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Problem Solving Skills of Mathematics Education Students with Lack Number Sense Ability

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Abstract: Problem-solving in mathematics is a fundamental skill that must be mastered by individuals. However, previous research indicates that students' problem-solving abilities tend to be moderate to low. Many factors contribute to this, including the factor of number sense ability. The objective of this study is to describe how number sense and problem-solving skills are used by mathematics education students with low number sense ability in solving linear programming problems using the simplex method. This research is qualitative descriptive involving 36 third-year mathematics education students as participants. Subjects were selected using purposive sampling technique based on a number sense ability test to identify those with low number sense ability. Participants were given an advanced test involving linear programming problems solved using the simplex method. The results indicate that mathematics education students with low number sense ability struggle to solve problems accurately. Stages of problem-solving that were not performed effectively included gathering information, planning, and implementing the plan. Furthermore, students did not engage in the reviewing stage. Moreover, students were not flexible in manipulating numbers, especially when dealing with large numbers. As a result, more time was required for computations, and the iteration process in the simplex method could not be effectively completed.

Keywords: linear programming, number sense, number sense ability, problem solving, simplex method.

▪ INTRODUCTION

Problem-solving is an important part of mathematics learning that is interrelated with the mathematics curriculum. In other words, problem-solving is a main focus in the mathematics curriculum and a fundamental need for students (Calor et al., 2020). This is because Problem-solving is not only the ultimate goal or benchmark for success in learning mathematics, but as the main tool for learning (NCTM, 2000). Therefore, learning mathematics at various levels of education needs to be pursued so that students that have the ability and solve problems skills and become good problem solvers. Because skill development is a process, these problem-solving skills must be acquired by the child in school and with increasing complexity as the child ages and progresses through school (Akyüz, 2020).

NCTM (2000) also stated that there are five fundamental abilities that a person must have in learning mathematics, namely the ability of problem-solving, reasoning and proof, connection, communication, and representation. Furthermore, NCTM (2000) also states that learning mathematics must ensure that someone can (1) build new mathematical knowledge by solving the problems, (2) solve problems that arise in mathematics and other fields, (3) apply and adapt various strategies that are suitable for problem-solving, and (4) observing and developing the process of solving mathematical problems.

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Problem-solving can be defined as an attempt to find a way out of a difficulty, to achieve a goal that cannot be achieved directly (Polya, 1973). In other words, problem-solving is an activity related to cognitive processes and requires mature thinking that is creative and solutive to find a way out of a problem and obstacles that must be resolved. The solution that must be found in a problem-solving process is often not known beforehand (Hikmah & Siswono, 2020). While the stages that must be done in the problem-solving process according to Polya (1973) such as: 1) understanding the problem; 2) dividing a plan; 3) carrying out the plan; and 4) looking back.

Problem-solving is a crucially important component for someone to master at every level of education (ACME, 2016). But in fact, research conducted by Arjudin et al. (2021) shows that in order for children to achieve higher levels of thinking, such the relational and extended abstract levels, they still require strong help and facilitation when they solve problems. In other research, the linear algebra problem-solving abilities of mathematics education students were mostly in the low and medium categories (Arjudin et al., 2020). This fact certainly raises concerns for researchers when giving non-routine questions to students. This exposure was also strengthened by the results of the PISA test in 2018 which showed that Indonesia's mathematical literacy ranked 73rd out of 79 participating countries with a score of 379 (OECD, 2019). PISA questions are mostly non-routine ones that measure the ability to reason, solve problems, argue, and communicate (Hidayati et al., 2020; Maulyda et al., 2019).

Many factors can cause the low problem-solving ability of the students, including the number sense ability factor. As we all know, learning mathematics cannot be separated from learning numbers. A person needs to have number sense as a basis for being able to solve more complex mathematical problems (Wulandari et al., 2021). Someone with number sensitivity will be more flexible in using their knowledge of numbers (Wulandari et al., 2020).

