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Unlocking Mathematical Creativity: How Students Solve Open-Ended Geometry Problems

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Abstract: Mathematical creativity has become increasingly significant in education, emphasizing originality, innovative solutions, and informed decision-making. However, a notable research gap exists in understanding how junior high school students creatively solve open-ended geometry problems. This study addressed this gap by exploring how students tackle such problems and constructing a mathematical creative process model. The research involved eight 7th-grade students from a public junior high school in North Kalimantan, Indonesia. A qualitative research approach, a case study strategy, was employed, utilizing observations, students' answer sheets, and interview-based tasks to gather detailed insights into the students' problem-solving processes. We implemented replicating the finding strategy and considered saturation to enhance the research quality. The findings revealed a six-phase model of the mathematical creativity process: reading, problem selection, and exploration; experiencing perception changes; looking for and generating ideas; undergoing incubation; implementing ideas; and verifying solutions. Self-regulation emerged as a crucial factor influencing student engagement and success in the creative process. Notably, the most creative student in this study demonstrated active actions during problem-solving through all phases, underscoring the importance of self-regulation. The study concludes that self-regulation and also incubation are pivotal in creative problem-solving. These insights provide valuable guidance for educators and researchers aiming to enhance mathematical creativity in the classroom, emphasizing the need for strategies that support self-regulation and innovative problem-solving abilities.

Keywords: geometry, mathematical creative process, open-ended problems, case-study.

▪ INTRODUCTION

Mathematical creativity has garnered increasing attention recently (Hetzroni et al., 2019; Joklitschke et al., 2022; Leikin & Guberman, 2023). Mathematical creativity is a cornerstone for nurturing innovative generations in global education, including in countries like Indonesia (Suherman & Vidákovich, 2022). Fostering this as a form of thinking or knowledge is crucial for further developing students' mathematical abilities and understanding. When students can approach mathematical situations fluently, flexibly, insightfully, and originally, they become more competent in using their mathematical knowledge and problem-solving skills in various mathematical tasks and challenges (Kattou et al., 2013). Newton et al. (2022) emphasize that mathematics is only complete with creative thinking and implies that by enhancing students' mathematical creativity, the overall mathematical ability of students can also be improved. In these regards, some researchers emphasize the importance of investigating creative processes among school students to identify effective ways to foster mathematical creativity (Bicer & Bicer, 2022; de Vink et al., 2022; Schindler & Lilienthal, 2020; Subanji et al., 2023). An individual's mathematical creative thinking is rooted in arriving at mathematical ideas, and improving mathematical creativity involves understanding this creative process (Pitta-Pantazi et al., 2018). However, despite the growing interest in mathematical

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creativity, a notable research gap exists in understanding how junior high school students engage in the creative process when solving open-ended geometry problems.

The definitions of mathematical creativity vary among researchers, and there is no universally accepted definition of mathematical creative thinking ability (Bruhn & Lüken, 2023; Haavold & Sriraman, 2022; Joklitschke et al., 2022; Pehkonen, 2019). In this research, we define mathematical creative thinking ability (MCTA) as students' ability to solve mathematical problems accurately, fluently, flexibly, originally, and elaboratively (Bruhn & Lüken, 2023; de Vink et al., 2022; Joklitschke et al., 2022; Kattou et al., 2013; Suherman & Vidákovich, 2022). Accurate means in line with mathematical ideas. Fluent refers to students' ability to generate various precise and complete answers in solving mathematical problems. Flexibility refers to students' ability to change their thinking path when encountering difficulties in solving mathematical problems, enabling them to find a solution. Original is an indicator of mathematical creative thinking seen in a student's ability to present answers unique or unfamiliar to other students when solving mathematical problems. Elaborative refers to the ability to process the method in detail for the mathematical problem. Furthermore, mathematical creative process is the stage conducted or experienced by students when solving open-ended problems to provide accurate, fluent, flexible, original, and elaborative solutions.

Open-ended problems, which allow for multiple solutions and require creative thinking, are considered suitable for assessing students' mathematical creative thinking (Bruhn & Lüken, 2023; de Vink et al., 2022; Meier & Grabner, 2022; Pehkonen, 2019; Schindler & Lilienthal, 2020; Suherman & Vidákovich, 2022). However, our preliminary study in a public junior high school in Indonesia revealed that students may find open-ended problem-solving challenging, particularly in abstract topics like geometry. While previous research on open-ended problem-solving has focused primarily on the end product (solutions), little attention has been given to the process leading to creative insights (Bicer & Bicer, 2022; Schindler & Lilienthal, 2020). In these regards, inquiring into how students in that school tackle open-ended geometry problems guides our research.

Some researchers have investigated the mathematical creative process in school mathematics: Schindler and Lilienthal (2020), de Vink et al. (2022), Bicer and Bicer (2022), and Subanji et al. (2023). However, in their qualitative investigation, they need to integrate replicating the finding[s] (Miles et al., 2014) or considering saturation to examine their finding[s] (Charmaz, 2014). In this regard, research on the mathematical creative process, particularly in geometry, is still needed. Integrating these strategies to examine qualitative research conclusions is highly beneficial for improving research quality. Hence, to address this gap, our qualitative research integrates replicating the findings to examine data saturation.

▪ METHOD

This research used an exploratory approach, making it appropriate for qualitative research (Creswell, 2018). Qualitative research serves as a means to explore and understand the perspectives of individuals or groups on specific issues (Creswell & Poth, 2016). Given the limited scope of the unit of analysis, a case study strategy was chosen (Creswell & Poth, 2016). It is important to note that the school's identity remained confidential, as agreed upon in the research ethics protocol.

Participants

The study involved eight 7th-grade students from a public junior high school in North Kalimantan, Indonesia, selected during the second semester of the 2022/2023 academic year. The school's 7th grade comprised nine classes, from Class 7.1 to Class 7.9, with 265 students in total. Participants were chosen purposively based on the criteria (Creswell & Poth, 2016): communication and mathematical abilities. Participants were required to be communicative and consisted of one high-achieving student and one lower-achieving student from different classes based on recommendations from the mathematics teachers (there were two mathematics teachers at this school). The participants were recruited in four iterations until data saturation (Charmaz, 2014), so the total number of participants was eight students (four pairs).

