25 (1), 2024, 419-439
Jurnal Pendidikan MIPA
e-ISSN: 2685-5488 | p-ISSN: 1411-2531
http://jurnal.fkip.unila.ac.id/index.php/jpmipa/

Identifying Learning Obstacles in Pre-Service Mathematics Teachers' Understanding of Circle Concepts: A Preliminary Study within A Didactical Design Research Framework

Tri Sedya Febrianti, Siti Fatimah*, Jarnawi Afgani Dahlan
Departement Of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

Abstract: Identifying Learning Obstacles in Pre-Service Mathematics Teachers' Understanding of Circle Concepts: A Preliminary Study within A Didactical Design Research Framework. Objective: This research aims to identify the types of learning obstacles that pre-service mathematics teachers encounter when solving problems related to circles. **Methods:** A qualitative research approach with a case study design was employed to obtain in-depth information regarding the learning obstacles faced by pre-service mathematics teachers. This involved administering a written test on circle material and conducting in-depth interviews using an interview guide to explore the participants' responses. **Findings:** The research was conducted from May 21 to June 14, 2024, at a national university in Java, Indonesia. The participants included 33 pre-service mathematics teachers enrolled in the Flat Analytical Geometry course (2nd semester) who had been exposed to circle material. While almost all 33 participants were able to provide answers, they were not always completely correct. The analysis focused on 8 participants selected for data confirmability through in-depth interviews. The diversity of learning obstacles (LOs) found among these 8 participants was representative of the LOs identified in the test results of all participants. The results indicate that all pre-service mathematics teachers experienced learning obstacles in solving circle-related problems. The study identified all three types of learning obstacles ontogenic, epistemological, and didactical among the participants. In summary, pre-service mathematics teachers face ontogenic, epistemological, and didactical learning obstacles when solving circle problems. **Conclusion:** The analysis of learning obstacles, which is part of Didactical Design Research (DDR), was conducted in this study. The results of this research can serve as a reference in creating learning designs that minimize learning obstacles related to the circle material experienced by pre-service mathematics teachers.

Keywords: learning obstacles, pre-service, mathematics teachers, circle, DDR.

▪ INTRODUCTION

The quality of mathematics education is influenced by many factors such as curriculum, facilities, learning environment, and teachers (Nurhasanah, 2018). All these factors are considered as integral parts of learning process in a classroom (Nurhasanah, 2018). In the classroom teaching process, teacher becomes the key person who is responsible for the students' learning process (Turnuklu & Yesildere, Sabandar & Nurhasanah in Nurhasanah, 2018). It can be stated that in order to have a good quality of mathematics education we need qualified mathematics teachers (Nurhasanah, 2018). Professional development programs need to be addressed not only for in-service teachers but also for preservice teachers to obtain qualified mathematics teachers (Nurhasanah, 2018). According to the Board of Studies Teaching & Educational Standards (BOSTES), pre-service teachers are students who are taking part in a program at the Educational Personnel Education Institute (BOSTES, 2011). In Indonesia, professional development

Siti Fatimah
*Email: sitifatimah@upi.edu

DOI: http://dx.doi.org/10.23960/jpmipa/v25i1.pp419-439
Received: 21 July 2024
Accepted: 20 August 2024
Published: 24 August 2024

programs for mathematics teachers are mostly focused on in-service mathematics teachers.

Two aspects of the professionalism of mathematics teachers are mastery of mathematics content and pedagogical skills in delivering that content. One of the essential mathematical concepts that must be mastered is geometry. Geometry is a branch of mathematics, and mastering geometric concepts can be understood as conceptual maturity. Conceptual maturity in geometry can be demonstrated by the ability to solve geometry-related problems. Geometry is an important subject for students, especially those training to become teachers, as they will eventually teach geometry-related topics. One of the topics in geometry is circles. According to Van de Walle, the study of circles is a critical aspect of mathematics due to its practical applications in daily life, making it an essential subject for students to learn. In Indonesia, circles are material that is always present in courses related to geometry, such as flat geometry and flat analytical geometry. Circles are also standard content in the elementary, middle, and high school curriculum. Thus, pre-service mathematics teachers need to master circle material well.

However, unfortunately, the geometric abilities of pre-service mathematics teachers are still relatively low, geometry is an abstract concept for pre-service mathematics teachers (Zamzam, 2020). Geometry learning at college level is included in a complex learning scheme (Pasandaran & Mufidah, 2020). On the one hand, pre-service mathematics teachers must have knowledge, analysis and skills in the field of algebra and on the other hand, they must also equip themselves with visual spatial, analytical and processing rigor skills accurately and effectively. Based on the results of observations made, there are several errors related to geometry courses, in one of university in Java pre-service mathematics teachers still have difficulty solving problems, the way of thinking is still theoretical, and they have difficulty understanding the material provided (Zamzam, 2020). Meanwhile, the research results explain that the facts obtained in geometry lectures include several tendencies: (1) 85% of pre-service mathematics teachers only read the material without identifying initial concepts; (2) pre-service mathematics teachers tend to only read certain parts such as formulas, example questions, and certain practice questions; (3) pre-service mathematics teachers need a dynamic visual display of the process of forming a conical curve; (4) pre-service mathematics teachers need a semi-cooperative learning scheme. This means that there are parts of the material that must be mastered individually and there are parts of the material that must be discussed in groups; (5) pre-service mathematics teachers need a learning frame or concept that can accommodate learning needs (Pasandaran & Mufidah, 2020). From several opinions that have been expressed, the opinion regarding the fact that pre-service mathematics teachers are still experience difficulties in geometry courses (Rahmawati, 2021).

Problem solving is an important and inseparable part in mathematics, circle material is no exception, so problem solving needs special attention from the lecturer. In teaching, lecturers need to design learning in such a way that the learning process takes place well and with purpose learning can be achieved as expected. In the past few years, a considerable number of researchers have been studying the effect of mathematics education, with a particular focus on elementary or high school students (Xiang, 2019). These studies examined the effects of multiple goals, such as students' learning initiative, teachers' affective support, teachers and students' attitudes, learning strategies relate to mathematics learning (Chen & Chang, 2018). One type of study that addresses this is Didactical Design Research (DDR). Didactical Design Research (DDR) emerged within the French didactic tradition in the early 1980s (Supriyadi et al., 2023). DDR evolved from the theory of didactic situations, which explores how teaching systems and learning processes can facilitate the dissemination of knowledge (Supriyadi et al., 2023). Theoretical and applied didactical research has used DDR. In the past three decades, scholars have tailored DDR to fit their theoretical frameworks and specific needs (Artigue & Trouche, 2021). In 2010, Suryadi from Universitas Pendidikan Indonesia (UPI) introduced the Indonesian adaptation of DD (Supriyadi et al., 2023). Grasping educational innovation and the innovative instructional practices of researchers forms the foundation of DDR (Prediger et al., 2015). DDR aids teachers in mastering teaching techniques and enhancing their pedagogical abilities. It encompasses various topic areas, methodologies, and reference theories (Riegel & Rothgangel, 2022), while DDR can help students learn.

DDR employs two research paradigms, namely interpretive and critical. The study of learning obstacles falls under the interpretive paradigm, indicating that in DDR, a preliminary study is necessary, with its output being an interpretation of the learning outcomes. Meanwhile, the results from the interpretive study can later be applied to the critical paradigm, which involves research that provides solutions in the form of learning design. The term learning obstacle was first introduced by Bachelard in 1938 in Brousseau (Suryadi, 2019). Learning objects have similar characteristics to knowledge, as stated by Brousseau in (Suryadi, 2019). With these similarities, Brousseau believes that an obstacle is a piece of knowledge (part of knowledge), with the meaning of knowledge that knowledge is formed through a process, not immediately existing and obtained (Suryadi, 2019). The process of acquiring knowledge is the same as the learning process so that obstacles that arise from the process of acquiring knowledge can be interpreted as learning obstacles. Learning obstacles arise as a result of difficulties. Learning difficulties can be caused by two factors, namely internal factors and external factors. Learning difficulties caused by internal factors include, for example, a child's mental ability not being able to understand a concept. Then, learning difficulties are caused by external factors, for example learning environmental factors and the design of teaching materials made by teachers. Therefore, learning obstacles arise from difficulties caused by external factors for students.