Number sense is a person's understanding of numbers and their operations that is flexibly used to develop strategies for problem-solving related to numbers (Gürefe et al., 2017; Maghfirah & Mahmudi, 2018; McIntosh et al., 1992; Wulandari et al., 2020, 2021). Meanwhile, according to NCTM (2000), number sense is related to a person's ability regarding the magnitude of numbers and their combinations. Furthermore, McIntosh et al. (1992) revealed that number sense refers to an intuitive sense for number estimation, the ability to calculate carefully and efficiently to detect errors and to recognize feasible results in a calculation. Good number sense skills make it easier for someone to learn various topics in mathematics (Cirino et al., 2015; Clements & Sarama, 2007). In addition, someone with good number sense skills will have high self-confidence in answering problem-solving questions (Byrnes & Wasik, 2009; Maloney & Beilock, 2012). In other words, someone who masters numbers well will be able to understand and use numbers effectively in everyday life.

Number sense has several characteristics. Singh (2009) stated that fluency in estimating and determining the magnitude of a number, flexibility in performing mental calculations, the ability to identify impossible calculation results, and the ability to comprehend and use various representations of numbers and operations are some traits of number sense. Someone who has good number sense has these characteristics. In addition, differences in each person's number sense ability can also be caused by the development of a person's number sense ability that goes hand in hand with age and the acquisition of experience and knowledge in daily activities (Schneider et al., 2017). Furthermore, number sense is grouped into 3 components, namely: (1) understanding and skills about numbers, (2) understanding and skills in using number operations, and (3) applying number skills and operations in calculations (Mcintosh et al., 1997).

The results of research conducted by Louange & Bana (2005) show that there is a strong correlation between number sense ability and problem-solving. Based on the results of observations of teachers and students who were research subjects, these results also confirmed, namely that performance in Problem-solving depends on students' proficiency in number sense. In addition, NCTM (2000) states that number sense and problem-solving are crucial in learning mathematics. Based on this explanation, it can be concluded that a well-developed number sense will positively impact students' mathematical logic intelligence, which can support their math problem-solving skills (Kamsari & Winarso, 2018).

▪ METHOD

Research Design and Procedure

A qualitative research strategy with descriptive research type to collect the data (Sugiyono, 2009). This study aims to describe how number sense ability and problem-solving skills were used by mathematics education students with a lack of number sense ability to solve linear programming problems using simplex methods. In this study, the research instrument was first developed, followed by a validation process involving two mathematics education lecturers at the Faculty of Teacher Training and Education (FKIP), Mataram University. Once the instrument was deemed valid, it was then used for data collection in the research. This research yielded an instrument with favorable attributes.

The research implementation process begins with administering a number sense ability test to categorize research subjects based on their number sense abilities. This is followed by providing a linear programming problem-solving test using the simplex method. Furthermore, to obtain in-depth and comprehensive information and to verify the results of the linear programming problem-solving test completed by the subjects, the researcher conducted interviews.

Participants

This research was conducted at faculty of teacher and training, University of Mataram involving 36 of 3rd year mathematics education students as a participant with purposive sampling. The purposive sampling method was to select subjects who met the criteria of low number perception ability. Then, in the interview stage, at least 2 students from each category of number sense ability are selected for further confirmation through interviews.

Instruments

The research stages were carried out by giving a number sense ability test then followed by a linear programming problem-solving test and interviews. Before data collection, the research team prepared a series of research instruments including a 30-item number sense test. This item tests were a modified number sense test based on Mcintosh et al., (1997) to assess students' sensitivity to numbers and the indicators of this test are shown in the following Table 1. Also, there was a 2-item linear programming problem-solving test and an interview guide. Subsequently, the instruments were validated involving two lecturers from the Mathematics Education Program to assess the appropriateness of the instruments developed by the research team.