Table 1. Participant groups, codes, abilities, classes, and sexes

Recruitment	Class	Participant Code	Mathematics Ability	Sex
Recruitment 1	7.9	P#1	High	Female
		P#2	Low	Male
Recruitment 2	7.8	P#3	High	Female
		P#4	Low	Male
Recruitment 3	7.4	P#5	High	Female
		P#6	Low	Male
Recruitment 4	7.1	P#7	High	Female
		P#8	Low	Male

Procedure

This qualitative inquiry implemented a case study approach, requiring the limitation of the unit of analysis (Merriam & Tisdell, 2016). The unit of analysis included all 7th-grade students at the selected public junior high school in North Kalimantan. However, the study focused on eight students from the 7th grade, as data saturation was achieved with this number of participants (Charmaz, 2014). This study consisted of multiple stages with iterations (see Figure 1). It is important to note that in the qualitative research paradigm, data collection and data analysis can be carried out simultaneously for certain stages of data collection (Miles et al., 2014).

The first step in data collection was recruiting participants. Recruitment began with two students from Class 7.9, selected based on recommendations from their mathematics teacher, which was aligned with the school's mathematics schedule. (Although the study was not conducted during regular mathematics lessons or in regular classrooms, school authorities requested that data collection be coordinated with the mathematics lesson schedule). The teachers made selections according to criteria specified by the researchers, focusing on communication and mathematical abilities, pairing one high-achieving student and one lower-achieving student.

Tasks were assigned to the selected participants, and observations were conducted as students worked on these tasks. Researchers documented all activities and later confirmed these observations during interviews. Immediately after task completion, interviews were conducted. Researchers confirmed participants' answer sheets and notes during these interviews and asked questions based on a pre-prepared interview guide. The interviews were flexible and evolved based on participants' responses.

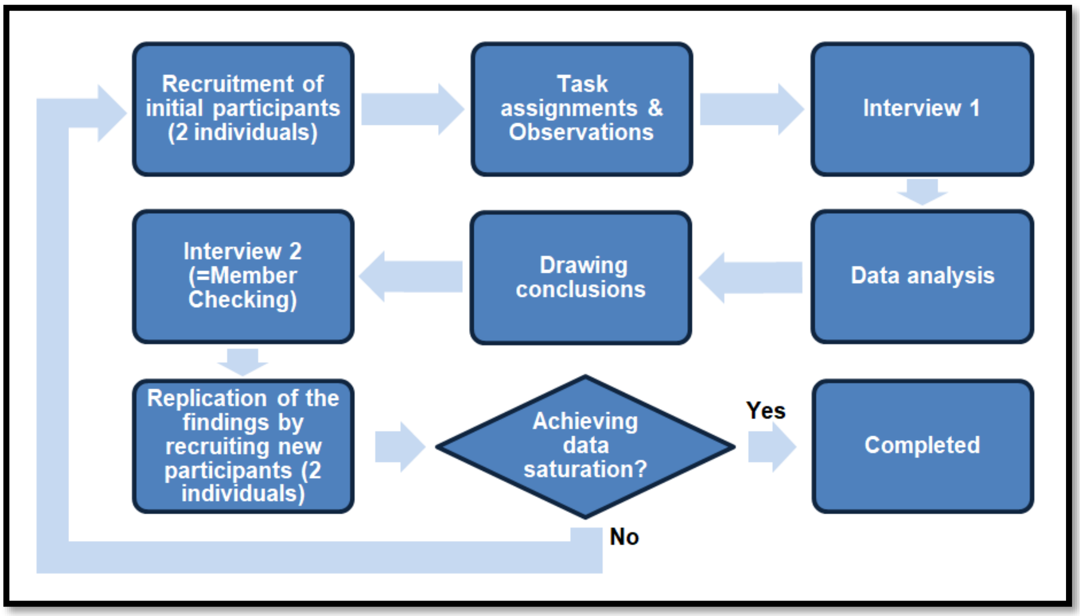
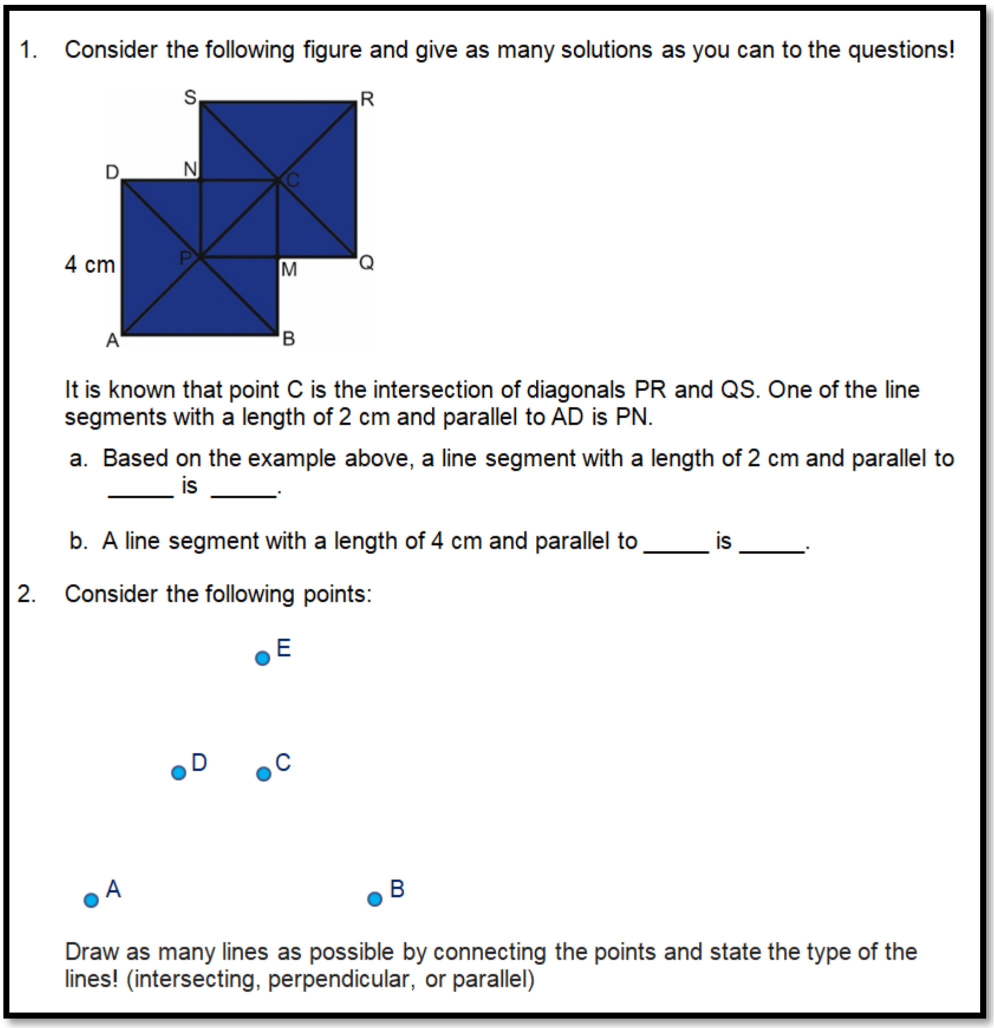


