedy et al., 2008; Puspitarani & Retnawati, 2020). Thus, evaluation instruments using diagnostic tests are one of the important evaluation tools developed by teachers or researchers to identify, strengths, weaknesses, understand learning difficulties in order to solve problems in learning, especially Pythagorean theorem material.

Some of the findings in Casanova et al. (2021) article recommend conducting further research on the development of comprehensive test instruments that can assess a wider coverage of specific geometry concepts and Hartmann et al. (2023) to develop instruments to diagnose and analyze the mathematical modeling process and identify its difficulties. Thus the evaluation instrument becomes one of the important learning tools to be designed and developed to measure, evaluate and be a consideration for conducting learning so that it is expected to optimize the learning process.

▪ CONCLUSION

Based on the results and discussion, it is found that the Pythagorean theorem material still has a low understanding, there are many misconceptions of understanding, difficulty finding and re-examining solutions, and previous concepts that are not mastered which has implications for the ability of abstraction that is not optimal, and. The quality of teaching apart from professional competence is also influenced by the ability to identify problems, as well as teachers' motivational beliefs in teaching. Technology integration can help increase learning motivation, streamline learning and students are happier with teachers using learning media, and teachers' beliefs and motivation to apply technological media have an important role in optimizing the learning process. In the realm of learning innovation, it was found that the problem-based learning model with a realistic mathematics approach proved to be valid and effective and encouraged active students and positive responses to the Pythagorean theorem material. Evaluation instruments must be valid, reliable, well designed to be an effective tool for optimizing the learning process, especially materials such as the Pythagorean Theorem and it is important to develop diagnostic instruments because they can identify student strengths and weaknesses and learning difficulties.

The results of this review of research on Pythagorean theorem material provide complete information to conduct more comprehensive research in the future. In developing learning tools, it is necessary to review the findings in the field starting from students' difficulties in understanding the concepts, principles and procedures for using the Pythagorean theorem in various problems. In this case, it is important to determine relevant learning strategies, models and approaches, use effective learning media and develop evaluation tools that are valid, practical and effective to use. Then the importance of research to design learning tools based on the findings in the field is very supportive to design effective and appropriate learning in improving student understanding of Pythagoras material. Our research has limitations that must be recognized by readers, namely limitations in accessing reputable journals on the web of science data base and accessing other reputable journals that are close access. Seeing the lack of media and the magnitude of its influence in learning, the author suggests the need for the development of innovative learning tools, especially effective and appropriate learning media in Pythagorean theorem material.

▪ REFERENCES

Adhitama, I., Sujadi, I., & Pramudya, I. (2018). Discover the pythagorean theorem using interactive multimedia learning. Journal of Physics: Conference Series, 1008(1). https://doi.org/10.1088/1742-6596/1008/1/012066

Alghadari, F., & Noor, N. A. (2020). Students depend on the Pythagorean theorem: Analysis by the three parallel design of abstraction thinking problem. Journal of Physics: Conference Series, 1657(1). https://doi.org/10.1088/1742-6596/1657/1/012005

Alok, K. (2011). Student evaluation of teaching: an instrument and a development process. International Journal of Teaching and Learning in Higher Education, 23(2), 226–235.

Arends. (2015). Larning to teach. NY: McGraw-Hill Education.

Arnellis, A., Syarifuddin, H., & Ismail, R. N. (2023). Optimizing students' mathematical critical and creative thinking skills through the flip-a-team model with e-learning. Al-Jabar : Jurnal Pendidikan Matematika, 14(1), 133–140. https://doi.org/10.24042/ajpm.v14i1.16904

Backfisch, I., Lachner, A., Hische, C., Loose, F., & Scheiter, K. (2020). Professional knowledge or motivation? Investigating the role of teachers' expertise on the quality of technology-enhanced lesson plans. Learning and Instruction, 66(December 2019), 101300. https://doi.org/10.1016/j.learninstruc.2019.101300

Borzekowski, D. L. G., Lando, A. L., Olsen, S. H., & Giffen, L. (2019). The Impact of an educational media intervention to support children's early learning in Rwanda. International Journal of Early Childhood, 51(1), 109–126. https://doi.org/10.1007/s13158-019-00237-4

Cai, J., & Ding, M. (2017). On mathematical understanding: perspectives of experienced Chinese mathematics teachers. Journal of Mathematics Teacher Education, 20(1), 5–29. https://doi.org/10.1007/s10857-015-9325-8

Casanova, J. R., Cantoria, C. C. C., & Lapinid, M. R. C. (2021). Students' geometric thinking on triangles: much improvement is needed. Infinity Journal, 10(2), 217–234. https://doi.org/10.22460/infinity.v10i2.p217-234

Changala, M., & Mbozi, E. H. (2017). Factors contributing to institutionalisation of the aged in old people's homes in zambia: Perspectives of the aged. South African International Conference on Education, September, 60–69.