Table 1. The distribution of items according to the component of number sense

Components	Question Number
Knowledge of facility with number	1. 2. 3. 5. 6. 7. 9. 20. 27
Knowledge of facility with operations	4. 10. 12. 13. 22. 16. 21. 23. 24. 29
Applying knowledge of facility with number and operations to computational settings	11. 14. 15. 17. 18. 19. 25. 26. 28. 30

Meanwhile, here is an example of the given problem-solving question:
"A jewelry store produces necklaces and bracelets using gold and platinum. The store has 18 ounces of gold and 20 ounces of platinum available. Each necklace requires 3 ounces of gold and 2 ounces of platinum, while each bracelet requires 2 ounces of gold and 4 ounces of platinum. The demand for bracelets is no more than 4 units. The profit from one necklace is $300, and the profit from one bracelet is $400. The store wants to determine the quantities of necklaces and bracelets to maximize profit. Based on the conditions above, then:
a. Formulate a linear programming model for this problem.
b. Solve this model using the simplex method."

This question will be described with Polya's indicators such as: Understanding the problem; Dividing a plan; Carrying out the plan; and Looking back.

Data Analysis

Data processing of test results and interviews in this study will be carried out after researchers receive all the necessary data and have been systematically organized. The data collected will be analysed using interactive data analysis (Miles & Huberman, 1992). The stages carried out include data reduction, data presentation, and conclusion drawing/verification. In other words, at the end of the research process, qualitatively the overall test results that have been processed are described to obtain an overview of linear programming problem-solving using the simplex method given to students with low number sense ability.

▪ RESULT AND DISSCUSSION

Understanding the Problem Stage

After understanding the problem, as represented in Figure 1, subject S-30 wrote down what information was given in the problem using the help of a table. However, this information was incomplete, including the absence of profit information for each necklace and bracelet sold. Besides this will be the objective function of the problem. Subject S-30 also did not include the third constraint information, namely "the demand for bracelets is no more than 4" in the table. Through the interview, Subject S-30 said that the information from the problem was understood and Subject S-30 knew that this linear programming problem asked to determine the maximum profit from the production. In other words, Subject S-30 collected information from the problem but did not do the stage of understanding the problem well. As a result, there was information that was missed and made the problem-solving process inappropriate until the end.

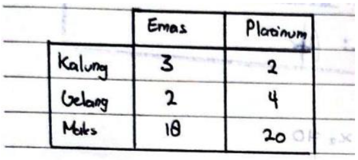


Figure 1. Answers to subject s-30 at the understanding the problem stage

Furthermore, due to the incomplete information at the understanding the problem stage, the results of the planning carried out were also not correct. Subject S-30 did not write down the information about "the requests for bracelets are no more than 4" at this stage. In the interview excerpt above, it is also known that Subject S-30 mentioned that the information was not used in the simplex method, but was used to check the final answer. Subject S-30 did not realize that the information was still part of the constraints in the given problem.

In the other hand, Subject S-23 did not write down any information given in the problem. Through interviews, Subject S-23 said that the information from the question was understood and that Subject S-23 knew the availability of each raw material to produce jewellery in the form of necklaces and bracelets and the limitations of each production. Besides that, Subject S-23Subject S-23 also knew that the problem asked to determine the maximum profit from the production. Thus, Subject S-23 immediately wrote the mathematical model of the linear programming problem given. However, this modelling was not correct. The correct inequality sign for both constraints is “$\leq$". Moreover, the information stated is not complete. In other words, Subject S-23 collected information from the question. However, S-23 did not carry out the stages of understanding the problem well and immediately made a plan. As a result, there is information that is missed and makes the problem-solving process inaccurate until the end.