Figure 1. Visualization of the research procedure

Collected data from observations and the first interview (Interview 1) were analyzed using thematic analysis (Creswell, 2018; Nowell et al., 2017). Themes related to the mathematical creative process were identified and categorized into various phases. These themes were examined during the second interview (Interview 2) for member checking. Researchers revised the conclusions based on the member-checking interview if any inappropriate interpretations, new codes, or themes were found (Nowell et al., 2017). After the initial analysis and member checking, the procedures were replicated by recruiting new participants, following the idea of iterative recruitment suggested by Miles et al. (2014) as a strategy to verify the drawn conclusion. This iterative process continued with new recruitments from different classes (7.8, 7.4, and 7.1) until data saturation was achieved in the fourth participant recruitment, the last participants from Class 7.1. In total, three replications were necessary to reach data saturation.

Instruments

This study utilized three primary research instruments: task assignments, observations, and semi-structured interviews to explore the creative problem-solving abilities of 7th-grade mathematics students when solving open-ended geometry problems. However, in the light of qualitative research, the key instruments in this research were the researchers themselves (Creswell & Poth, 2016).


1. Consider the following figure and give as many solutions as you can to the questions!

It is known that point C is the intersection of diagonals PR and QS. One of the line segments with a length of 2 cm and parallel to AD is PN.

a. Based on the example above, a line segment with a length of 2 cm and parallel to _____ is _____.

b. A line segment with a length of 4 cm and parallel to _____ is _____.

2. Consider the following points:

Draw as many lines as possible by connecting the points and state the type of the lines! (intersecting, perpendicular, or parallel)

Figure 1. Tasks: open-ended geometry problems

Tasks

Two open-ended mathematics problems were designed to assess students' MCTA:

- Problem 1: Focused on creative problem-solving related to the segment of parallel states of lines.
- Problem 2: Focused on identifying the types of line relationships: intersecting, perpendicular, or parallel (see Figure 1).

In line with our conceptualization in the introduction, each problem included five indicators to measure MCTA:

- Accurate: Participants should provide solutions consistent with mathematical principles.
- Fluent: Participants should generate various precise and complete answers to the problems.
- Flexible: Participants should adapt their thinking paths when encountering difficulties, enabling them to find solutions.
- Original: Participants should present unique or unfamiliar answers compared to other students.

- Elaborative: Participants should detail their methods for solving the problems.

These tasks were based on fundamental geometry, focusing on lines and angles, but they were new to the students, ensuring they were non-routine. The tasks were collaboratively created by an assessment expert, Author 1, Author 2, and the school's mathematics teachers. Students' answer sheets when dealing with the tasks were utilized to gather our data regarding MCTA.

Observation Sheets

As participants worked on the tasks, observations were conducted to document their problem-solving processes. To facilitate this data collection technique, we prepared observation guidelines focused on how participants tackled the open-ended problems, including their reading, analyzing, exploring, planning, implementing plans, and verifying solutions activities (Schoenfeld, 2022).

Semi-Structured Interviews Guidelines

Semi-structured interviews (Creswell & Poth, 2016) provided flexibility in question formulation and occurred immediately after task completion. Each participant underwent two interviews:

- Interview 1: Addressed the problem-solving done by participants. In this interview, we prepared guidelines to confirm and explore our main observational aspects: reading, analyzing, exploring, planning, implementing plans, and verifying solutions. Researchers cross-verified answer sheets and observation notes during this interview.
- Interview 2: Aimed at verifying the themes that emerged from the initial data analysis. Participants were asked to reflect on and confirm their experiences with mathematical creative processes, such as reading, exploring problems, experiencing perception changes, obtaining ideas, implementing ideas, verifying solutions, and self-regulation.

These instruments—tasks, observations, and interviews—were crucial for collecting comprehensive data on students' mathematical creative processes. Their combined use ensured a robust and in-depth exploration of the research questions, enhancing the study's validity and reliability (Merriam & Tisdell, 2016).

Data Analysis

The qualitative data analysis in this study followed Creswell's model (2018). Transcriptions of verbatim interviews and field notes were meticulously scrutinized to enhance the researchers' understanding of the collected data (Nowell et al., 2017). Data coding was performed iteratively, with text segments labeled by keywords or phrases to develop focused codes (Creswell, 2018). These codes were then grouped into themes, with each code rigorously examined for consistency with the actual data (Creswell & Poth, 2018). Here is an example of how coding and theme generation were conducted:

Table 2. Example of coding in data analysis

Trancript	Code	Theme
Researcher: "*Earlier, while you were working on the problem, I was observing you. When you moved from problem one to problem two without finishing, why was that?*"	Reading the problems, selecting a problem, studying	Reading, choosing a problem, and

Trancript	Code	Theme
Participant: "*It was because I didn't understand problem one. I wasn't sure what the question was asking. So, I looked at problem two, but I didn't understand that either. So, I went back to problem one and read the information again. Oh... that's how it's done.*" **Researcher**: "*Oh, so after reading problem one and not understanding it, you looked at problem two, but still didn't understand it. Then you went back to problem one and finally understood it?*" **Participant**: "*Yes.*" **Researcher**: "*So, how did you suddenly understand it after reading it several times?*" **Participant**: "*Because I looked at the problem, and it was asking for a specific line segment. Then I checked the information, and that's how it should be done. So, I followed the example.*"	the given example in the problem, reading the example	exploring the chosen problem

Peer debriefing involving Author 1 and Author 2 was employed to verify the developed themes (Creswell, 2018). Member checking was conducted by confirming themes from the initial interviews (Interview 1) during the Interview 2. Data analysis commenced after the first interview with the initial recruitment of participants and continued iteratively throughout the study. Peer debriefing and member checking were used to verify and refine themes, ensuring the robustness of the research findings (Creswell & Poth, 2016).

