



## Students' Thinking Process in Solving Mathematics Problems Oriented to Higher Order Thinking Skills

Kristoforus Djawa Djong<sup>1\*</sup>, Yutri Kormalina Lusi<sup>1</sup>, Meryani Lakapu<sup>1</sup> & Ilda Guterres<sup>2</sup>

<sup>1</sup>Department of Mathematics Education, Universitas Katolik Widya Mandira, Indonesia

<sup>2</sup>Department of Mathematics Education, Universidade Oriental Timor Loros'ae, Timor Leste

**Abstract:** An educator needs to know the thinking process of their students in order to facilitate the transfer of knowledge and skills. HOTS can play a very big role in supporting students' academic achievement, with HOTS students are able to solve problems, and reflect on their problem-solving experience. The purpose of this research is to describe how students' thought process in solving HOTS-oriented mathematics problems. The method used in this research. The subjects in this study consisted of 7 students who were selected using purposive technique. This research starts from the preparation stage, the research implementation stage and the completion stage. The instrument in this research is HOTS-oriented test questions used to determine the thinking process of high, medium and low ability students. The results of this study show that a person's ability or thought process depends on the level of the problem given. Of the seven subjects in working on two different HOTS problems, it was found that there were 2 high ability students who were able to fulfill the indicators of analysis, evaluation and creation. Three other students in question number 1, met all indicators, but in the second question only met the analysis and evaluation indicators. While the other two students in question number 1 met all the indicators, but in the second question did not meet any of the indicators. The results of this study can be used as a reference for lecturers in choosing media, models and learning methods used to transfer knowledge and skills to students.

**Keywords:** thinking process, HOTS, math problems.

### ■ INTRODUCTION

Currently, reforms in education are taking place in various countries, including Indonesia. The overhaul occurs in the evaluation system which leads to the assumption that learning, especially mathematics learning, should not only be oriented towards mathematical abilities and basic knowledge, but can be more focused on developing students' skills in solving new, non-routine problems so that mathematics learning can reach the totality of the dynamics of students' thinking processes.

The reform is characterized by the implementation of learning and evaluation that is more oriented towards Higher Order Thinking Skills (HOTS). HOTS can play a huge role in supporting students' academic achievement, with HOTS students are able to solve problems, and reflect on their problem-solving experience. This makes students more confident in developing higher order thinking skills (Hmelo & Ferrari, 1997).

In today's rapidly changing world, students are not only required to acquire knowledge but also learn skills that help them synthesize and generate knowledge. In addition, by learning skills students acquire new ways of thinking for their personal character growth. Thus, higher order thinking skills are highly valued (Lo & Feng, 2020).

Many people use the term HOTS to describe a form of complex thinking that demands high cognitive processes. It is a term developed to combine two different perspectives on critical thinking from the field of philosophy, which sees it as evaluation

or judgment, and from the field of psychology, which sees it as problem-solving. HOTS is defined as thinking that occurs "when a person takes new information and information stored in memory and interconnects and/or reorganizes and extends this information to achieve a goal or find possible answers in a confusing situation". As the historical development of the term indicates, HOTS encompasses evaluation or judgment, problem solving, as well as creative thinking and decision making. As HOTS resulted from the merging of two different perspectives on critical thinking, it also includes broadly defined critical thinking as a component. Reasoning or productive thinking is part of problem solving, as it is used to integrate past experiences that have not been related to find solutions to new challenges.

In general, a person's thinking ability consists of Lower Order Thinking Skills (LOTS) and HOTS. Based on Bloom's revised taxonomy, LOTS includes the ability to remember, understand, and apply, while HOTS includes the ability to analyze, evaluate, and create. Although included in LOTS, the ability to remember, understand, and apply is very influential on HOTS. In research (Livnat, HersHKovitz, & Tabach, 2020) on students' activities in online learning based on thinking levels, found that students' performance and completion rates on the LOT applets were overall higher than those of the HOT applets, which, combined with other findings, may point to meta-cognitive or motivational processes involved, so this is an important input for teachers who use information technology in learning.