At this stage, students with the low number sense ability category did not write the information that should be in the understanding problem stages. Some students did not write down what information was given in the problem and some students wrote in the table as a representation, but were incomplete. Through interviews, students said that the information from the problem was understood so that they could immediately write it down in the mathematical model of the given linear programming problem. When information is not understood completely, it is very possible for errors to occur in solving problems (vanLehn, 2011). Also, representation is employed to assist in remembering, understanding, reasoning, and communicating about objects and relationships among items depicted spatially (Zwartjes, 2018). However, this modelling was not yet correct and the information written down was not entirely written, including the information regarding "the demand for bracelets is not more than 4" which was missed. From the interviews, students do not realize that the information is still part of the constraints in the given problem. In other words, students collect information from the question by only reading the question but do not understand the question well and immediately carry out the planning stage. As a result, there is information that is missed and makes the problem-solving process inaccurate to the end. The importance of the information collection stage is that if even one piece of information is missing, it can affect the entire step of solving the maths problem. However, this phase gains significance as it paves the way for subsequent stages. The more comprehensive the understanding of the problem at this stage, the simpler it becomes to address and resolve it later on (Kusuma et al., 2024).

Dividing a Plan Stage

At the stage of dividing a plan shown in Figure 2, Subject S-30 first made an example in a written mathematical model. Next, Subject S-30 changed the previously created mathematical model into standard form and Subject S-30 made a plan to test whether the linear programming problem was simplex feasible or not before determining a solution using the simplex method. However, it should be noted that the process carried out by Subject S-30 was correct before using the simplex table, but the work results were not correct due to the incomplete constraints of the problem given and errors in providing additional variables when creating the standard form. It should be enough to add the slack variables, namely x_3 and x_4, but Subject S-30 did not check the feasibility through the matrix and vector space after adding the slack variables. Subject S-30 immediately added artificial variables in the form of x_5 and x_6. As a result, the solution method used must involve the Big-M method. Even though the given linear programming problem can be solved using the ordinary simplex method. This will certainly influence determining the optimal solution to a given linear programming problem.

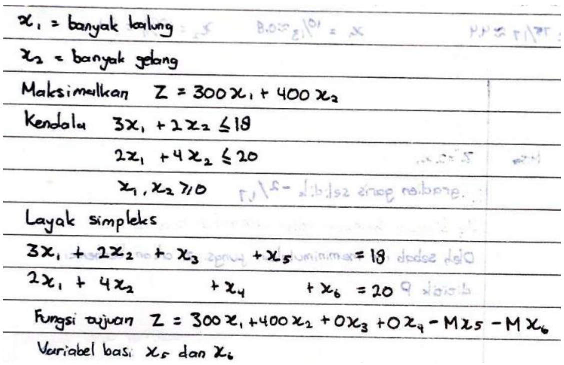


Figure 2. Answers to subject s-30 at the planning stage

Besides, as shown at Figure 3, by missing the understanding the problem stages, the results of the dividing a plan were also incorrect. Subject S-23 did not write down the information about "the request for bracelets is no more than 4" at this stage. Subject S-23 also stated that this information was not used in this simplex method when confirmed by interviews. Subject S-23 did not realize that the information was still part of the constraints in the given problem. Subject S-23 also did not make a simile first in the mathematical model written down, so the variables x_1 dan x_2 were not known to represent certain information. In addition, the variable restriction “x_1,x_2≥0” was also not written by Subject S-23.

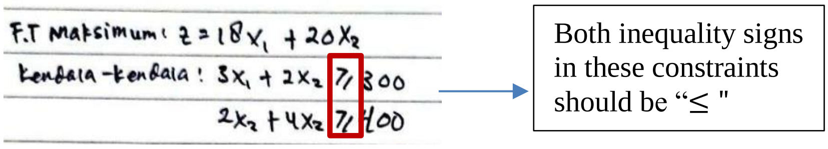


Figure 3. Answers of subject s-23 at the dividing a plan stage (part i)

In addition, the dividing a plan stage seen in Figure 4 also shows that Subject S-23 changed the mathematical model made previously into a standard form. Furthermore, Subject S-23 made plans to test whether the linear programming problem was simplex feasible before determining the solution using the simplex method. However, it should be noted that the process carried out by Subject S-23 was correct before using the simplex method, it is just that the work results were inaccurate due to the incomplete constraints of the given problem and errors in determining the inequality sign on the constraints. Consequently, the solution method used must involve the Big-M method. Even though the linear programming problem given is sufficient to be solved using the usual simplex method, this will certainly affect determining the optimum solution of the given linear programming problem.