The data analysis process began with a preparation stage, which involved transcribing interviews and compiling field notes derived from observations. The researchers repeatedly read the transcriptions to ensure a deep understanding of the data. Each participant's statement was labeled with codes reviewed multiple times. Creating themes involved repeatedly reading the coded data to ensure the codes matched the interview excerpts. These codes were then grouped into themes, described in detail. The interpretation of the themes required providing detailed explanations and comparing them with related research literature. A follow-up interview (Interview 2) was conducted to ensure the findings accurately reflected the participants' experiences. This stage also allowed further exploration of the findings, and participants were encouraged to comment on the results.

Replicating the findings was essential for achieving data saturation. This stage involved confirming the findings with new participants from different classes. Participant recruitment was repeated four times, with the same activities as the initial recruitment. During the interviews, we confirmed the findings. Data saturation was indicated when the developed themes and codes recurred consistently, representing the study's final findings. Strategies to replicate findings and achieve data saturation enhanced our research quality (Miles et al., 2014).

Research Quality

The quality of the research in this study focuses on validity and reliability. Validity in qualitative research involves checking the accuracy of the findings using strategies such as triangulation, member checking, peer debriefing, and replication of findings (Creswell, 2018). Triangulation in this study involved confirming the validity of data obtained from observations and students' answer sheets through interviews. Member checking was conducted through follow-up interviews with participants to verify the accuracy of the findings. Researchers provided detailed perspectives and explanations for the developed themes to ensure the study's realism and comprehensiveness. Peer debriefing, involving discussions between authors, enhanced the accuracy of the thematic analysis. Replicating findings by recruiting new participants from different classes to achieve data saturation further strengthened the study's validity and reliability (Miles et al., 2014).

As defined by Creswell (2018), reliability in qualitative research means demonstrating that the approach is consistent. Reliability strategies in this study included listening to audio recordings and repeatedly reading the transcripts. The researcher meticulously developed and reviewed codes to avoid deviations from their intended meanings. Both authors and two external researchers reviewed codes, themes, and descriptions. Before peer review, the researchers explained the study's objectives, provided interview transcripts and coding results, and developed themes. The external researchers reviewed the description of each developed code. Additionally, participants reviewed the developed descriptions of the themes through member checking, ensuring the findings accurately reflected their experiences.

▪ RESULT AND DISSCUSSION

The research findings highlighted students' mathematical creative process when they worked on open-ended problems. We developed a model of the student's mathematical creative process by analyzing observation data, participants' answer sheets, and interviews. In contrast to Subanji et al. (2023), who promote a mathematical creative process model using problem posing, our model is rooted in problem-solving activities. It revealed that the student's mathematical creative process was complex and non-linear, not linear stages as proposed by Wallas (1926). The non-linear nature of the creative process aligns with Bicer and Bicer's (2022) findings that young students' creative processes are unpredictable and influenced by external factors such as teachers, peers, and the environment.

Table 2. Themes of students' mathematical creative process

Code	Phase/Theme	Description	N
Reading the problems, studying figures, studying the example in the given problem, reading the example, selecting a problem	Reading, choosing a problem, and exploring the chosen problem	The student reads given problems thoroughly, selects a problem to work on, and explores the problem.	8
Encountering difficulties, initially finding the problem is challenging, the problem appears easier after comprehension, enthusiastic after understanding the problem	Experiencing perception changes	After understanding the problem, the student changes her/his perception from her/his initial impression of the problem to a more confident stance.	3
Connecting given information with instruction or question, linking the	Looking for and obtaining ideas	The student connects problem-related information	4

Code	Phase/Theme	Description	N
given example or figure with instruction or question, linking problem-solving experience with the given problem, connecting the given figure with example		and links their classroom-acquired knowledge to solve problems.	
Facing challenges in articulating segment or line position, gaining insights from Problem 1, linking the given example in Problem 1 to answers regarding line positions in Problem 2	Experiencing incubation	The student initially struggles but subsequently overcomes difficulties through sudden insights, leading to the generation of diverse solutions.	1
Solving Problem 1, stating the positions or line type of the connected lines, linking examples to the final solution	Implementing ideas	The student implements ideas or insights they have gained to solve the problem.	3
Checking answers by connecting them with the given example, checking all the steps during problem-solving	Verifying the solution	The student verifies the final solutions by associating them with available examples in the problem or checking all the steps in problem-solving.	4
Feeling apprehensive, fearing errors when connecting lines, fearing inadequate answer sheets, feeling uncertain when providing only one answer	Doing self-regulation	The student engages in self-regulation throughout the problem-solving process.	3

Our study identified several vital phases in the mathematical creative process: reading, choosing a problem, exploring the chosen problem, experiencing perception changes, looking for and obtaining ideas, experiencing incubation, implementing ideas, and verifying the solution (see Table 2). Concurrently, students demonstrated their capacity for self-regulation as they navigated these phases. These discerned phases encapsulate the salient themes that emerged from this research. Our mathematical creative process model is generated by combining all phases each participant undergoes.

Comparing various problem-solving models in mathematical creativity, such as those by Wallas (1926), Schindler and Lilienthal (2020), and Bicer and Bicer (2022), our findings reveal both commonalities and distinctions. Wallas' (1926) model, originally not specific to mathematics education but later adopted by mathematics education researchers (Pitta-Pantazi et al., 2018), follows the classic four-stage structure: preparation, incubation, illumination, and verification. Sriraman et al. (2011) note that the creative processes of mathematicians follow the Wallas model. Bicer and Bicer's (2022) model introduces the "inception" phase for the initial generation of ideas, while Schindler and Lilienthal's (2020) model emphasizes finding an entry point into a problem.

Table 2. Mathematical creative process and number of participants

Mathematical Creative Phase	Participants							
	P#1	P#2	P#3	P#4	P#5	P#6	P#7	P#8

Phase 1: Reading, choosing a problem, and exploring the chosen problem	✓	✓	✓	✓	✓	✓	✓	✓
Phase 2: Experiencing perception changes	✓		✓					
Phase 3: Looking for and obtaining ideas	✓		✓				✓	✓
Phase 4: Experiencing incubation	✓							
Phase 5: Implementing ideas	✓		✓		✓		✓	✓
Phase 6: Verifying the solution	✓		✓				✓	✓
Self-regulation	✓		✓				✓	

Our findings offer a fresh perspective by highlighting the “experiencing perception changes” phase during the initial stages of mathematical problem-solving, emphasizing the need to reevaluate and reshape one’s understanding of a problem. This phase, distinctively termed in our research, is pivotal to student success and is not explicitly detailed in the problem-solving or creative processes outlined by Wallas (1926), Schindler and Lilienthal (2020), or Bicer and Bicer (2022). It is closely tied to self-efficacy, significantly influencing students’ actions, expectations, and efforts. Our participants’ experiences illustrate how initial difficulties can give way to confidence as they gain a deeper understanding of the problem.