Chi, M. T. H., & VanLehn, K. A. (2012). Seeing Deep Structure From the Interactions of Surface Features. Educational Psychologist, 47(3), 177–188. https://doi.org/10.1080/00461520.2012.695709

Deshpande, D. S., Riccomini, P. J., Hughes, E. M., & Raulston, T. J. (2021). Problem solving with the pythagorean theorem: a think aloud analysis of secondary students with learning disabilities. Learning Disabilities, 19(1), 23–47.

Dodeen, H. (2013). College students ' evaluation of effective teaching : Developing an instrument and assessing its psychometric properties. Research in Higher Education Journal, 21, 1–12.

Durksen, T. L., Klassen, R. M., & Daniels, L. M. (2017). Motivation and collaboration: The keys to a developmental framework for teachers' professional learning. Teaching and Teacher Education, 67, 53–66. https://doi.org/10.1016/j.tate.2017.05.011

Dwijayanti, N., Suharta, I. G. P., & Sariyasa. (2017). Bruner's cognitive stages and their effects on the understanding of fraction concept. International Research Journal of Engineering, IT and Scientific Research, 3(4), 37–46. https://doi.org/10.21744/irjeis.v3i4.497

Eccles, J. S., & Roeser, R. W. (2011). Schools as developmental contexts during adolescence. Journal of Research on Adolescence, 21(1), 225–241. https://doi.org/10.1111/j.1532-7795.2010.00725.x

Farjon, D., Smits, A., & Voogt, J. (2019). Technology integration of pre-service teachers explained by attitudes and beliefs, competency, access, and experience. Computers and Education, 130, 81–93. https://doi.org/10.1016/j.compedu.2018.11.010

Güner, N. (2018). How to teach the pythagorean theorem: an analysis of lesson plans. Ankara Universitesi Egitim Bilimleri Fakultesi Dergisi, 51(1), 119–141. https://doi.org/10.30964/auebfd.405041

Hartmann, L. M., Krawitz, J., & Schukajlow, S. (2023). Posing and solving modelling problems—extending the modelling process from a problem posing perspective. Journal Fur Mathematik-Didaktik, 44(2), 533–561. https://doi.org/10.1007/s13138-023-00223-3

Henderson, C., Turpen, C., Dancy, M., & Chapman, T. (2014). Assessment of teaching effectiveness: Lack of alignment between instructors, institutions, and research recommendations. Physical Review Special Topics - Physics Education Research, 10(1), 1–20. https://doi.org/10.1103/PhysRevSTPER.10.010106

Hendriana, H., Putra, H. D., & Hidayat, W. (2019). How to design teaching materials to improve the ability of mathematical reflective thinking of senior high school students in Indonesia? Eurasia Journal of Mathematics, Science and Technology Education, 15(12). https://doi.org/10.29333/ejmste/112033

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. American Educational Research Journal, 42(2), 371–406. https://doi.org/10.3102/00028312042002371

Hornstra, L., Mansfield, C., van der Veen, I., Peetsma, T., & Volman, M. (2015). Motivational teacher strategies: the role of beliefs and contextual factors. Learning Environments Research, 18(3), 363–392. https://doi.org/10.1007/s10984-015-9189-y

Husna, A., Zulkardi, Z., Putri, R. I. I., Susanti, E., & Nusantara, D. S. (2024). Hypothetical learning trajectory of linear equation systems with three variables: The context of typical snacks riau islands. Jurnal Math Educator Nusantara: Wahana Publikasi Karya Tulis Ilmiah Di Bidang Pendidikan Matematika, 10(1), 12–22. https://doi.org/10.29407/jmen.v10i1.21550

Jung, E., Zhang, Y., & Chiang, J. (2019). Teachers' mathematics education and readiness beliefs, and kindergarteners' mathematics learning. International Journal of Education in Mathematics, Science and Technology, 7(2), 137–154. https://doi.org/10.18404/ijemst.552416

Juniati, D., & Budayasa, I. K. (2023). Geometry learning strategies with optimised technology to improve the performance of undergraduate mathematics students. World Transactions on Engineering and Technology Education, 21(1), 26–31.

Kennedy, L. M., Tipps, S., & Johnson, A. (2008). Guilding children's learning of mathematics. Thomson Wadsworth.

Knezek, G., & Christensen, R. (2016). Extending the will, skill, tool model of technology integration: adding pedagogy as a new model construct. Journal of Computing in Higher Education, 28(3), 307–325. https://doi.org/10.1007/s12528-016-9120-2

Letchumanan, M., Mohamad, M., & Davrajoo, E. (2019). Understanding the reasons that hinder mobile e-book use of university mathematics students. Malaysian Journal of Mathematical Sciences, 13(2), 265–281.