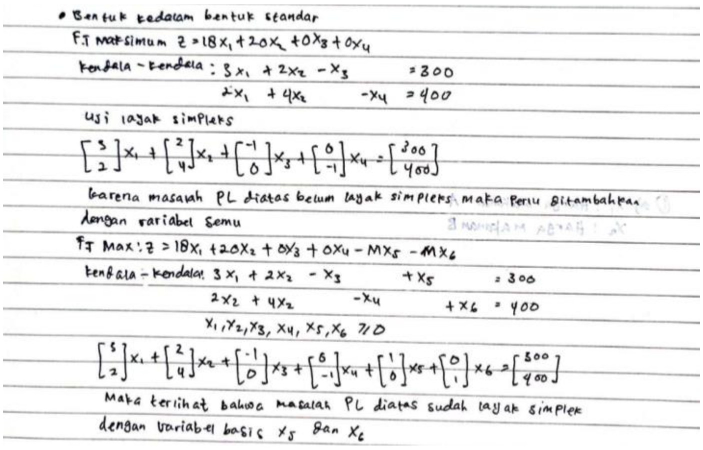


Figure 4. Answers to subject s-23 at the planning stage (Part II)

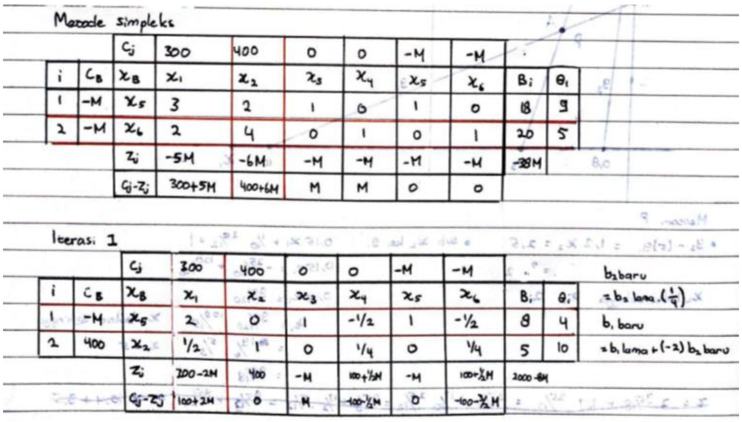
Based on Figure 4, it can be said that Subject S-23 was able to develop a plan to solve the given linear programming problem following the stages in solving linear programming problems using the simplex method. However, this process still obtained incorrect results due to errors during the understanding the problem stage.

In general, because the first stage is missing, the results of the dividing a plan were also not appropriate. Some students did not make a supposition first in the mathematical model written down. Thus, variables used were not known to represent specific information, and there was no restriction on the value of the variables. However, some students write suppositions to be used on the mathematical model. Furthermore, this stage also shows that students can change the mathematical model made previously into a standard form and then test whether the linear programming problem is suitable for simplex before solving it using the simplex method. However, it should be noted that the process carried out by students was correct before using the simplex method. It's just that the work results were not accurate due to the incomplete constraints of the given problem, errors in determining the inequality sign on the constraints, and errors in providing additional variables, namely quasi variables, when making the standard form. As a result, the solution method used must involve the Big-M method. Even though the given linear programming problem can be solved using the ordinary simplex method. It certainly will influence determining the optimal solution to a given linear programming problem. This is consistent with research findings by Star & Rittle-Johnson (2008) which indicate that collected information can serve as clues for using more efficient strategies. It encourages various ways to help students recognize, positively evaluate, and implement efficient steps when asked to solve problems.