HOTS is very important for prospective mathematics teacher students so that in this research (Richland & Begolli, 2016). The assessment focused on policies to support students' higher-order thinking, including requiring assessments that measure these skills; meaningfully disseminating assessment data to help improve teachers, schools or curricula; and designing professional development that explicitly addresses these skills. Yaniawati explained in the research that HOTS plays an important role in improving the effectiveness of the learning process. HOTS can be said to be able to lead students to success in learning. Therefore, lecturers become facilitators who provide support, source information and direct students to get real learning experiences as an effort to develop HOTS. A person has a high level of thinking ability, characterized by critical questions that are expressed to explore information about material that has not been mastered (Burns & Reis, 1991). Students' mastery of mathematical content is necessary so that they can process information logically and analytically (Hadi, 2021).

Students who have HOTS skills must be able to remember formulas, besides that, they must also be able to understand problems and apply, and be able to analyze mathematical problems. Not only that, students are also able to evaluate work results, and create new creations. (Richland & Begolli, 2016) emphasized in her research that mathematics teachers should highlight the alignment between the definition of higher order thinking psychologically and educational goals. Teachers should also create active learning because when students are active, their HOTS will increase (Madhuri, Kantamreddi, & Goteti, 2012).

From the above problems, this research was carried out with the aim of describing how students' thinking processes in solving HOTS-oriented mathematical problems, the benefit is that lecturers can facilitate well the process of transferring knowledge and skills.

▪ **METHOD**

**Participants**

The subjects in this study were 7 students selected from all students of the mathematics education study program at Widya Mandira Catholic University batch 2020/2021, 2021/2022 and 2022/2023. Three students were selected using a purposive technique where the researcher determined high, medium and low ability subjects. Three people had the highest IPK ( $IPK \geq 3.5$ ) in 3 different batches, 2 others had IPKs in the medium category, namely ( $3 \leq IPK < 3.5$ ) in 3 different batches and 2 others were in the low category, namely ( $IPK < 3$ ) in 3 different batches.

**Research Design and Procedures**

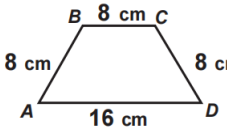
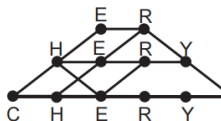
This type of research is descriptive qualitative to provide an overview of the thought process of students in solving HOTS-oriented mathematics problems. This method can provide a scientific description of the object studied and can pay attention to the quality of the study.

This research starts from the preparation stage, the research implementation stage and the completion stage. In the preparation stage, the researcher prepared all the administration needed to collect data including the HOTS test questions. In the implementation stage, the researcher determined the subjects, gave the questions and conducted interviews with the subjects who had worked on the questions. Then in the completion stage, the researcher analyzed and processed the research results.

**Instrument**

The instruments in this study are HOTS-oriented test questions used to determine the thinking process of high, medium and low ability students. The HOTS questions consisted of 2 numbers taken from the question Mathematical Olympiads for Elementary and Middle Schools Mathematical Olympiads for Elementary and Middle School, Olympiad Problems 2020-2011 Division E. The two questions prepared contain indicators of analysis, evaluation and creation (Table 1). In addition, the HOTS indicators used were adopted from the indicators formulated by Anderson and Krathwohl (Table 2).

**Tabel 1.** Test questions

No.	Questions
1	<p><b>3E Time: 6 minutes</b></p> <p>Two bugs walk from point <i>A</i> to point <i>D</i> along the sides of figure <i>ABCD</i>. They start and finish together. The first bug walks from <i>A</i> to <i>B</i> to <i>C</i> to <i>D</i> at an average speed of 3 centimeters per second. The second bug walks directly from <i>A</i> to <i>D</i>. What is the average speed of the second bug?</p> 
2	<p><b>4E Time: 7 minutes</b></p> <p>Cheryl traces her name, <b>CHERYL</b>, by following the lines shown. She can change direction only at a letter. How many different paths can trace her name?</p> 