Li, X., Zhou, Y., & Chen, M. (2021). Validation and Implementation of Hawgent on Pythagoras Theorem. Journal of Physics: Conference Series, 2123(1). https://doi.org/10.1088/1742-6596/2123/1/012042

Lidya, A. N., & Istikomah.Endang. (2021). Powtoon-based mathematics learning media: Validation stage of the Pythagorean theorem. Mathematics Research and Education Journal, 5(1), 1–7. https://doi.org/10.25299/mrej.2021.vol5(1).9206

Llinares, S. (2021). Instructional quality of mathematics teaching and mathematics teacher education. Journal of Mathematics Teacher Education, 24(1), 1–3. https://doi.org/10.1007/s10857-021-09488-2

Lomos, C., Luyten, H., Boualam, R., Lomos, C., Luyten, H., Boualam, R., & Fischbach, A. (2021). Teachers as key actors in Implementing Information and Communications Technology (ICT) in Teaching and learning - findings from ICILS 2018.

Maharani, S. D., Putri, R. I. I., Syawaliyah, A. F., & Safitri, M. L. O. (2024). Interactive learning media: Literacy and numeracy Limas House Balaputera Dewa Museum in phase B. Research and Development in Education (RaDEn), 4(1), 617–629. https://doi.org/10.22219/raden.v4i1.32832

Mart, C. T. (2017). Student evaluations of teaching effectiveness in higher education. International Journal of Academic Research in Business and Social Sciences, 7(10). https://doi.org/10.6007/ijarbss/v7-i10/3358

Meryansumayeka, Zulkardi, Putri, R. I. I., & Hiltrimartin, C. (2022). Designing geometrical learning activities assisted with ICT media for supporting students' higher order thinking skills. Journal on Mathematics Education, 13(1), 135–148. https://doi.org/10.22342/jme.v13i1.pp135-148

Moher, D., Liberati, A., Tetzlaff, J., & Altman, D. G. (2010). Preferred reporting items for systematic reviews and meta-analyses: The PRISMA statement. International Journal of Surgery, 8(5), 336–341. https://doi.org/10.1016/j.ijsu.2010.02.007

Nurwita, F., Kusumah, Y. S., & Juandi, D. (2023). Developing learning media based on Android application for improving math problem solving skill of junior high school students on Pythagorean Theorem. AIP Conference Proceedings.

O'Keefe, L., Rafferty, J., Gunder, A., & Vignare, K. (2020). Delivering high-quality instruction online in response to COVID-19 faculty playbook with support from the (Online Learning Consortium (ed.)).

Pan, G., Shankararaman, V., Koh, K., & Gan, S. (2021). Students' evaluation of teaching in the project-based learning programme: An instrument and a development process. International Journal of Management Education, 19(2), 100501. https://doi.org/10.1016/j.ijme.2021.100501

Pauli, C., Lipowsky, F., & Reusser, K. (2024). Capturing the subject-specific quality of mathematics instruction: How do expert judgments relate to students' assessments of the quality of their own learning and understanding? ZDM - Mathematics Education, 0123456789. https://doi.org/10.1007/s11858-024-01561-3

Pratama, G. S., & Retnawati, H. (2018). Urgency of higher order thinking skills (HOTS) content analysis in mathematics textbook. Journal of Physics: Conference Series, 1097(1). https://doi.org/10.1088/1742-6596/1097/1/012147

Puspitarani, & Retnawati, H. (2020). Diagnosis of learning difficulties in mathematics for students resolving problems related to material in the pythagorean theorem for 8th grade students in SMP 1 Todanan and SMP Muhammadiyah 9 Todanan, academic year 2018/2019. Journal of Physics: Conference Series, 1581(1). https://doi.org/10.1088/1742-6596/1581/1/012026

Putra, I. S. (2020). Improve student's learning using media for understanding and interest in pythagorean theorem learning. Vygotsky, 2(2), 66. https://doi.org/ 10.30736/vj.v2i2.222

Rahayu, E. G. S., Juandi, D., & Jupri, A. (2021). Didactical design for distance concept in solid geometry to develop mathematical representation ability in vocational high school. Journal of Physics: Conference Series, 1882(1). https://doi.org/10.1088/1742-6596/1882/1/012077

Rewah, V., Sulangi, V., & Salajang, S. (2021). Development of learning devices with the PBL model using the Pythagoras theorem of RME approach. Journal of Physics: Conference Series, 1968(1). https://doi.org/10.1088/1742-6596/1968/1/012050

Richland, L. E., & Begolli, K. N. (2016). Analogy and higher order thinking: learning mathematics as an example. Policy Insights from the Behavioral and Brain Sciences, 3(2), 160–168. https://doi.org/10.1177/2372732216629795