In this research, focus on learning obstacles is the obstacles that arise when forming mathematical concepts. Learning obstacles consist of three types, namely: (1) ontogenic obstacles, (2) didactical obstacles, and (3) epistemological obstacles (Suryadi, 2019). Ontogenic learning obstacles are obstacles related to students' mental readiness and cognitive maturity in receiving knowledge. This is caused by a mismatch in the level of difficulty or thinking demands faced by students in didactic situations. Suryadi in his research found that ontogenic obstacles can be divided into three types, namely ontogenic obstacles which are psychological, instrumental and conceptual (Suryadi, 2019). Ontogenic psychological obstacles are students' unpreparedness regarding motivation and interest in the material being studied (Suryadi, 2019). Ontogenic instrumental obstacles are students' unpreparedness regarding tennis which is key to a learning process, which can be revealed, for example, through responses and errors in the student's solution process (Suryadi, 2019). Meanwhile, conceptual ontogenic obstacles are students' unpreparedness related to previous learning experiences, such as a lack of mastery of basic concepts and material supporting prerequisites. Didactic learning obstacles are obstacles caused by didactic systems such as sequence factors and/or curriculum stages including their presentation in classroom learning (Suryadi, 2019). Didactical obstacles must be considered based on the sequence of material, both structurally (connections between concepts) and functionally (continuity of thought processes); and the stages of presenting the material, whether it is not detailed enough or, on the contrary, too detailed (Suryadi, 2019). Epistemological learning obstacles are obstacles due to students' limited understanding and mastery of something (concept, problem, or other) that is only associated with a certain context. This type of learning obstacle is easy to find where students can work on questions according to the examples and forms provided by the teacher or textbook, but have difficulty solving questions in other forms or contexts (Suryadi, 2019). Thus, efforts to understand and explore learning obstacles must take into account all points of view and their interrelationships.

There has been a lot of research on students' learning obstacles at middle and high school levels, as carried out by (Fauzi et al., 2023); (Hariyani et al., 2022); (Hendriyanto et al., 2024); (Puspita et al., 2023); (Sari et al., 2024); (Utami, 2023); (Wang et al., 2021). On the other hand, there is only one study that addresses learning barriers for pre-service mathematics teachers, specifically focusing on obstacles related to high-order thinking skills in solving complex problems. This research found that pre-service mathematics teachers face difficulties in connecting geometry material, including circle-related concepts (Hadi et al., 2020). There is currently a lack of detailed research on learning obstacles faced by pre-service mathematics teachers when solving circle problems. Therefore, it is crucial to conduct research on the learning obstacles of pre-service mathematics teachers in geometry, particularly regarding circle material.

Such research is important for advancing further studies, specifically in developing learning designs through Didactical Design Research (DDR). DDR aims to create effective learning designs that enhance pre-service teachers' abilities to solve circle problems. In other words, this research serves as preliminary work for DDR, with the goal of significantly improving the implementation of circle material in teaching. To address the research objectives, this study will focus on answering the research question: "What are the learning obstacles of pre-service mathematics teachers in solving circle problems?"

▪ METHOD

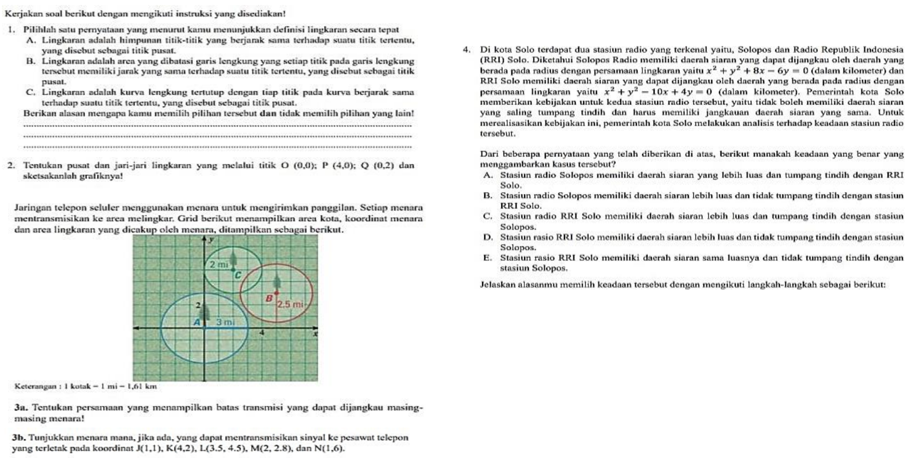
Research Design

This study used a qualitative approach. Qualitative research has a relationship with the ideas or views of the subject under study (Fauzi & Suryadi, 2020; Sugiyono, 2017). One features of qualitative research is representing the views and perspectives of the people (labeled as the participants) in a study (Yin, 2011). There are variations in qualitative research, one of them is case study. In this research, the method employed was a case study. Case studies are research methods to explain certain phenomena, such as individuals, programs, processes, and others (Salimi et al., 2021). Therefore, this case study was deemed an appropriate method to explore in detail students' learning obstacles in solving circle problems.

Data Collecting

A total of 33 pre-service mathematics teachers from a national university in Central Java participated in the study. The researchers selected this group because they had already been taught the circle material. The study employed both test and non-test instruments. The test instrument consisted of questions related to conceptual understanding and problem-solving of circle material, while the non-test instrument involved in-depth interviews to enhance the data obtained.

The researchers investigated pre-service mathematics teachers' learning obstacles by administering tests to the 33 pre-service mathematics teachers who had studied circle material. The test comprised 5 questions designed to assess pre-service mathematics teachers' understanding of circle definitions, the equation of a circle, and their ability to apply circle concepts to real-world problems. The indicator is displayed as follows (1) pre-service mathematics teachers are able to accurately explain their understanding of the definition of a circle, (2) pre-service mathematics teachers are able to understand the characteristics of the general form of the circle equation, (3) pre-service mathematics teachers are able to use their understanding of the elements of a circle to solve problems. The questions are illustrated in Figure 1 below.



Kerjakan soal berikut dengan mengikuti instruksi yang disediakan!

1. Pilihlah satu pernyataan yang menurut kamu menunjukkan definisi lingkaran secara tepat
   A. Lingkaran adalah himpunan titik-titik yang berjarak sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.
   B. Lingkaran adalah area yang dibatasi garis lengkung yang setiap titik pada garis lengkung tersebut memiliki jarak yang sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.
   C. Lingkaran adalah kurva lengkung tertutup dengan tiap titik pada kurva berjarak sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.

   Berikan alasan mengapa kamu memilih pilihan tersebut dan tidak memilih pilihan yang lain!

   ....

   ....

   ....

2. Tentukan pusat dan jari-jari lingkaran yang melalui titik O (0,0); P (4,0); Q (0,2) dan sketsakanlah grafiknya!

Jaringan telepon seluler menggunakan menara untuk mengirimkan panggilan. Setiap menara mentransmisikan ke area melingkar. Grid berikut menampilkan area kota, koordinat menara dan area lingkaran yang dicakup oleh menara, ditampilkan sebagai berikut.

Keterangan : 1 kotak = 1 mi = 1,61 km

3a. Tentukan persamaan yang menampilkan batas transmisi yang dapat dijangkau masing-masing menara!

3b. Tunjukkan menara mana, jika ada, yang dapat mentransmisikan sinyal ke pesawat telepon yang terletak pada koordinat J(1,1), K(4,2), L(3.5, 4.5), M(2, 2.8), dan N(1,6).

4. Di kota Solo terdapat dua stasiun radio yang terkenal yaitu, Solopos dan Radio Republik Indonesia (RRI) Solo. Diketahui Solopos Radio memiliki daerah siaran yang dapat dijangkau oleh daerah yang berada pada radius dengan persamaan lingkaran yaitu $x^2 + y^2 + 8x - 6y = 0$ (dalam kilometer) dan RRI Solo memiliki daerah siaran yang dapat dijangkau oleh daerah yang berada pada radius dengan persamaan lingkaran yaitu $x^2 + y^2 - 10x + 4y = 0$ (dalam kilometer). Pemerintah kota Solo memberikan kebijakan untuk kedua stasiun radio tersebut, yaitu tidak boleh memiliki daerah siaran yang saling tumpang tindih dan harus memiliki jangkauan daerah siaran yang sama. Untuk merealisasikan kebijakan ini, pemerintah kota Solo melakukan analisis terhadap keadaan stasiun radio tersebut.