However, there were still errors in determining the initial base variable due to several things, namely the lack of information used as a mathematical model, the wrong sign of the inequality in the constraints, the appearance of quasi-variables, and conducting incomplete testing stages to ensure the feasibility of the standard form of the linear programming problem. Therefore, an error occurs in determining the initial base variable. Students utilize the information at their disposal but struggle to establish connections between these pieces of information, leading to irrelevant conclusions (Chick, 1998)

Carrying Out the Plan

Based on Figure 2, it also shows the work of Subject S-30 at the stage of dividing a plan. Subject S-30 carried out incomplete testing stages to ensure the feasibility of the standard form in the linear programming problem being prepared. Thus, an error occurs in determining the initial base variable due to the artificial variables that appeared. After conducting the feasibility test, Subject S-30 continued the stage of dividing a plan by performing the Big-M method. However, Subject S-30 was also only able to perform up to the first iteration due to running out of time when processing the value at the Elementary Row Operation stage from the iteration 0 table to be filled in the first iteration table. This also implies the inflexibility of Subject S-30 in performing a number-counting operation. Therefore, it took a longer time to calculate each element needed. Although Subject S-30 was able to write the Elementary Row Operation formula used in the first iteration table. This explanation can be seen more clearly in Figure 5 below.


Metode simpleks

		C_j	300	400	0	0	-M	-M		
i	C_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	B_i	θ_i
1	-M	x_5	3	2	1	0	1	0	18	9
2	-M	x_6	2	4	0	1	0	1	20	5
		z_j	-5M	-6M	-M	-M	-M	-M	-38M	
		$C_j - z_j$	300+5M	400+6M	M	M	0	0		

Iterasi 1

		C_j	300	400	0	0	-M	-M			b_2 baru
i	C_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	B_i	θ_i	$= b_2$ lama $.(\frac{1}{4})$
1	-M	x_5	2	0	1	-1/2	1	-1/2	8	4	b_1 baru
2	400	x_2	1/2	1	0	1/4	0	1/4	5	10	$= b_1$ lama $+ (-2)\, b_2$ baru
		z_j	200-2M	400	-M	100+½M	-M	100+½M	2000-8M		
		$C_j - z_j$	100+2M	0	M	-100-½M	0	-100-½M			

Figure 5. Answers to subject s-30 at the implementing plan stage

Furthermore, Figure 4 also shows the results of Subject S-23's work at the carrying out the plan stage. Subject S-23 carried out the testing stage to ensure the feasibility of the standard form of the linear programming problem. Even though the feasibility conclusion was correct. However, errors still occur in determining the initial base variable due to a lack of information and errors in inequality signs made in the first stage. In other hand, after carrying out the feasibility test, Subject S-23 continued the stages of implementing the plan by using the Big-M method. However, Subject S-23 could only carry out the first iteration due to running out of time when processing the values at the Elementary Row Operation stage from iteration table 0 to then be filled in the first iteration table. It also implies the inflexibility of Subject S-23 in carrying out a number calculation operation. Therefore, it takes longer to calculate each element needed. Even though Subject S-23 can already write the Elementary Row Operation formula used in the first iteration table, the exposure can be seen more clearly in Figure 6 (a) and (b) below.