Rudi, R., Suryadi, D., & Rosjanuardi, R. (2022). Didactical transposition within reflective practice of an Indonesian mathematics teacher community: A case in proving the Pythagorean theorem topic. Southeast Asian Mathematics Education Journal, 12(1), 65–80. https://doi.org/10.46517/seamej.v12i1.132

Rudi, Suryadi, D., & Rozjanuardi, R. (2020). Teacher knowledge to overcome student errors in pythagorean theorem proof: a study based on didactic mathematical knowledge framework. Proceedings of the 7th Mathematics, Science, and Computer Science Education International Seminar, MSCEIS 2019. https://doi.org/10.4108/eai.12-10-2019.2296511

Sachdeva, S., & Eggen, P.-O. (2021). Learners' critical thinking about learning mathematics. International Electronic Journal of Mathematics Education, 16(3), em0644. https://doi.org/10.29333/iejme/11003

Sadrina, S., Mustapha, R., & Ichsan, M. (2018). The evaluation of project-based learning in Malaysia: propose a new framework for polytechnics system. Jurnal Pendidikan Vokasi, 8(2), 143–150. https://doi.org/10.21831/jpv.v8i2.19100

Sari, N. M., Yaniawati, P., Darhim, & Kartasasmita, B. G. (2019). The effect of different ways in presenting teaching materials on students' mathematical problem solving abilities. International Journal of Instruction, 12(4), 495–512. https://doi.org/10.29333/iji.2019.12432a

Sembiring, M. B., & Amaliah, R. (2017). Finding students' missing concept about Pythagorean theorem n mathematics. JPMS, 12(2), 70–77.

Singh, J., Evans, E., Reed, A., Karch, L., Qualey, K., Singh, L., & Wiersma, H. (2022). Online, hybrid, and face-to-face learning through the eyes of faculty, students, administrators, and instructional designers: Lessons learned and directions for the post-vaccine and post-pandemic/covid-19 world. Journal of Educational

Technology Systems, 50(3), 301–326. https://doi.org/10.1177/00472395211063754

Swartz, M. K. (2011). The PRISMA statement: A guideline for systematic reviews and meta-analyses. Journal of Pediatric Health Care, 25(1), 1–2. https://doi.org/10.1016/j.pedhc.2010.09.006

Torres-Carrión, P. V., González-González, C. S., Aciar, S., & Rodríguez-Morales, G. (2017). Methodology for a systematic seview. 2018 IEEE Global Engineering Education Conference (EDUCON), 1364–1373. https://doi.org/10.1109/educon.2018.8363388

Towe, M. M., & Julie, H. (2020). Developing learning trajectories with the RME of phytagorean theorem. Journal of Physics: Conference Series, 1470(1). https://doi.org/10.1088/1742-6596/1470/1/012027

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). Elementary and middle school mathematics: Teaching developmentally (10th ed.). Pearson Education UK.

Wittmann, E. C. (2021). Connecting mathematics and mathematics mducation. Department of Mathematics Technical University of Dortmund, Germany. https://doi.org/10.1007/978-3-030-61570-3

Yakar, E. A., & Yilmaz, S. (2022). Mathematical thinking processes for the pythagorean theorem of the secondary school students 1. 3(2), 87–99.

Yang, X., & Kaiser, G. (2022). The impact of mathematics teachers' professional competence on instructional quality and students' mathematics learning outcomes. Current Opinion in Behavioral Sciences, 48, 1–8. https://doi.org/10.1016/j.cobeha.2022.101225

Zazkis, D., & Zazkis, R. (2016). Prospective teachers' conceptions of proof comprehension: Revisiting a proof of the Pythagorean theorem. International Journal of Science and Mathematics Education, 14(4), 777–803. https://doi.org/10.1007/s10763-014-9595-0

Zhang, S., & Liu, Q. (2019). Investigating the relationships among teachers' motivational beliefs, motivational regulation, and their learning engagement in online professional learning communities. Computers and Education, 134(October 2018), 145–155. https://doi.org/10.1016/j.compedu.2019.02.013

Zulfah, Astuti, Insani, S. U., Zulhendri, & Akbar, P. (2019). Development of open-ended based mathematics problem to measure high-level thinking ability. Journal of Physics: Conference Series, 1315(1), 0–8. https://doi.org/10.1088/1742-6596/1315/1/012047

Zulkardi, Putri, R. I. I., & Wijaya, A. (2020). Two decades of realistic mathematics education in Indonesia. International Reflections on the Netherlands Didactics of Mathematics: Visions on and Experiences with Realistic Mathematics Education, 325–340. https://doi.org/10.1007/978-3-030-20223-1_9