The self-regulation phase aligns with metacognition, encompassing information-processing abilities, task-handling strategies, monitoring, and self-regulation. This phase, integrating with the aspect of originality, underlines the significance of self-regulation in creative mathematical thinking. Losenno et al. (2020) demonstrate the significance of emotion regulation, specifically cognitive reappraisal, in promoting self-regulated learning in elementary-aged children. It underscores that cognitive reappraisal positively influences all phases of self-regulated learning and that effective enactment of self-regulated learning strategies contributes to improved mathematics problem-solving outcomes. Our finding also aligns with Schoenfeld’s (2020) idea, stating the significance of monitoring and self-regulation in metacognition, where proficient problem solvers continually adjust their strategies and stay open to alternatives.

Participant P#1 completed both Problem 1 and Problem 2, navigating all phases of the mathematical creative process to solve the problems. She was the only participant who experienced the incubation phase, a stage present in the creative process models of Wallas (1926), Schindler and Lilienthal (2020), and Bicer and Bicer (2023) for generating ideas when facing fixation. Despite solving both problems correctly, Participant P#1 provided only one answer for Problem 1, parts (a) and (b), indicating a lack of fluency from the perspective of MCTA.

Participant P#3 also went through all the creative processes in solving the problems, except for the incubation phase. She solved both problems correctly and in various ways, suggesting that mathematical creativity is influenced by both mathematical and general creativity (Leikin & Guberman, 2023; Schoevers et al., 2020). As demonstrated by Participant P#1, individual inspiration can be rooted in this stage, as suggested by Bicer and Bicer (2023) and Levenson (2011).

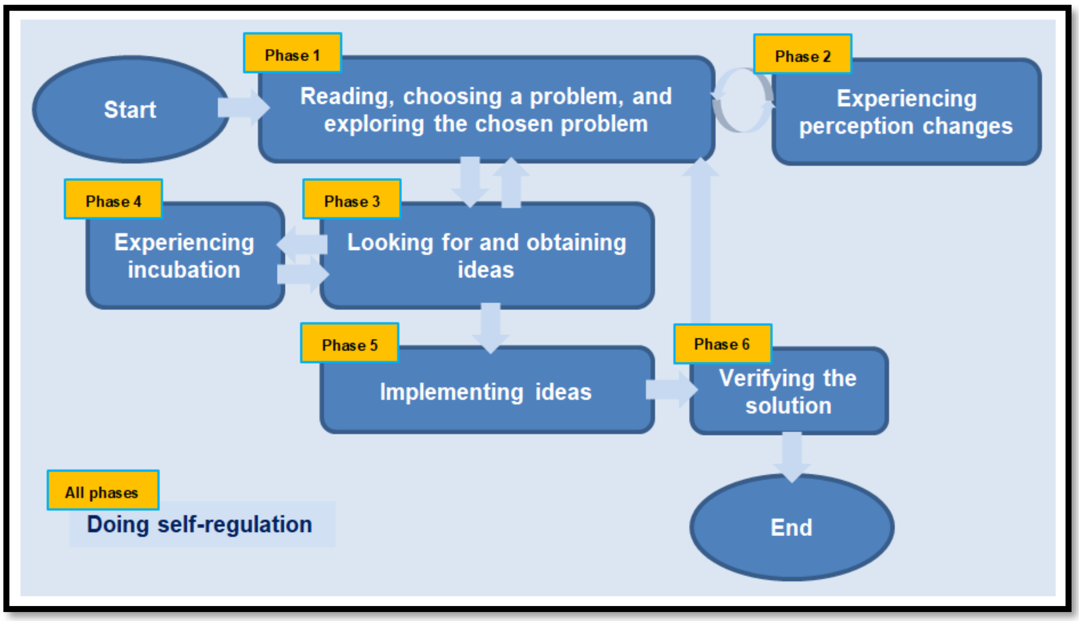


Figure 2. Visualization of mathematical creative process

Participant P#5 went through only two phases: reading, choosing a problem, exploring the chosen problem, and implementing ideas. She correctly answered only Problem 1a. Participant P#7 followed all phases of the mathematical creative process except for experiencing perception changes. Although she provided various solutions for Problem 1, she failed to provide a solution for Problem 2. Participant P#8 was an exception in the low mathematical abilities group. Despite being identified by his math teacher as a low-ability student, his MCTA was much better than that of Participants P#2, P#4, and P#6. His work was even better than that of Participant P#5, who was from the high-ability student group.

These findings underscore the complexity and non-linear nature of the mathematical creative process, as highlighted in previous research (Bicer & Bicer, 2022; Pitta-Pantazi et al., 2018). They also emphasize the role of individual differences and external factors in shaping students' creative thinking in mathematics.

Reading, Choosing a Problem, and Exploring the Chosen Problem

The phase of reading, choosing a problem, and exploring the chosen problem involves students thoroughly reading both problems and deciding which one to work on while studying them. In this phase, students aim to comprehend and explore the problems' problematic situations (Pitta-Pantazi et al., 2018; Schoenfeld, 2022; Sitorus & Masrayati, 2016). Observations indicated that when students first encounter these open-ended problems, they thoroughly read both Problem 1 and Problem 2. When asked, the students explained that they did this to determine which problem was easier to work on by connecting the given problems with their prior knowledge and experiences, particularly in their mathematics lessons with their teachers, similar to Bicer and Bicer's (2022) findings. For instance, in Interview 2, Participant P#4 said: "I read the problems first. I look for which problem is easier to work on. The instructions are here: work on the questions that seem easier. So, I read first (=read each problem)." [P#4, Int. 2] Afterward, students move on to the next step, deciding which problem to work on. Here is a conversation between the researcher and Participant P#5:

Researcher: "Well, when working on Problem 1, then Problem 2, and back to Problem 1. Why?"
Participant: "Because this one looks harder, I did not understand. I want to make sure which one is easier. Turns out, this one (=Problem 1) is easier than this one [Participant pointed Problem 2]." [P#5, Int. 1]

Furthermore, students explored the selected problem. The reading, analyzing, and exploring phase is similar to the preparation phase of the creative process proposed by Wallas (1926) and the looking-for-a-start phase described by Schindler and Lilienthal (2020). Bicer and Bicer (2022) refer to this phase as inception, as it might be the first time these students were asked to solve problems creatively by employing as many solutions as possible. Analyzing the problem, as part of this first phase, involves studying examples or figures available in the situation. For instance, Participant P#8 said, "Here, it is asked to find the side (with a length of) 4 cm. This is 4 cm, right? So, it is half of this. From P to N and M to C." [P#8, Int. 1] Participant P#3 mentioned, "Before solving (the problem), I first look at the picture." [P#3, Int. 1] It means that she was paying attention to the given figure.