   Dari beberapa pernyataan yang telah diberikan di atas, berikut manakah keadaan yang benar yang menggambarkan kasus tersebut?
   A. Stasiun radio Solopos memiliki daerah siaran yang lebih luas dan tumpang tindih dengan RRI Solo.
   B. Stasiun radio Solopos memiliki daerah siaran lebih luas dan tidak tumpang tindih dengan stasiun RRI Solo.
   C. Stasiun radio RRI Solo memiliki daerah siaran lebih luas dan tumpang tindih dengan stasiun Solopos.
   D. Stasiun rasio RRI Solo memiliki daerah siaran lebih luas dan tidak tumpang tindih dengan stasiun Solopos.
   E. Stasiun rasio RRI Solo memiliki daerah siaran sama luasnya dan tidak tumpang tindih dengan stasiun Solopos.

   Jelaskan alasanmu memilih keadaan tersebut dengan mengikuti langkah-langkah sebagai berikut:

Figure 1. The test instruments of circle materials

This study not only focuses on the students' test answers but also deeply confirms the phenomena behind the students' test responses through the non-written test (interviews). The interview guidelines are illustrated in Figure 2 below.

Both research instruments, which consist of a circle material proficiency test and a non-test (in the form of an interview guide), were independently developed by the researcher and have been validated by one lecturer from a state university in West Java and one lecturer from a state university in Central Java. After validation, the instruments were piloted to test their readability. This pilot test was conducted with pre-service mathematics teachers enrolled in a plane geometry course at a state university in West Java. The results of the pilot test indicated that some questions needed editorial revisions, resulting in a research instrument that was suitable for use with the research subjects, namely pre-service mathematics teachers taking an analytic plane geometry course at a state university in Central Java.

- Peneliti bertanya mengenai butir soal *learning obstacle*. Adapaun pertanyaan umum yang dapat diajukan sebagai berikut:
  1. Bagaimana pendapatmu mengenai soal tes ini? Apakah kamu dapat menjawab seluruh soal yang diberikan?
  2. Nomor berapa saja yang dapat kamu kerjakan dan tidak dapat kamu kerjakan?
  3. Informasi apa yang dapat kamu peroleh dari soal tersebut?
- Apabila partisipan tidak memahami soal yang diberikan atau salah dalam menjawab soal tersebut, maka pertanyaan umum yang dapat diajukan adalah sebagai berikut:
  1. Kesulitan apa yang kamu alami ketika mengerjakan soal tersebut?
  2. Bagaimana pemahaman kamu mengenai materi pada soal tersebut (soal yang tidak dipahami/ adanya kesalahan dalam menjawab)?
- Untuk memverifikasi alur pembelajaran yang terjadi di kelas dan mengetahui alur berpikir mahasiswa, diajukan pertanyaan umum sebagai berikut:
  1. Coba ceritakan bagaimana proses pembelajaran materi lingkaran yang dilakukan oleh dosen?
  2. Kesulitan apa yang kamu alami dalam mempelajari materi lingkaran?

Figure 2. The in-depth interviews guidelines

The written test was administered to the pre-service mathematics teachers on May 21, 2024, while the non-written test (in-depth interviews) was conducted from May 27 to June 14, 2024. After the written test was conducted, pre-service mathematics teachers' responses were analyzed based on the types of learning obstacles (ontogenical, epistemological, and didactical) identified. After analyzing the pre-service mathematics teachers' test results, the researcher examined and grouped the tendencies of the obstacles identified among the 33 pre-service mathematics teachers. To further investigate these obstacles, the researcher selected a representative sample of pre-service mathematics teachers to participate in non-test methods, specifically in-depth interviews. In this research, pre-service mathematics teachers' learning obstacles involved both a question-taking test and in-depth interviews, which are complementary rather than separate. This approach aligns with the case study objective of focusing on individual incidents to gain a comprehensive understanding. In-depth interviews were essential for obtaining detailed information about the learning obstacles experienced by pre-service mathematics teachers.

Data Analysis

Data analysis in this study used the Miles and Huberman model, carried out in three stages: (1) data reduction, where the researchers recorded all pre-service mathematics teachers' responses in answering questions related to circle material; (2) data presentation, the researchers began to classify the types of pre-service mathematics teachers responses and identify them based on their obstacles; (3) drawing conclusions, where the researchers analyzed in detail the types of pre-service mathematics teachers' learning obstacles in circle material based on the theory of learning obstacles and then draws conclusions (Miles, M. B., & Huberman, 1994).

The research starts with data collection. The data collected includes written tests on circle material and in-depth interviews regarding the learning obstacles identified from the written tests. Next, data reduction was carried out. Out of the total 33 pre-service mathematics teachers who completed the written test on circle material, the data was reduced to 8 pre-service mathematics teachers who were selected for in-depth interviews. This reduction was based on the patterns of learning obstacles identified from the written test results. Subsequently, the written test data and the interviews of these 8 pre-service mathematics teachers proceeded to the data presentation stage. In the data presentation stage, the findings and discussion of the results from the written tests and interviews with the 8 pre-service mathematics teachers are presented. The findings section solely presents the data on the learning obstacles identified. The discussion then links these findings with the written test responses, interview results, and relevant theories related to the types of learning obstacles discovered. In the conclusion stage, key steps are undertaken to summarize and reflect on the findings. This is followed by assessing the validity of the findings to ensure they are credible and supported by the data.

In this research, data validity is ensured through credibility, transferability, dependability, and confirmability. Credibility involves verifying the accuracy and authenticity of data through strategies like member checking and triangulation. This qualitative data presented is validated through method triangulation. Triangulation means being able to look at the same phenomenon, or research topic, through more than one source of data (Abdalla et al., 2018). Information coming from different angles may be used to confirm, develop, or illuminate the research problem (Abdalla et al., 2018). Triangulation can combine methods and collection sources of qualitative and quantitative data (written tests, interviews, questionnaires, observation and field notes, and documents, among others). This research uses two methods to collect data: written tests and interviews. The use of multiple methods can even help researchers discover misleading dimensions in a phenomenon (Abdalla et al., 2018).

Transferability assesses the applicability of findings to other contexts by providing detailed descriptions of the studied population. Transferability in this research is demonstrated by using the same data collection methods with student groups from different demographics, namely pre-service mathematics teachers at state universities in West Java and Central Java.

Dependability relates to the reliability of the research process, ensuring that decisions and methods are well-documented and replicable. Dependability in this research is demonstrated through the detailed process outlined in writing the article. This includes explaining the specific objectives of the research, discussing the selection criteria and rationale for choosing participants, describing the data collection methods and their duration, outlining how the data was reduced or transformed for analysis, discussing the interpretation and presentation of the findings, and detailing the techniques used to establish data credibility.

Confirmability requires researchers to be reflective, recognize their biases and maintaining transparency about how these may impact the research. Confirmability in this research is clearly demonstrated by conducting interviews to confirm pre-service mathematics teachers' responses. Collectively, these criteria help establish the trustworthiness and relevance of qualitative findings (Thomas & Magilvy, 2011).

▪ RESULT AND DISSCUSSION

Data Reduction

The research was conducted from May 21 to June 14, 2024. During this period, 33 pre-service mathematics teachers from the Flat Analytical Geometry course (2nd semester), who had received instruction on circle material, participated in the study. These participants were selected based on criteria related to learning obstacle (LO) analysis and were assigned codes from M1 to M33.

Almost all 33 participants were able to provide answers, although they were not always correct. The focus of the LO analysis was on 8 participants, as only these individuals were selected for the data confirmability process through in-depth interviews. The selection of these 8 participants was based on their test work data. The diversity of LO types found among these 8 participants was representative of the LO data observed in the test results of all participants involved in the research.

Data Presentation

The 8 participants include M1, M2, M5, M9, M17, M19, M20, M30. The LO findings can be summarized as follows in Table 1.

Table 1. The data of learning obstacles (LO) Of 9 participants

The Type Of LO		Participants Code	Question Number	The Finding Results
Ontogenic Obstacles	**Psychological**	M17	1-4	The participant was not motivated to learn mathematics because M17 did not understand the entire mathematics material which was caused by errors in the learning process M17 had experienced.
	Conceptual	M19	1	Lack of mastery of basic concepts and supporting prerequisites for circle material, namely curves.
		M20	2	Lack of mastery of the basic concept of the position of a point on a circle.
		M5	3a	Lack of mastery of the concept of circle radius.
			4	Lack of mastery of concepts related to differences in the meaning of evaluating the location of a point on a circle and the area of the circle.
		M1	3b	Does not understand the concept of the general form of circle equations.
	Instrumental	M17	1	Misunderstanding of the definition of a circle, participants assume a circle is a set of points that are the same distance from a certain point without justifying the number of members of the set.
		M2	2	Limited understanding of the "meaning" of circle equations.
		M30	3b	Not able to understand the elements that form a circle.