**Table 2.** HOTS indicator

<b>Aspect</b>	<b>Indicator</b>
Analyze	✓ Students are able to examine and analyze information appropriately
	✓ Students are able to determine what is asked correctly
	✓ Students are able to connect information from a problem
Evaluate	✓ Students are able to choose the right solution method
	✓ Students are able to re-examine work appropriately
Create	✓ Students is able to plan an appropriate and coherent solution according to the problem
	✓ Students able to produce existing elements into one unit
	✓ Students are able to make decisions/conclude appropriately

After conducting the test, the researcher communicated directly with the subject to dig up information related to his thought process in solving HOTS-oriented math problems and to clarify the results of the test question work. In this study, researchers used interview guidelines that were not arranged systematically and completely, the interview guidelines only contained general questions that would be asked. During the interview, the questions adjusted the test results.

### **Data analysis**

Data collection techniques in this study were tests and interviews conducted in an unstructured manner. The data collected was tested for validity using triangulation techniques, namely comparing test results and interview results. Data analysis in this study followed the analysis model written by model Miles and Huberman that is (1) data reduction, at this stage data analysis refers to the process of sharpening information, classifying information and discarding raw data that is not used from test results and interviews obtained from the field about students' thinking processes in solving HOTS-oriented mathematics problems given to the subject; (2) data presentation, at this stage activities are carried out to classify and identify data to draw conclusions. The presentation of data carried out in this study is classifying and describing based on indicators of students' thinking processes in solving HOTS-oriented mathematics problems and then expressed in tabular form; and (3) drawing conclusions, at this stage drawing conclusions from the data that has been collected in the form of the results of students' work. From the results of students' work, researchers describe the thinking process of students in solving HOTS-oriented mathematics problems. The following is the formula used for the percentage of correct students:

$$P = \frac{R}{SM} \times 100\%$$

Description:

P : Percentage

R : Student score

SM : Total score

▪ **RESULT AND DISSCUSSION**

The student thinking process test using HOTS questions was given to 7 students. Three students were selected using a purposive technique where the researcher determined high, medium and low ability subjects. Three people had the highest IPK ( $IPK \geq 3.5$ ) in 3 different batches, 2 others had IPKs in the medium category, namely ( $3 \leq IPK < 3.5$ ) in 3 different batches and 2 others were in the low category, namely ( $IPK < 3$ ) in 3 different batches. All students worked on 2 numbers of HOTS questions then interviewed and then student answers and interview results were analyzed to draw conclusions. The following is the percentage of correct answers from 7 students (Table 3). This percentage is based on the final answer.

**Table 3.** Subject data description

No	Category	Student Code	Percentage Correct (%)	
			Question Number 1	Question Number 2
1	High	ST1	100	89
2	High	ST2	100	100
3	High	ST3	100	0
4	Medium	SS1	100	100
5	Medium	SS2	100	0
6	Low	SR1	100	0
7	Low	SR2	100	0

**Students' Thinking Process about HOTS Problem Number 1**

The first analysis focuses on the thought process of 7 students about HOTS question number 1. Students were asked to determine the average speed of the second insect, if it is known that the distance traveled by the 2 insects is different but the travel time is the same. The following is the grouping of answers from the seven students:

**Table 4.** Grouping of answers to question number 1

No	Grup	Number of Students	Category	Indicator Met
1	Grup 1	1	1 Tinggi	Analyze, Evaluate, Create
2	Grup 2	6	2 Tinggi. 2 Sedang. 2 Rendah	Analyze, Evaluate, Create

Of the seven students, one person worked using the concept of comparing the distance traveled by insect 1 and the distance traveled by insect 2 as well as the speed of insect 1 and the speed of insect 2, thus directly obtaining the average speed of insect 2 is 2 (Figure 1).