Experiencing Perception Changes

The perception changes phase occurs when students shift from their initial impression of a problem to a revised understanding after deeper engagement This phase is not identified in the creative process models of Wallas (1926), Shindler and Lilienthal (2020), and Bicer and Bicer (2022). However, because of its significance in the coded data, this phase is elevated to an independent theme in our study. Initially, participants often found open-ended problems daunting and confusing: "When I read the problems, I thought, 'How do I solve this?'" [P#1, Int. 1] However, after engaging in the initial phase of the mathematical creative process, they discovered that these problems were more manageable than initially perceived: "When I first read it, maybe I did not pay much attention, so (the problem seemed) difficult. However, after rereading it, (I) could do it!" [P#7, Int. 1]

Participants expressed satisfaction and enthusiasm upon understanding the problem and linking classroom-acquired knowledge to the problem. Participant P#3 explained: "At first (when looking at the problem), I felt overwhelmed. It seemed too difficult. The problem was tough. Look! Boxes, pictures, like this, it is hard! After rereading it, I could deal with it!" [P#3, Int. 1] Participant P#3 successfully answered Problem 1 with various solutions and Problem 2, demonstrating how this perceptual shift facilitated effective problem-solving.

Our findings resonate with Meier and Grabner's (2022) research, suggesting that mathematical creativity enhances cognitive abilities and positively influences students' self-efficacy. This phenomenon underscores that mathematical creativity is influenced by cognitive and affective factors (Shindler & Lilienthal, 2020; Bicer & Bicer, 2022). Students' perceptions are rooted in their beliefs about learning mathematics and significantly impact their problem-solving approaches (Schoenfeld, 2022).

Looking for and Obtaining Ideas

The seeking initial ideas for the problem-solving phase is where students integrate problem-related information and connect their acquired class knowledge to formulate solutions. This stage closely aligns with Phase 1 and resonates with Bicer and Bicer's (2023) findings, which suggest that participants draw on prior knowledge and experiences to prepare for problem-solving. This finding is also echoed in Sitorus and Masrayati's (2016) creative process stage, emphasizing that students recall prior knowledge and envision mathematical connections after gathering information and framing problems. Participants in this phase build upon insights gained in the first phase, applying class knowledge or insights obtained earlier to generate problem-solving approaches. For example, Participant P#7 described her approach to solving Problem 1 by connecting the provided example with the question:

[A]ccording to the problem, what was asked is (the side with the length of) 2 cm. So, I followed the pattern. (The segment from) P to N was automatically divided into two, right? So, it was 2 cm. I followed the pattern (=connecting the question with the given example in the problem). [P#7, Int. 1]

Similarly, Participant P#3 relied on class content to tackle Problem 2: "This problem is the same as the material we covered in class!" She derived her solution strategy from her classroom knowledge, requiring additional time to develop her ideas: "Then, (the problem) took a long time for me to work on. Then, I looked at the example above: 'Oh, this is how it is done!'" [P#1, Int. 1] This phenomenon mirrors Schindler and Lilienthal's (2020) findings, where participants experienced "Aha!" moments characterized by sudden clarity and emotional responses.

Participant P#1 uniquely underwent the incubation phase, experiencing illumination as she suddenly grasped how to approach the problems (Pitta-Pantazani et al., 2018). Some participants revisited the first phase, as depicted in the mathematical creative process model. Participant P#3, for instance, was initially perplexed by the letters in Problem 1's picture, prompting a return to the initial phase to clarify understanding.

Experiencing Incubation

Experiencing incubation is when students encounter difficulties in problem-solving but overcome them through sudden insights, leading to diverse solutions. In this phase, the cognitive work occurs subconsciously (Pitta-Pantazani et al., 2018). In this study, only one participant underwent this phase, achieving the highest score in mathematical creativity based on the test. Bicer and Bicer (2023) emphasize that incubation involves students' unconscious, mathematical thought processes, similar to the participants in this study who set aside the current problem and engaged with other tasks.

Participant P#1 experienced this phase when confronted with Problem 2 concerning the relationship between two line segments. "After answering the first one, I thought about working on Problem 2." [P#1, Int. 1] The initial idea to solve the problem emerged from recent math class teachings: "I remembered what the teacher taught yesterday. I was recalling what was conveyed by the teacher. Then, I got an idea." Subsequently, the participant successfully found a solution. Moreover, Participant P#1 had a sudden insight by intuitively connecting an example from Problem 1 with Problem 2:

When I looked at the picture in Problem 1: 'How come? Can the short segment, which is half the length of the longer (segment), be called parallel?' Then, I looked at Problem 2. What is it? Um... This (segment) is shorter too. This (segment) is longer, (both line segments) can also be called parallel! [P#1, Int.2]

Observations revealed that while working on Problem 2, Participant P#1 frequently referred to Problem 1 for new insights. When asked about this, the participant confirmed deriving new ideas from Problem 1:

Researcher: *"How do you get the idea of parallel lines?"*
Participant: *"I got this idea." [Pointing to the Figure in Problem 1]*
Researcher: *"Oh, I see... When you worked on Problem 2, did you always look back at Problem 1?"*
Participant: *"Yes." [P#1, Int. 1]*

The incubation phase is unique, with only one of the eight participants experiencing it in this study. This finding underscores that the process of solving mathematical problems can vary significantly among individuals. It aligns with the perspectives of Wallas (1926), Schindler and Lilienthal (2020), and Bicer and Bicer (2023), all of whom highlight incubation as a critical stage in the creative process. In this study, incubation is interpreted as a spontaneous thought process that generates new ideas without conscious planning, exemplified by Participant P#1's experience.

Implementing Ideas

Implementing ideas or insights is when students apply the strategies they have developed to solve the problem. Participant P#3 explained, "I stated in terms of the definitions, line A is equal to line B, line A is equal to line C, like that." [P#3, Int. 1] This participant used definitions to solve Problem 2 by identifying the types of line positions. Similarly, P#1 described her approach as follows: "After I got the idea, I answered on the answer sheet." [P#1, Int. 1] Once she formulated a solution strategy, she wrote it down. Participant P#8 connected his learned examples from the problem: "This is 4 cm, so it is already here. From A to D, Q to R!" [P#8, Int. 1] This participant utilized examples given in the problem to arrive at a final solution.