Epistemological Obstacles	M9	3a	Limited context for applying the general form of circle equations due to the minimal variety of practice questions received.
Didactical obstacles	M9	1	The definition of a circle shown in teaching materials is only limited to the context of a set or position of points.
		2	Minimal variety of practice questions presented in teaching materials.
	M20	3a	Only accept practice questions that are application of formulas, not application of circle concepts in the learning process.
		3b	
		4	

The Detailed Findings Of Ontogenic Obstacles Of The Psychological Type

LO ontogenic type which is psychological in solving circle problem number 1- 4 experienced by M17. M17 did not write any answers on the answer sheet, but instead wrote the following statement:

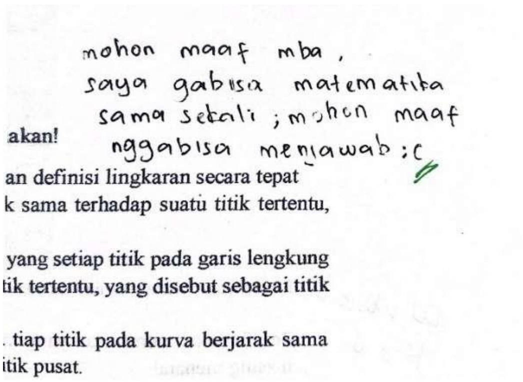


Figure 3. The statement of M17

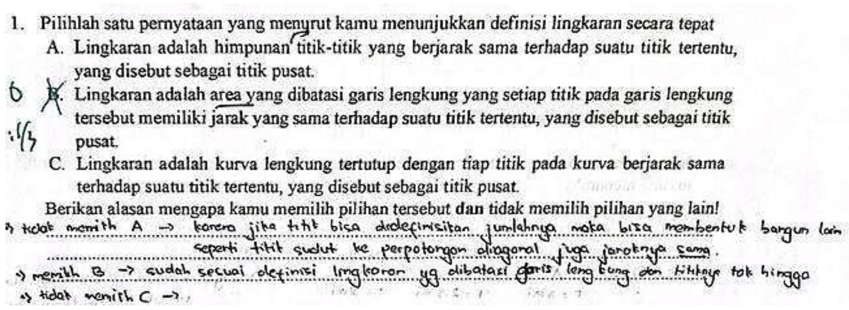
From this statement, it can be concluded that M17 was unable to solve all the available problems because M17 felt could not do mathematics. Furthermore, interviews were conducted to identify the factors that caused M17 to convey this.

Interviewer : Can you explain why you wrote this?
M17 : I find it difficult to understand the questions because I don't like mathematics.

Based on the interview results, it can be concluded that M17 experienced psychological learning obstacles due to a lack of motivation to study mathematics. This lack of motivation stems from not understanding fundamental mathematics concepts, leading M17 to rely on memorization rather than true comprehension. In educational psychology, motivation theory explores what drives individuals to engage with and develop an interest in an activity (Pantziara & Philippou, 2015). Motivation significantly influences mathematics learning outcomes (Pantziara & Philippou, 2015). Therefore, the solution to overcome this obstacle is to cultivate pre-service mathematics teachers' motivation before starting the lesson and to conduct mathematics teaching in a way that engages pre-service mathematics teachers in understanding the basic concepts, allowing them to construct their own understanding in an activity that makes them more interested in learning mathematics.

The Detailed Finding Of Ontogenic Obstacles Of The Conceptual Type

In question number 1, M19 experienced learning obstacles due to a lack of mastery of basic concepts and supporting prerequisites related to circular material, specifically curves. This obstacle impairs M19's ability to justify answer option C, making it difficult to evaluate the answer choice optimally. The data from M19's test answers and interviews are shown below.


1. Pilihlah satu pernyataan yang menurut kamu menunjukkan definisi *lingkaran secara tepat*
   A. Lingkaran adalah himpunan titik-titik yang berjarak sama *terhadap suatu titik tertentu*, yang disebut sebagai titik pusat.
   B. Lingkaran adalah area yang dibatasi garis lengkung *yang setiap titik pada garis lengkung* tersebut memiliki jarak yang sama terhadap suatu *titik tertentu, yang disebut sebagai titik* pusat.
   C. Lingkaran adalah kurva lengkung *tertutup dengan tiap titik pada kurva berjarak sama* terhadap suatu titik tertentu, yang disebut sebagai titik pusat.

   Berikan alasan mengapa kamu memilih pilihan tersebut dan tidak memilih pilihan yang lain!

-> tidak memilih A -> karena jika titik bisa didefinisikan jumlahnya maka bisa membentuk bangun lain seperti titik sudut ke perpotongan diagonal juga jaraknya sama.
-> memilih B -> sudah sesuai definisi lingkaran yg dibatasi garis lengkung dan titiknya tak hingga
-> tidak memilih C ->

Figure 4. The M19's answer of the test number 1

It can be seen that M19 was able to provide reasons why he chose and did not choose options B and A, but was unable to provide reasons why he did not choose option C. This was later confirmed in the interview session as follows.

Interviewer : You can't justify option C because you never learned about curves or what?

M19 : I don't know at all, maybe because I never studied about it.

Based on the data, we can conclude that M19 cannot answer the question correctly because M19 experiences ontogenic conceptual learning obstacles due to a limited understanding of concepts related to curves.

In question number 2, M20 experiences conceptual ontogenic learning obstacles due to a lack of mastery of the basic concept of a circle, particularly the distinction between points located on the circle and points inside the circle. In this case, M20 incorrectly assumes that a known point through the circle is the center point, even though, conceptually, a point on the circumference of the circle is located on the circle, not inside it.

The answer clearly shows that M20 cannot solve the problem because M20 does not correctly determine the position of the point on the circle. This issue arises from M20's lack of a solid conceptual understanding of circles. Next, take note to the figure 6 that display the answer of M5 in question number 3a.

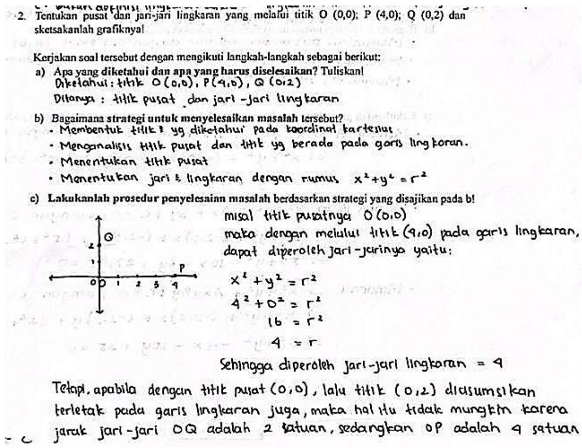


Figure 5. The M20's answer of the test number 2

Next, take note to the figure 6 that display the answer of M5 in question number 3a.

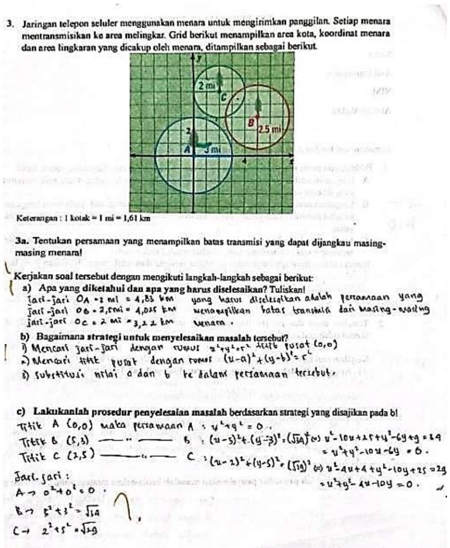
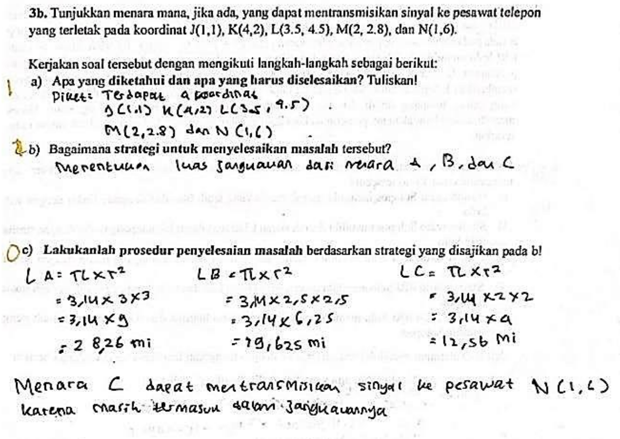


Figure 6. The M5's answer of the test number 3a

Based on the displayed data, M5 experiences conceptual ontogenic learning obstacles due to a lack of mastery of the concept of the radius of a circle. M5 does not have a good understanding of what a circle's radius is, as confirmed by the in-depth interview session. In the question number 3b, below are the results of test work and interviews from M1.