Menggunakan perbandingan yang diketahui

Jarak yang ditempuh Serangga I (A-B-C-D) = 24 cm

Jarak yang ditempuh Serangga II (A-D) = 16 cm

Kecepatan rata-rata Serangga I = 3 cm per detik

Misalkan kecepatan rata-rata serangga II = S, maka didapatkan

$$\frac{24}{16} = \frac{3}{S}$$

$$S = \frac{16 \times 3}{24}$$

$$S = 2 \text{ cm per detik}$$

Jadi, serangga kedua kecepatan rata-ratanya yaitu 2 cm per detik.

**Figure 1.** Student answer (high)

The other six students worked in a longer way, one example of student work can be seen in (Figure 2) which is to first determine the travel time of the first and second insects and then substitute it in the speed formula (Second insect) to determine the average speed of the second insect. A low-performing student was wrong in writing the unit of speed. When interviewed, it was found that the student could not distinguish between units of speed and distance when working on the problem (Figure 3).

Serangga I	Serangga II
* Jarak tempuh = $\overline{AB} + \overline{BC} + \overline{CD}$ = 8 cm + 8 cm + 8 cm = 24 cm	Jarak tempuh = $\overline{AD} = 16 \text{ cm}$
membuktikan waktu tempuh 3 cm/detik	karena serangga II dan I memulai dan mengakhiri pada Maka
Maka waktu tempuh yg dibutuhkan oleh Serangga I dalam jarak 24 cm	Waktu tempuh Serangga II Jarak/detik (cm/detik)
= $\frac{\text{Jarak tempuh}}{\text{Waktu tempuh}} = \frac{24 \text{ cm}}{3 \text{ cm/detik}} = 8 \text{ detik}$	= $\frac{\text{Jarak tempuh serangga II}}{\text{Waktu tempuh serangga I}}$ = $\frac{16 \text{ cm}}{8 \text{ detik}} = 2 \text{ cm/detik}$
	Jadi kecepatan serangga II rata-rata 2 cm/detik

**Figure 2.** Student answer (high)

①. Dik: Jarak A ke B ke C =  $8 + 8 + 8 = 24$

$v_1 = 3 \text{ cm/s}$ , jarak mewakili 1 cm/s.

Dit:  $v_2 = \dots$

Penyelesaian:

Jarak A ke D = 16 cm

$$v = \frac{16}{8} = 2 \text{ cm/s}$$

**Figure 3.** Student answer (low)

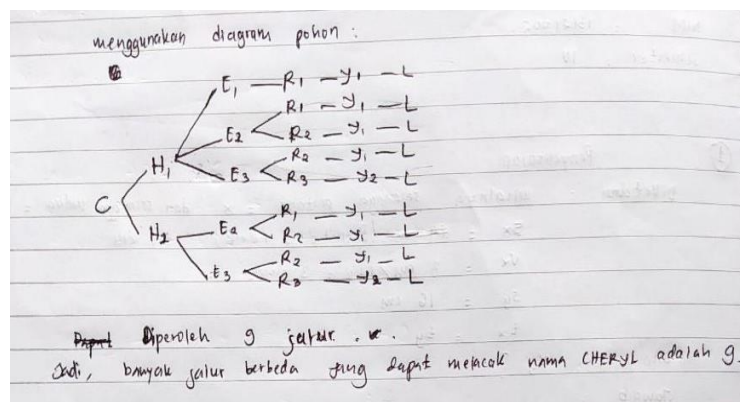
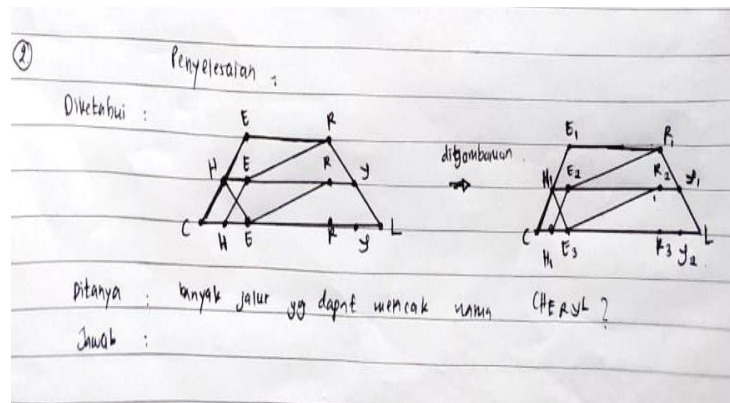
**Students' Thinking Process about HOTS Problem Number 2**