This phase involves implementing initial ideas to solve the problem, as observed in our study. Like Schindler and Lilienthal's (2020) findings, our participants verified their solutions step-by-step as they implemented their ideas. This iterative approach aligns with Bicer and Bicer's (2022) step-by-step process, while Wallas (1926) integrates it into the verification phase of creative problem-solving.

Verifying the Solution

The final phase of the mathematical creative process is validating solutions, akin to the models proposed by Wallas (1926), Schindler and Lilienthal (2020), and Bicer and Bicer (2023). This phase involves students verifying their final solutions by comparing them with the examples provided in the problem. Here is an excerpt from a conversation between the researcher and Participant P#1:

Researcher: *"Did you check your work?"*
Participant: *"I did... In the part, um... what is it... I reread the example. Is my answers correct? Like the example or not!"*
[...]
Researcher: *"How do you check it? Do you finish all of them or one by one?"*
Participant: *"Checked one by one." [P#1, Int. 1]*

Participant P#1 validated her solution by comparing it to the given examples, a method similar to that observed in Schindler and Lilienthal's (2020) research, where participants ensured their work by comparing solutions sequentially. Validating the solution was the final step for Participant P#1 after completing each problem. Participants P#3, P#7, and P#8 similarly validated their solutions by revisiting the examples and pictures provided in the problem. "I ensured this answer by looking at the examples and pictures." [P#3, Int. 1] This phase marks the culmination of the participants' efforts to ensure the correctness of their written answers, reflecting their comprehensive understanding of the problem-solving process.

Doing Self-Regulation

Self-regulation is a crucial phase where students manage and control themselves during problem-solving. For instance, in Phase 1—Reading, choosing a problem, and exploring the chosen problem—the data collected, particularly from observations, revealed that participants effectively managed how to approach the given problems by considering time as a limited resource, as emphasized by Schoenfeld (2022) in problem-solving contexts. This activity indicated that students engaged in self-regulation concerning time allocation.

Furthermore, insights from Interview 1 were designed to gather metacognitive data about participants' awareness and understanding of their thought processes during problem-solving. Participants were asked to describe what they were doing, why they were doing it, how it contributed to finding a solution, and its effectiveness (Schoenfeld, 2022). For instance, Participant P#3 explained her approach to Problem 1: *"The problem asked for parallel lines. So, I chose parallel lines. I was uncertain if I provided only one solution, so I wrote down all possible solutions." [P#3, Int. 1]*

This finding illustrates the emotional expressions (affective statements) and the participants' awareness and understanding of their self-regulation, echoing key characteristics observed in the illumination phase of the creative process, as highlighted by Bicer and Bicer (2023). Schindler and Lilienthal (2020) underscore that these characteristics demonstrate the inherently creative nature of the participants' problem-solving processes.

The Process and the Aspects of Mathematical Creative Thinking Ability

MCTA plays a pivotal role in the creative process (Pitta-Pantazi et al., 2018) as students engage with open-ended problems. Mathematical creative thinking involves a series of actions students undertake when approaching open-ended geometry problems, from reading, choosing a problem, and exploring the chosen problem to verifying the solution phase. Meanwhile, MCTA refers to students' capacity for fluency, flexibility, originality, and elaboration when tackling the problems.

Our data indicate that all participants, including those with high mathematical abilities, showed a need for improvement in their MCTA. For example, two top-performing participants, Participant P#1 and Participant P#3, still demonstrated areas for growth. This situation aligns with research findings suggesting that Indonesian students' performance in mathematical problems requiring MCTA falls below the international average (Ramadhanta et al., 2024). Moreover, our research underscores that students have not fully mastered the MCTA indicators, contributing to their overall lower levels of MCTA proficiency (Febrianingsih, 2022).

Participant P#1's work revealed a lack of fluency in dealing with Problem 1, as she provided only one solution for two questions in Problem 1: a) line segment with a length of and parallel to is and 2) line segment with a length of and parallel to is. Here are examples of other solutions for Problem 1:

(a) (line segments with a length of 2 cm and parallel to) AD is NS, BM, or MC.
(line segments with a length of 2 cm and parallel to) AB is PM or NC.
(line segments with a length of 2 cm and parallel to) BC is PN or NS. [....]

(b) (line segments with a length of 4 cm and parallel to) AD is PS or QR.
(line segments with a length of 4 cm and parallel to) PN is AD, BC, or QR.
(line segments with a length of 4 cm and parallel to) MC is AD, PS, or QR [....]

Participant P#1 provided more than one solution for Problem 2, and her solutions were original because they differed from Participant P#3's: 1) CA and DB are intersected, and 2) AB is parallel to DC. Her final answer for question 1b was similar to Participant P#1's, but she connected each pair of points separately. Her works were elaborate because she provided detailed solutions for both problems. She could rephrase the questions or instructions and provide easily understandable answers. Participant P#3 exhibited greater fluency than P#1 when dealing with Problem 1. She was able to provide multiple solutions. Her solutions for Problem 1a were line segments with a length of 2 cm and parallel to AB was PM or MC, and for Problem 1b, line segments with a length of 4 cm and parallel to AD were PS, BC, or QR. Like Participant P#1, she gave elaborative solutions to Problems 1 and 2. However, she provided only one complete solution for Problem 2, similar to Participant P#1's. Here are examples of other solutions for Problem 2: Line EC and DC are perpendicular (at point C); lines AE, CE, and BE are intersected (at point E).