Interviewer : Why here do you write point N within the range of tower C, why don't you check all the points, that's all you check?

M1 : I realized this towards the end. What I mean is that earlier, I only sketched out the area because I understood the extent of the range but not the radius. I assumed that by writing this one sentence, you would already grasp my idea. However, it seems that there wasn't enough time, or I'm not sure about my answer. I forgot to clarify this point.



3b. Tunjukkan menara mana, jika ada, yang dapat mentransmisikan sinyal ke pesawat telepon yang terletak pada koordinat J(1,1), K(4,2), L(3.5, 4.5), M(2, 2.8), dan N(1,6).

Kerjakan soal tersebut dengan mengikuti langkah-langkah sebagai berikut:

a) Apa yang diketahui dan apa yang harus diselesaikan? Tuliskan!

Diket: Terdapat 4 koordinat J(1,1) K(4,2) L(3.5, 4.5) M(2,2.8) dan N(1,6)

b) Bagaimana strategi untuk menyelesaikan masalah tersebut?

Menentukan luas jangkauan dari menara A, B, dan C

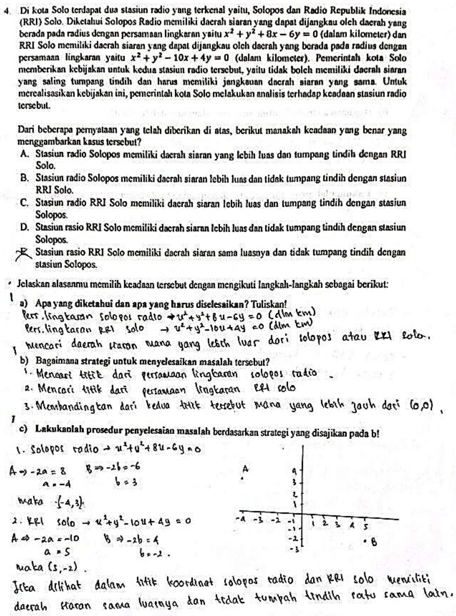
c) Lakukanlah prosedur penyelesaian masalah berdasarkan strategi yang disajikan pada b!

L A = π × r²	L B = π × r²	L C = π × r²
= 3,14 × 3 × 3	= 3,14 × 2,5 × 2,5	= 3,14 × 2 × 2
= 3,14 × 9	= 3,14 × 6,25	= 3,14 × 4
= 28,26 mi	= 19,625 mi	= 12,56 mi

Menara C dapat mentransmisikan sinyal ke pesawat N(1,6) karena masih termasuk dalam jangkauannya

Figure 7. The M1's answer of the test number 3b

Based on the displayed data, M1 experienced conceptual ontogenic learning obstacles due to a lack of mastery of the concepts related to differentiating between evaluating the position of a point on a circle and understanding the area of the circle. In the question number 4, below are the results of test work and interviews from M5.



4. Di kota Solo terdapat dua stasiun radio yang terkenal yaitu, Solopos dan Radio Republik Indonesia (RRI) Solo. Diketahui Solopos Radio memiliki daerah siaran yang dapat dijangkau oleh daerah yang berada pada radius dengan persamaan lingkaran yaitu $x^2 + y^2 + 8x - 6y = 0$ (dalam kilometer) dan RRI Solo memiliki daerah siaran yang dapat dijangkau oleh daerah yang berada pada radius dengan persamaan lingkaran yaitu $x^2 + y^2 - 10x + 4y = 0$ (dalam kilometer). Pemerintah kota Solo memberikan kebijakan untuk kedua stasiun radio tersebut, yaitu tidak boleh memiliki daerah siaran yang saling tumpang tindih dan harus memiliki jangkauan daerah siaran yang sama. Untuk merealisasikan kebijakan ini, pemerintah kota Solo melakukan analisis terhadap keadaan stasiun radio tersebut.

Dari beberapa pernyataan yang telah diberikan di atas, berikut manakah keadaan yang benar yang menggambarkan kasus tersebut?

A. Stasiun radio Solopos memiliki daerah siaran yang lebih luas dan tumpang tindih dengan RRI Solo.
B. Stasiun radio Solopos memiliki daerah siaran lebih luas dan tidak tumpang tindih dengan stasiun RRI Solo.
C. Stasiun radio RRI Solo memiliki daerah siaran lebih luas dan tumpang tindih dengan stasiun Solopos.
D. Stasiun rasio RRI Solo memiliki daerah siaran lebih luas dan tidak tumpang tindih dengan stasiun Solopos.
E. Stasiun rasio RRI Solo memiliki daerah siaran sama luasnya dan tidak tumpang tindih dengan stasiun Solopos.

Jelaskan alasanmu memilih keadaan tersebut dengan mengikuti langkah-langkah sebagai berikut:

a) Apa yang diketahui dan apa yang harus diselesaikan? Tuliskan!

Pers. lingkaran solopos radio → x² + y² + 8x − 6y = 0 (dlm km)
Pers. lingkaran RRI solo → x² + y² − 10x + 4y = 0 (dlm km)
Mencari daerah siaran mana yang lebih luas dari solopos atau RRI solo.

b) Bagaimana strategi untuk menyelesaikan masalah tersebut?

1. Mencari titik dari persamaan lingkaran solopos radio.
2. Mencari titik dari persamaan lingkaran RRI solo
3. Membandingkan dari kedua titik tersebut mana yang lebih jauh dari (0,0).

c) Lakukanlah prosedur penyelesaian masalah berdasarkan strategi yang disajikan pada b!

1. Solopos radio → x² + y² + 8x − 6y = 0
A ⇒ −2a = 8, a = −4; B ⇒ −2b = −6, b = 3
maka (−4, 3)
2. RRI solo → x² + y² − 10x + 4y = 0
A ⇒ −2a = −10, a = 5; B ⇒ −2b = 4, b = −2
maka (5, −2)

Jika dilihat dalam titik koordinat solopos radio dan RRI solo memiliki daerah siaran sama luasnya dan tidak tumpah tindih satu sama lain.

Figure 8. The M5's answer of the test number 3b

Interviewer : Here in essence we are asked to determine which circle A and B are wider and whether they overlap or not, is that so? Can you explain your process for answering these two things?

M4 : Yes, I already understand the meaning of the problem here, from the equations I know I can use to determine the center point and radius of the circle, but I forgot the formula for finding the radius of a circle using this equation.

Interviewer : What factors do you think caused you to forget the formula?

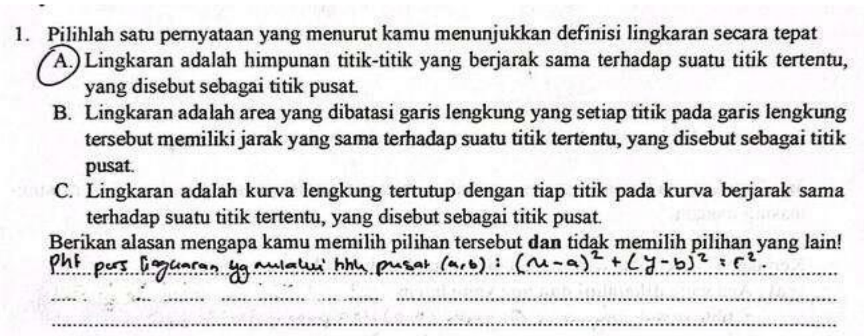
M4 : It's just a lack of practice.

Based on the displayed data, M5 experienced conceptual ontogenic learning obstacles due to a lack of mastery of concepts related to the general form of the circle equation, particularly in determining the radius from the general equation of a circle.