The second analysis focuses on the thought process of 7 students regarding HOTS question number 2. Students are asked to determine the number of different paths to track CHERYL's name, if it is known that the distance traveled by the 2 insects is different but the travel time is the same. The following is a grouping of answers from the seven students:

**Table 5.** Grouping of answers to question number 2

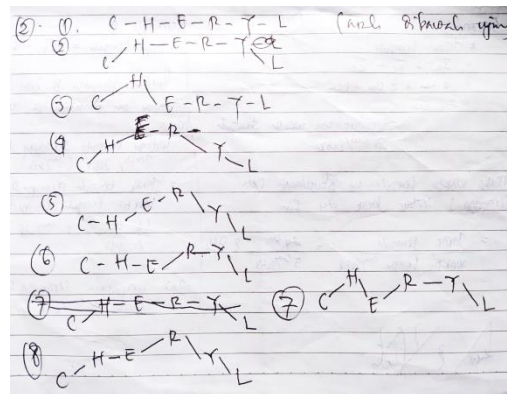
No	Grup	Number of Students	Category	Indicator Met
1	Grup 1	1	1 Tinggi	Analyze, Evaluate, Create
2	Grup 2	1	1 Tinggi	Analyze, Evaluate, Create
3	Grup 3	3	1 Tinggi, 1 Sedang, 1 Rendah	Analyze, Evaluate
4	Grup 4	2	1 Sedang, 1 Rendah	Analyze, Evaluate, Create

The first group consisted of 1 high ability student. This student first gave a more specific code to each letter of CHERYL's name then determined the number of different paths using the tree diagram correctly (Figure 4). When interviewed about how confident his answer was, this subject said that he was very sure that there was no breakthrough or no missed path because he had numbered the paths beforehand.



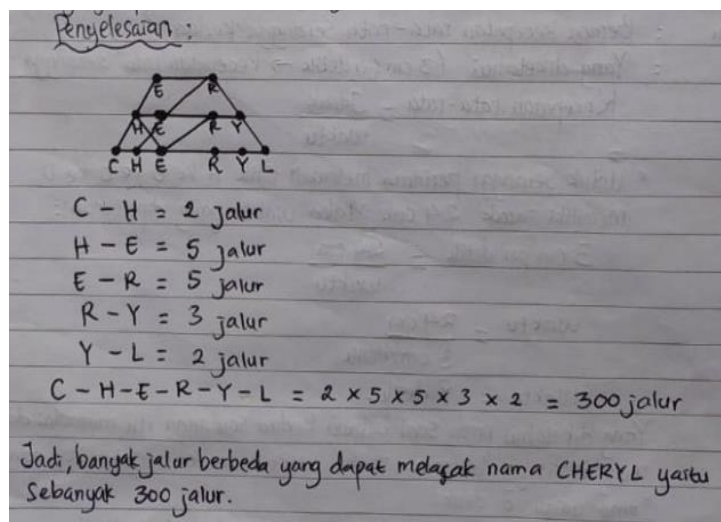
**Figure 4.** Student answer (high)

The second group consisted of 1 high ability student. This subject worked manually, in this case connecting each letter in CHERYL's name on a different path without giving a code. When interviewed that: "do you think there is no repetition when you work this way", the subject said that: "after I tried to connect each dot by dot I tried to look again to see if there were any similarities, but I didn't find that problem". With his way of working and after checking many times, the subject was sure that there were 8 different paths that could be used to trace CHERYL's name.



**Figure 5.** Student answer (high)

The third group consisted of 1 high ability student, 1 medium ability student and 1 low ability student. These three subjects worked in the same way, determining the number of paths of two adjacent letters and multiplying the number of paths. For example: C to H is 2 paths, H to E is 5 paths, E to R is 5 paths, R to Y is 3 paths and Y to L is 2 paths. The number of paths obtained is 300. The high student was not so sure about the number of paths obtained, but justified his method. After trying to use the manual method, he obtained the answer 9, but was confident in his first way of working. Medium and low students believed that there was no decomposition because it was already multiplied.