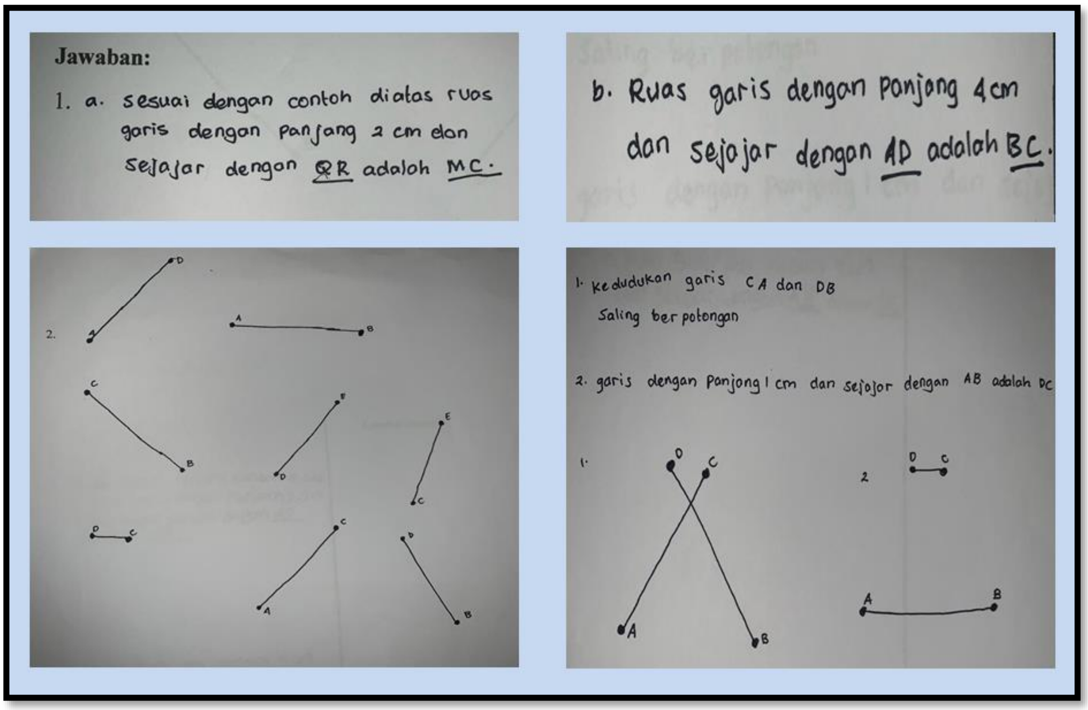


Figure 3. Participant P#1 works

While working on the problems, Participant P#1 faced difficulties when determining the positions of lines. Participant P#1 said:

I also read the part I struggled with (types of lines). I still need to solve a problem like Problem 2. I was still thinking [...] After I drew out the first one (=drawing a line), I thought about drawing out the second one. [P#1, Int. 1]

Participant P#1 attempted to overcome this difficulty by linking the problem with the knowledge they had acquired: "I was thinking about what was taught by the teacher yesterday. I felt like recalling what the teacher said yesterday!" [P#1, Int. 2]. She was able to change her thinking path when encountering difficulties in solving mathematical problems, enabling her to find a solution. This participant got the idea to add her solutions from Problem 1. Participant P#1 linked the solution of Problem 2 with the given example in Problem 1. Furthermore, in Problem 2, Participant P#1 connected the lines separately, which differed from other participants. Based on Participant P#1's answer sheet, she could answer Problem 2 accurately, fluently, flexibly, originally, and elaborately.

To foster mathematical creativity in junior high school, educators should create an environment that encourages curiosity and exploration, as Haavold et al. (2020) highlighted. This objective can be achieved by presenting mathematics as a puzzle-solving endeavor rather than a memorization task. As recommended by the same source, incorporating inquiry-based instruction empowers students to explore mathematical concepts through problem-solving tasks, nurturing creativity in terms of fluency, flexibility, and novelty. Additionally, educators should integrate abductive reasoning and unconventional mathematics problems to stimulate creative thinking, as suggested by various researchers.

Diverse problem types, as mentioned by de Vink et al. (2022), should be incorporated into teaching strategies to enhance creativity further. Assessments should prioritize speed and accuracy and include creative tasks, ensuring the recognition and development of student's creative potential, as proposed by Kattou et al. (2013). Moreover, teachers should guide students through the mathematical creativity process and encourage the exploration of open problems, fostering an open-minded and enthusiastic approach to mathematics (Pehkonen, 2019). By implementing these practices, educators can cultivate mathematical creativity in junior high school students, preparing them for the demands of the modern world. It is crucial to embrace diversity in learning styles and provide tailored support to students with varying mathematical achievements, as de Vink et al. (2022) emphasized. This approach ensures that all students have the opportunity to develop their creative problem-solving skills. Finally, integrating creativity into teaching, assessment, and educational provision across the curriculum, as advocated by Newton et al. (2022), will create a holistic learning environment that nurtures mathematical creativity and equips students with valuable skills for the future.

Nonetheless, our research has limitations. Although we recorded videos of our participants while they were dealing with the tasks, we did not conduct Stimulated Recall Interviews (SRI) during our interviews, as Bicer and Bicer (2022) did, due to the limited time allocated by the school for setting up the research. Future studies could incorporate SRI to enhance data triangulation. Based on our findings, further research should focus on teaching methods to improve students' MCTA. Additionally, incorporating intercoders in data analysis could enhance research reliability (Creswell, 2018). It is also essential to acknowledge that our data source is based on students' experiences and perceptions, which may contain biases. Despite these limitations, our research provides valuable insights into the intricate process of mathematical creativity. By drawing on established models and incorporating novel phases such as "experiencing a change in perception," we enhance our understanding of how students tackle open-ended mathematical problems. Through rigorous quality assurance measures, we strive to contribute to the field of mathematics education and inspire further investigations into this fascinating cognitive journey.

▪ CONCLUSION

Our research highlights the complex, non-linear nature of the mathematical creative process in solving open-ended problems. We developed a model based on problem-solving activities, identifying key phases: reading, choosing a problem, exploring, experiencing perception changes, looking for ideas, incubation, implementing ideas, and verifying solutions. These phases emphasize the importance of self-regulation and metacognition.

Comparing various models of mathematical creativity, such as those by Wallas (1926), Schindler and Lilienthal (2020), and Bicer and Bicer (2022), our findings reveal both commonalities and distinctions. Our model introduces the phrase "experiencing perception changes" during the initial stages of mathematical problem-solving, emphasizing the need to reevaluate and reshape one's understanding of a problem. This phase, distinctively termed in our research, is pivotal to student success and closely tied to self-efficacy, significantly influencing students' actions, expectations, and efforts.

The self-regulation phase aligns with metacognition, encompassing information-processing abilities, task-handling strategies, monitoring, and self-regulation. This phase underscores the significance of self-regulation in creative mathematical thinking, aligning with Schoenfeld's (2020) idea that proficient problem solvers continually adjust their strategies and stay open to alternatives.

Our research has limitations. Due to time constraints, we did not conduct Stimulated Recall Interviews (SRI), which could enhance future studies. Future research should focus on teaching methods to improve students' mathematical creative thinking ability and incorporate intercoders for data analysis reliability. Despite these limitations, our research provides valuable insights into the mathematical creative process, enhancing our understanding of how students tackle open-ended problems and contribute to mathematics education.

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