The findings reveal that pre-service mathematics teachers experienced ontogenic conceptual learning obstacles due to a lack of mastery of fundamental concepts related to circles, such as curves, the distinction between points on and inside the circle, and the radius. For instance, M19 struggled to justify answer choices involving curves, M20 misidentified a point on the circle as the center, and M5 lacked a proper understanding of the circle's radius. Additionally, M1 faced difficulties differentiating between evaluating a point's position on a circle and understanding its area. This obstacle suggests that strengthening pre-service mathematics teachers' foundational understanding is essential to overcoming these learning challenges. One of the best ways to overcome this obstacle is by integrating a learning approach that is closely connected to scholarly knowledge of circle concepts. For example, using classic sources like The Elements by Euclid can provide a strong foundation in geometric concepts, including a deep understanding of circles. By referring to this work, pre-service mathematics teachers can develop a better grasp of the basics of curves, the position of points on a circle, and the concept of the radius, all of which are essential elements in mastering circle material.

The Detailed Finding Of Ontogenic Obstacles Of The Instrumental Type

In the question number 1, below are the results of test work and interviews from M17.


1. Pilihlah satu pernyataan yang menurut kamu menunjukkan definisi lingkaran secara tepat
   A. Lingkaran adalah himpunan titik-titik yang berjarak sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.
   B. Lingkaran adalah area yang dibatasi garis lengkung yang setiap titik pada garis lengkung tersebut memiliki jarak yang sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.
   C. Lingkaran adalah kurva lengkung tertutup dengan tiap titik pada kurva berjarak sama terhadap suatu titik tertentu, yang disebut sebagai titik pusat.

Berikan alasan mengapa kamu memilih pilihan tersebut **dan** tidak memilih pilihan yang lain!

pht pers lingkaran yg melalui titik pusat (a,b) : $(x-a)^2 + (y-b)^2 = r^2$.

Figure 9. The M17’s answer of the test number 1

In the interview, M17 initially chose answer option A, believing it best describes a circle. When asked to explain their reasoning, M17 stated that a circle is defined by its equation with center coordinates (a, b) and described it as a set of points equidistant from the center. However, M17 struggled when asked whether three points equidistant from each other could define a circle and how to find the circle's equation in such a case. M17 admitted to not knowing how to proceed with these questions. This indicates that while M17 understands the basic definition of a circle and its equation, they lack a deeper understanding of how to apply these concepts to determine a circle's equation from specific points or shapes. This reveals a gap in applying theoretical knowledge to practical problems involving circles. Next, take note to the display of M2’s answer test number 2 and the result of the interviews with M2.

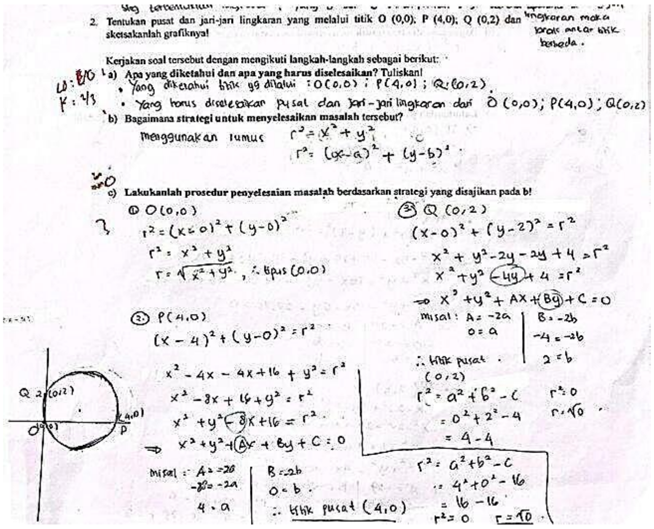


Figure 10. The M2’s answer of the test number 2

Interviewer : For example, in the equation of a circle with center (a, b), the formula is $(x-a)^2 + (y-b)^2 = r^2$. That's true, isn't it?
M2 : Yes, ma'am, that's right.
Interviewer : If we observe (a, b) where the center point is then the equation is right? So, if you input coordinate points O, P, Q into points (a, b) then you assume that the circle is centered at points O, P, and Q. Not a circle that passes through points O, P, Q.
M2 : Oh yes, but I'm often confused about the difference, sis, when should I enter coordinates into (x, y) and when into (a, b).
Interviewer : Alright.

Based on the displayed data, M2 experienced ontogenic instrumental learning obstacles because he could not properly understand the meaning of the circle equation. This is evident from M2 incorrectly placing the coordinates O, P, and Q where they should not be—the location of the circle's center. In reality, O, P, and Q are points on the circle, not the center of the circle. In question number 3b, M30 experienced LO in the form of instrumental ontogeny, the data are shown below.

Q : Do you understand the question?
M30 : I don’t understand what I’m being asked to do.

Based on the data, M30 did not fully grasp the meaning of the question. M30 misunderstood the question as being standalone and unrelated to the previous question. As a result, M30 attempted to find the distance between points to determine the shortest distance between them, assuming that a very short distance meant that they could transmit signals to each other. M30 did not critically examine the terms in the question, such as coordinate points and towers. Therefore, M30 experiences ontogenic instrumental learning obstacles.

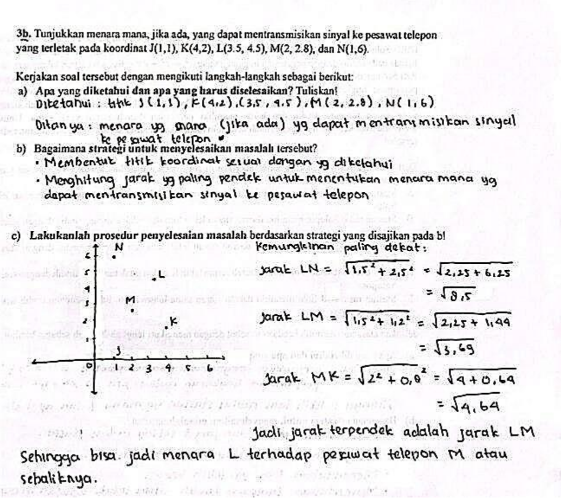


Figure 11. The M30's answer of the test number 3b

The findings reveal that in question number 1, M17 experienced ontogenic instrumental learning obstacles due to a misunderstanding of the definition of a circle. While M17 correctly defined a circle as a set of points equidistant from a central point and was able to state the circle equation, M17 struggled with the concept of how to apply this definition to find a circle's equation from a given set of points. This indicates that although M17 could articulate the standard definition and equation of a circle, their comprehension and application of these concepts were limited. Similarly, in question number 2, M2 faced ontogenic instrumental learning obstacles by misplacing coordinates into the circle equation, confusing the center of the circle with points on the circle. M2's difficulty in distinguishing when to use coordinates for the circle's center versus points on the circle demonstrates a lack of understanding of the equation's application. In question number 3b, M30 experienced instrumental ontogenic learning obstacles by failing to grasp the meaning of the question and confusing the problem's requirements with unrelated concepts, such as the distance between points, instead of focusing on the relevant geometric terms. These cases collectively highlight the importance of a deeper understanding of both the definitions and applications of geometric concepts to effectively solve problems related to circles. To address these issues, pre-service mathematics teachers should be guided to construct their understanding of circles so that they can fully grasp all related concepts. This approach involves helping pre-service mathematics teachers meaningfully interpret all elements associated with circles, such as the definition, the circle equation, and the roles of various points. By actively engaging in constructing and refining their understanding, pre-service mathematics teachers can better comprehend and apply the concepts related to circles, overcoming the instrumental and ontogenic learning obstacles identified.

The Detailed Finding Of Epistemological Obstacles

In the question number 3a, below are the results of test work and interviews from M9.

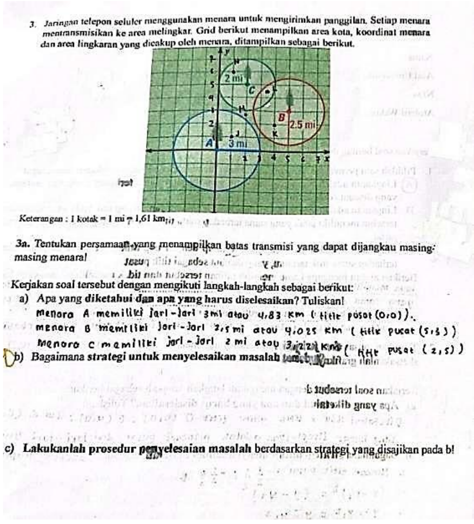


Figure 12. The M9’s answer of the test number 3a

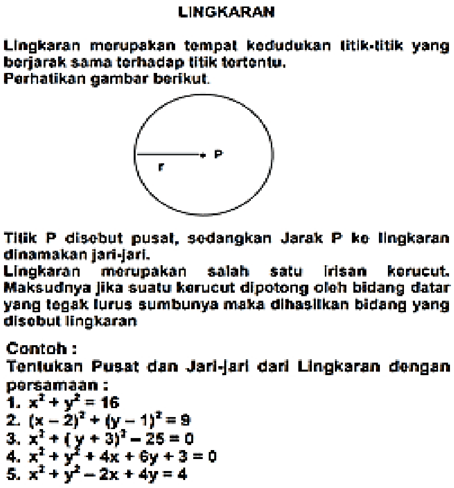
In the interview, M9 confirmed understanding question 3a and mentioned that they typically approach such problems by comprehending the context and the illustrations provided. However, M9 was unable to solve the problem, identifying their primary issue as a lack of understanding of the term transmission limit. When prompted, M9 acknowledged that their inability to grasp the meaning of transmission limit prevented them from completing the problem. This highlights that M9's difficulty stemmed from a misunderstanding of specific terminology rather than a lack of general comprehension of the material or problem-solving approach.