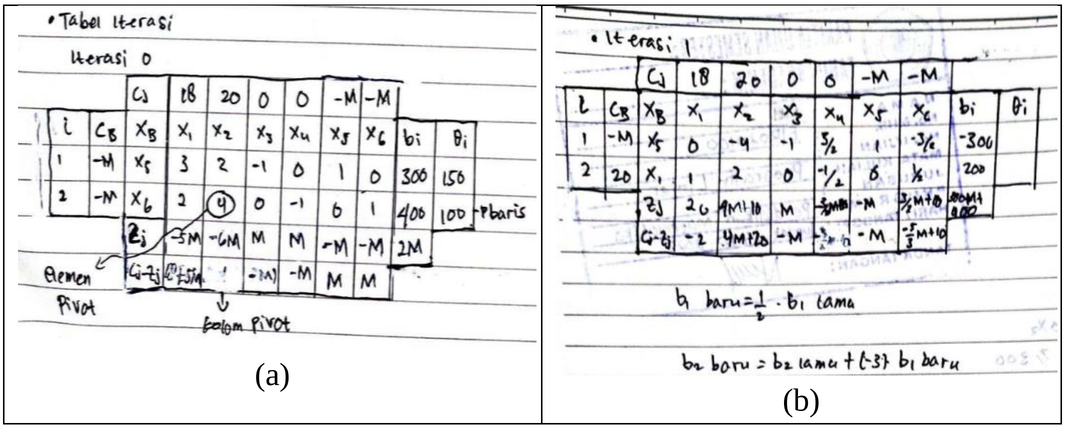


Figure 6. (a) Answers to subject s-23 at the implementing plan stage part i; (b) answers to subject s-23 at the implementing plan stage part II

Second, students carried out the Big-M method only up to the first iteration because they ran out of time when processing the values at the Elementary Row Operation stage from the 0-iteration table to be filled in the first iteration table. This implied inflexibility in carrying out number calculation operations that involve large numbers. It would take a longer time to calculate each element needed. Although, the students can write the Elementary Row Operation formula used in the first iteration table. Students with low number sense ability tend to be reluctant to do problem-solving if they feel it requires a long calculation process (Nugraha & Mulhamah, 2017). Besides that, the students with low number sense skills also tend not to be interested in calculations involving numbers (Carlyle & Mercado, 2012).

Looking Back Stage

Through the interview, it was found that both of them, Subject S-30 and Subject S-23, did not perform the stage of looking back the answers written down. Subject S-23 mentioned that the stage was not applied because the iteration had not finished. In other words, Subject S-23 did not check every step of the work done. So, it can be concluded that students with low number sense ability categories did not do the looking back phase.

The students stated they did not complete this stage because they had not completed the iteration. In other words, students do not check every step of the work. The tendency for someone not to check the answer anymore is because they feel confident with the process that they have worked on. It is true that Setiawan's et al., 2021 research shows that students are still relatively low in validating the conclusions drawn.

In general, it said that students with a low number sense ability category still cannot do problem-solving appropriately. The stages of problem-solving that did not do well are the understanding the problem, dividing a plan, and carrying out the plan stage. Furthermore, students also did not perform the looking back stage. Besides that, students are also not flexible in operating numbers, especially when large numbers are involved. As a result, it takes more time to perform the Elementary Row Operation and the iteration process in the simplex method cannot be completed properly. In addition, number sense is crucial to solving math problems (Miftahurrohmah et al., 2020). That's because a person's problem-solving ability by number sense which includes a person's ability to think creatively, rationally, effectively, and flexibly (Bütüner, 2018; Nugraha & Mulhamah, 2017). It might be said that low number sense ability can lead to low solving ability in students.

▪ CONCLUSION

Students with low number sense abilities solve linear programming problems using the simplex method with the following steps: 1) At the understanding the problem stage, students collect information from the question by reading but do not write the information that should be and do not understand the problems well and immediately carry out the planning stage. As a result, there is information that is missed and makes the problem-solving process inaccurate until the end. 2) In the dividing a plan stages, students do not make a permutation before making a mathematical model and immediately change it to a standard form for the simplex feasibility test. Moreover, there are no restrictions on variable values. 3) At the stage of carrying out the plan, it was not correct due to an error in the inequality sign in the constraints, and there was an error in providing additional variables when carrying out the simplex feasibility test. As a result, an error occurs in determining the initial basis variable, and the solution method used must involve the Big-M method. Even though the given linear programming problem can be solved using the ordinary simplex method. 4) The students with the low number sense ability category did not perform the looking back stage. It is due to the unfinished iteration process and not checking every step of the work done. In general, it can be stated that students with a low number sense ability category are still unable to solve problems accurately. The stages of problem-solving that did not work well were the understanding the problem, dividing a plan, and carrying out the plan stage. Furthermore, students also did not perform the looking back stage. Besides that, students are also not flexible in operating numbers, especially when large numbers are involved. As a result, it takes more time to perform the Elementary Row Operation and the iteration process in the simplex method cannot be completed properly.

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