**Figure 6.** Student answer (high)



The fourth group consisted of 1 medium ability student and 1 low ability student. These two subjects answered 9 lines and 12 lines respectively without giving any explanation. The information obtained during the interview was that they answered by guessing without any reason.



**Figure 7.** Student answer (medium dan low)

The thought process of the seven subjects in solving HOTS-oriented math problem number 1 is very good. In problem number 1, all students were able to analyze, evaluate and create, where they were able to examine and parse information appropriately, were able to determine what was asked correctly and were able to connect the known information from a problem, they were able to choose the right solution method and were able to check or recheck their work process appropriately, were able to plan the right and coherent solution according to the problem, they were also able to produce existing elements into a single unit and were able to make decisions or conclude appropriately. Although they use manual methods, it can be said that they have broad thinking because they can solve new problems/challenges (Heong, et al., 2011).

The thought process of the seven subjects in solving HOTS-oriented math problem number 2 was quite good. In problem number 2, 2 high ability students were able to fulfill all three indicators, although the second high student made an operation error. Three students (high, medium and low) were only able to analyze and evaluate, while the other 2 students did not meet any of the indicators because they wrote answers by guessing the numbers. In certain cases, students' scores on multiple-choice questions were greater than those on description questions, due to the guessing factor (Tan, Thibault, Chew, & Rajalingam, 2022). Two things that caused the subject to guess the final answer were when he felt stuck and the time provided was over (Nagy, Ulitzsch, & Lindner, 2022).

In this study, in general, it can be said that the subject can or does not fulfill the indicator depending on the level of the problem given. In the interview, it was found that the subject could explain his work well. But at certain levels, the subject may or may not be able to give the right reasons for their work (Tibbits, 2016), because if the subject sees the problem / his work many times, it will automatically have a new perspective continuously so that this is what makes him feel less confident to give reasons (Rubel, 2023).

Students who were given scaffolds demonstrated higher-order thinking skills more frequently than students who did not receive scaffolds (Giacumo, Savenye, & Smith, 2012). Some ways to improve HOTS are to improve concepts, techniques, and skills as well as to maximize the potential of students or college students in learning and utilize higher order thinking skills such as analysis, synthesis and evaluation (Madhuri, Kantamreddi, & Goteti, 2012), teachers or lecturers can use PBL in learning (Moallem, 2019) (Aba-Oli, Koyas, & Husen, 2024), implementing inquiry learning (Lu, Pang, & Shadiev, 2022), using technology in learning (Fidan & Fidan, 2022), improving teacher performance (Siswono, 2014), teaching materials are designed to emphasize information gathering activities, remembering, organizing skills rather than focusing, integrating,

evaluating, and analyzing skills (Zoller, Lubezky, Nakhleh, Tessier, & Dori, 1995), (Zohar, Schwartz, & Tamir, 2007).

#### ■ CONCLUSION

Based on the results and discussion, it can be concluded that a person's ability or thought process depends on the level of the problem given. Of the seven subjects in working on two different HOTS problems, it was found that there were 2 high ability students who were able to fulfill the indicators of analysis, evaluation and creation. Three other students in question number 1, met all indicators, but in the second question only met the analysis and evaluation indicators. While the other two students in question number 1 met all the indicators, but in the second question did not meet any of the indicators.

Based on the results of this study, theoretical implications can be stated, namely the thought process of each student is different, both in terms of solving problems and other things, but the results obtained in solving the problem are not much different. For this reason, researchers who will conduct research related to the thinking process of students in solving HOTS-oriented mathematics problems in the future must increase their activeness, sense of inspiration, confidence and cooperate with research informants or research respondents to explore information more thoroughly so that it can help smooth the research in knowing how the thinking process of each subject to be studied.

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