The findings reveal that in question number 3a, M9 exhibited an epistemological learning obstacle due to a lack of understanding of the term transmission limit within the context of circle definitions. Although M9 was able to comprehend the question's context and illustrations, M9 struggled with solving the problem because M9 did not grasp that the transmission limit referred to determining the equation of a circle. This misunderstanding highlights a gap in M9's conceptual knowledge of how specific terms relate to the circle's properties and equations. To address this, it is crucial to ensure that pre-service mathematics teachers have a clear understanding of terminology and its application in geometric contexts to effectively tackle related problems. To address this epistemological learning obstacle, lecturers can enhance pre-service mathematics teachers' understanding by introducing a variety of problems related to circles. This approach helps pre-service mathematics teachers become familiar with different applications of circle concepts in real-life situations and sharpens their problem-solving skills. By exposing pre-service mathematics teachers to diverse scenarios and problem types, lecturers can deepen their comprehension of how terminology and concepts apply to practical problems, ultimately improving their ability to solve related questions effectively.

The Detailed Finding Of Didactical Obstacles

Through interviews with participants and lecturers, it was found that lecturers used handouts to teach circle material. Upon reviewing these handouts, the researcher identified several weaknesses that contributed to didactical obstacles for participants.

Specifically, the definition of a circle was limited to a set of points, and the range of practice questions provided was insufficient. There were no questions that involved applying circles to real-world problems. Below is an excerpt from the handouts used by lecturers in teaching circle material.



LINGKARAN

Lingkaran merupakan tempat kedudukan titik-titik yang berjarak sama terhadap titik tertentu.
Perhatikan gambar berikut.

Titik P disebut pusat, sedangkan Jarak P ke lingkaran dinamakan jari-jari.
Lingkaran merupakan salah satu irisan kerucut. Maksudnya jika suatu kerucut dipotong oleh bidang datar yang tegak lurus sumbunya maka dihasilkan bidang yang disebut lingkaran

Contoh :
Tentukan Pusat dan Jari-jari dari Lingkaran dengan persamaan :
1. $x^2 + y^2 = 16$
2. $(x - 2)^2 + (y - 1)^2 = 9$
3. $x^2 + (y + 3)^2 - 25 = 0$
4. $x^2 + y^2 + 4x + 6y + 3 = 0$
5. $x^2 + y^2 - 2x + 4y = 4$

Figure 13. The part of the handouts

The review of handouts used by lecturers to teach circle material revealed several didactical obstacles that impact pre-service mathematics teachers' learning experiences. Interviews with participants and lecturers indicated that these handouts presented a limited definition of a circle, merely defining it as a set of points. This narrow approach does not encompass the full scope of circle concepts, potentially leading to incomplete understanding among pre-service mathematics teachers. Additionally, the handouts lacked a variety of practice questions, particularly those that apply circle concepts to real-world scenarios. The absence of practical applications means pre-service mathematics teachers might struggle to connect theoretical knowledge to practical uses, hindering their ability to solve complex problems. To address these issues, it is crucial for lecturers to expand the definition of a circle to include its mathematical properties and applications, and to incorporate a broader range of practice problems. This should include real-world problems that require pre-service mathematics teachers to apply their understanding of circles, thereby enhancing their conceptual grasp and problem-solving skills. Additionally, tools like GeoGebra can support pre-service teachers' understanding of circles and help overcome didactical learning obstacles.

Research on learning obstacles at the middle and high school levels, conducted by (Fauzi et al., 2023); (Hariyani et al., 2022); (Hendriyanto et al., 2024); (Puspita et al., 2023); (Sari et al., 2024); (Utami, 2023); (Wang et al., 2021), shows that many students still face difficulties with circle material. One approach to address this issue is the role of mathematics teachers. However, research on pre-service mathematics teachers also indicates that they face learning obstacles in studying circle material. This finding aligns with Jannah, et al (2019), who identified two cognitive levels in Mathematics Education Departments: remembering and understanding, as well as three types of learning obstacles (didactical, cognitive, and epistemological). Noto, et al (2019) also found that pre-service teachers struggle with mastering geometry material, identifying five types of difficulties related to epistemology in geometry transformations: a) applying concepts; b) visualizing geometric objects; c) determining principles; d) understanding problems; and e) mathematical proofs. These finding also aligns with the result that the preservice teachers mostly experienced difficulty in defining the sphere (Ünlü, 2022). In addition, it was determined that most preservice teachers provided either insufficiently detailed or incorrect answers for the concepts of circle, circular region, and sphere (Ünlü, 2022). When the examples of the concepts were examined, it was determined that only a few of the preservice teachers gave appropriate examples of the related concepts (Ünlü, 2022).

This presents a significant problem that needs immediate attention. For school students to gain a solid understanding, mathematics teachers must be experts in their field, with no learning obstacles impeding their teaching. This is equally true for pre-service mathematics teachers who will eventually become mathematics educators. A potential solution to this issue is integrating prior knowledge with new knowledge in the learning process to improve cognitive levels (Wang et al., 2023). Teachers should provide formal definitions based on students' needs (Jannah et al., 2019). This approach could also address the findings from this study, considering that pre-service mathematics teachers experience ontogenic conceptual learning obstacles due to inadequate understanding of the definition of a circle. Lecturers should guide them in understanding the definition of a circle using reliable source books, such as Euclid's The Elements. Additionally, tools like GeoGebra can support pre-service teachers' understanding of circles and help overcome didactical learning obstacles. Once pre-service teachers have a solid grasp of the concept of a circle (its definition, elements, and equations), they should be exposed to a variety of real-world problems to broaden their contextual understanding (Lee & Dietiker, 2022; Madsen et al., 2024). This approach also helps address epistemological obstacles. Efforts to overcome learning obstacles are closely related to the implementation of the learning process. This research aims to inspire other researchers to develop learning designs that can effectively address learning obstacles in circle material. In this context, the learning design research referred to is Didactical Design Research (DDR).

Data Conclusion

Based on the findings presented it can be concluded that all participants in the research experienced various types of learning obstacles. The study identified all types of learning obstacles—ontogenic, epistemological, and didactical. Therefore, it can be concluded that pre-service teachers still face learning obstacles in solving circle problems. These obstacles stem from a lack of motivation to learn mathematics, limited learning tools, insufficient understanding of the concept of circles, difficulties in interpreting the elements of circles, and limited context in applying circles to solve real-world problems.

▪ CONCLUSION

In Indonesia, circles are a fundamental topic in courses related to geometry, including flat geometry and flat analytical geometry. They are also standard content in the curriculum for elementary, middle, and high school students. Therefore, pre-service mathematics teachers need to have a thorough understanding of circle material.

Problem-solving is a crucial and integral part of mathematics education, and circle material is no exception. Thus, problem-solving requires special attention from lecturers. In teaching, lecturers must design learning experiences that are effective and purposeful, ensuring that the learning objectives are achieved. When learning is conducted in a one-directional manner, it can reduce opportunities for pre-service mathematics teachers to develop their own knowledge and can result in less meaningful learning. This can lead to various learning obstacles.

Learning obstacles are a preliminary research area in Didactical Design Research, as introduced by Suryadi (2019). While there has been substantial research on learning obstacles in mathematics for school students, including those related to circle material, there has been limited research on learning obstacles experienced by pre-service teachers in solving circle problems. It is crucial to assess whether pre-service mathematics teachers have mastered circle material effectively, as they will eventually become mathematics teachers. This assessment aims to address issues observed in the field, where many school students face learning obstacles in solving circle problems. Therefore, research into learning obstacles in pre-service mathematics teachers is highly important.

This study, conducted using a qualitative case study approach, reveals that all pre-service mathematics teachers involved experienced learning obstacles in solving the circle problems provided. The research identified all types of learning obstacles: ontogenic, epistemological, and didactical. In other words, pre-service mathematics teachers faced ontogenic, epistemological, and didactical learning obstacles in solving circle problems.

Efforts to address learning obstacles are closely related to the implementation of the learning process. This research aims to inspire other researchers to develop learning designs that can effectively overcome learning obstacles in circle material. In this context, the learning design research referred to is Didactical Design Research (DDR).

▪ REFERENCES

Abdalla, M. M., Oliveira, L. G. L., Azevedo, C. E. F., & Gonzalez, R. K. (2018). Quality in Qualitative Organizational Research: types of triangulation as a methodological alternative. Administração: Ensino e Pesquisa, 19(1), 66–98. https://doi.org/10.13058/raep.2018.v19n1.578

Artigue, M., & Trouche, L. (2021). Revisiting the French didactic tradition through technological lenses. Mathematics, 9(6), 1–19. https://doi.org/10.3390/math9060629

BOSTES. (2011). Australian professional standards for teachers.

Chen, Y., & Chang, C. (2018). The impact of an integrated robotics stem course with a sailboat topic on high school students ' perceptions of integrative STEM , interest , and career orientation robots in the STEM curriculum. 14(12).

Fauzi, I., Arisetyawan, A., & Chano, J. (2023). Integration of Indonesian culture in the didactic design of the concept of fractions in elementary schools. International Online Journal of Primary Education, 12(4), 263–284. https://doi.org/10.55020/iojpe.1336481

Fauzi, I., & Suryadi, D. (2020). The analysis of students' learning obstacles on the fraction addition material for five graders of elementary schools. Al Ibtida: Jurnal Pendidikan Guru MI, 7(1), 33. https://doi.org/10.24235/al.ibtida.snj.v7i1.6020

Hadi, W., Faradillah, A., Makmur, C., & Blok, G. (2020). *Hambatan mahasiswa calon guru matematika dalam menyelesaikan masalah bermuatan high-order thinking skills universitas muhammadiyah prof . Dr . Hamka , jakarta timur , indonesia* [challenges faced by prospective mathematics teachers in solving problems involving high-order thinking skills at universitas muhammadiyah prof. Dr. Hamka, east jakarta, indonesia]. AKSIOMA: Jurnal Program Studi Pendidikan Matematika, 9(3), 662–670.

Hariyani, M., Herman, T., Suryadi, D., & Prabawanto, S. (2022). International Journal of Educational Methodology, 8(3), 505–515. https://www.ijem.com/exploration-of-student-learning-obstacles-in-solving-fraction-problems-in-elementary-school

Hendriyanto, A., Suryadi, D., Juandi, D., Dahlan, J. A., Hidayat, R., Wardat, Y., Sahara, S., & Muhaimin, L. H. (2024). The didactic phenomenon: Deciphering students' learning obstacles in set theory. Journal on Mathematics Education, 15(2), 517–544. https://doi.org/10.22342/jme.v15i2.pp517-544

Jannah, U. R., Nusantara, T., Sudirman, Sisworo, Yulianto, F. E., & Amiruddin, M. (2019). Student's learning obstacles on mathematical understanding of a function: A case study in Indonesia higher education. TEM Journal, 8(4), 1409–1417. https://doi.org/10.18421/TEM84-44

Lee, K., & Dietiker, L. (2022). Mathematically captivating learning experiences. 1580–1587.

Madsen, J. K., George, N. L., & Fernandes, A. C. (2024). The pedagogical use of didactic classes for teaching cognitive psychology. Teaching and Learning Inquiry, 12, 1–16. https://doi.org/10.20343/teachlearninqu.12.15

Miles, M. B., & Huberman, A. M. (1994). An expanded sourcebook: Qualitative data analysis (2nd ed.). SAGE Publications.

Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of pre-service mathematics teachers on transformation geometry. Journal on Mathematics Education, 10(1), 117–125. https://doi.org/10.22342/jme.10.1.5379.117-126

Nurhasanah, F. (2018). Mathematical abstraction of pre-servise mathematics teachers in learning non-conventional mathematics concepts. Universitas Pendidikan Indonesia.

Pantziara, M., & Philippou, G. N. (2015). Students' motivation in the mathematics classroom. revealing causes and consequences. International Journal of Science and Mathematics Education, 13(S2), 385–411. https://doi.org/10.1007/s10763-013-9502-0

Pasandaran, R. F., & Mufidah, M. (2020). *Studi kasus pembelajaran geometri analitik* [Case study of learning analytic geometry]. Pedagogy: Jurnal Pendidikan Matematika, 5(2), 91–105. https://doi.org/10.30605/pedagogy.v5i2.413

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: an overview on achievements and challenges. ZDM - Mathematics Education, 47(6), 877–891. https://doi.org/10.1007/s11858-015-0722-3

Puspita, E., Suryadi, D., & Rosjanuardi, R. (2023). The effectiveness of didactic designs for solutions to learning- obstacle problems for prospective mathematics teacher

students: case studies on higher-level derivative concepts. Mathematics Teaching-Research Journal, 15(3), 5–18.
Rahmawati. (2021). *Representasi mahasiswa dalam menyelesaikan madalah geometri analitik.*
Riegel, U., & Rothgangel, M. (2022). Designing research in subject matter didactics. results and open questions of a Delphi study. Results and Open Questions of a Delphi Study (RISTAL), 5, 56–77.
Salimi, M., Dardiri, A., & Sujarwo, S. (2021). The profile of students' social skills of Bengawan Solo Elementary Nature School. European Journal of Educational Research, volume-10-(volume-10-issue-1-january-2021), 211–226. https://doi.org/10.12973/eu-jer.10.1.211
Sari, A. D., Suryadi, D., & Dasari, D. (2024). Learning obstacle of probability learning based on the probabilistic thinking level. Journal on Mathematics Education, 15(1), 207–226. https://doi.org/10.22342/jme.v15i1.pp207-226
Sugiyono. (2017). *Metode penelitian pendidikan: Pendekatan kuantitatif, kualitatif dan R&D.* Alfabeta.
Supriyadi, E., Suryadi, D., Turmudi, T., Prabawanto, S., Juandi, D., & Dahlan, J. A. (2023). Didactical design research: a bibliometric analysis. Journal of Engineering Science and Technology, 18(3), 153–160.
Suryadi, D. (2019). Monograf : didactical design research (DDR). Gapura Press.
Thomas, E., & Magilvy, J. K. (2011). Qualitative rigor or research validity in qualitative research. Journal for Specialists in Pediatric Nursing, 16(2), 151–155. https://doi.org/10.1111/j.1744-6155.2011.00283.x
Ünlü, M. (2022). Investigation of preservice mathematics teachers' concept definitions of circle, circular region, and sphere. International Journal of Mathematical Education in Science and Technology, 53(7), 1787–1814. https://doi.org/10.1080/0020739X.2020.1847334
Utami, N. S. (2023). Students ' learning obstacles in solving early algebra problems : a focus on functional thinking. 395–412.
Wang, F., Huang, Z., Liu, Q., Chen, E., Yin, Y., Ma, J., & Wang, S. (2023). Dynamic cognitive diagnosis: an educational priors-enhanced deep knowledge tracing perspective. IEEE Transactions on Learning Technologies, 16(3), 306–323. https://doi.org/10.1109/TLT.2023.3254544
Wang, X., Liu, X. Y., Jia, S., Jiao, R., Zhang, Y., Tang, L., Ni, X., Zhu, H., Zhang, F., Parpura, V., & Wang, Y. F. (2021). TESOT: a teaching modality targeting the learning obstacles in global medical education. Advances in Physiology Education, 45(2), 333–341. https://doi.org/10.1152/ADVAN.00191.2020
Xiang, J. (2019). Factors affecting the learning effect of advanced mathematics among chinese college students in social science majors. 15(11).
Yin, R. K. (2011). Qualitative Research From Start To Finish. The Guilford Press.
Zamzam, K. F. (2020). *Pengembangan modul geometri berbasis reciprocal teaching untuk meningkatkan kemampuan komunikasi matematis mahasiswa calon guru* [Development of a geometry module based on reciprocal teaching to improve the mathematical communication skills of pre-service mathematics teachers]. Jurnal Cendekia : Jurnal Pendidikan Matematika, 4(1), 365–373. https://doi.org/10.31004/cendekia.v4i2.226