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## Mathematical Representation of High School Students in Understanding the Concept of Arithmetic Sequences and Series

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**Abstract:** This study aims to describe the mathematical representation ability of class XI MA Sumber Payung students in understanding the concepts of arithmetic sequences and series. This type of research is descriptive qualitative research. The results obtained by students with high math skills in understanding the concepts of arithmetic sequences and series are in the form of representations of mathematical equations/expressions, where students can create mathematical equations or models from other representations given, solve problems involving mathematical expressions and representations of words/written texts. Students can write down what is known and asked in a data or information, so that in answering questions using words or written text. Likewise for students with low mathematical abilities, representation in understanding the concept of arithmetic sequences and series is in the form of representations of mathematical equations/expressions and representations of words/text.

**Keywords:** Representation, understanding arithmetic concepts, sequences and series.

**Abstrak:** Penelitian ini bertujuan untuk mendeskripsikan kemampuan representasi matematis siswa kelas XI MA Sumber Payung dalam memahami konsep barisan dan deret aritmetika. Jenis penelitian ini adalah penelitian kualitatif deskriptif. Hasil yang diperoleh siswa berkemampuan matematika tinggi dalam memahami konsep barisan dan deret aritmetika adalah berupa representasi persamaan/ekspresi matematika, dimana siswa bisa membuat persamaan atau model matematika dari representasi lain yang diberikan, menyelesaikan masalah dengan melibatkan ekspresi matematis dan representasi kata-kata/teks tertulis siswa bisa menuliskan apa yang diketahui dan yang ditanyakan dalam suatu data atau informasi, sehingga dalam menjawab soal dengan menggunakan kata-kata atau teks tertulis. Begitu juga pada siswa berkemampuan matematika rendah representasi dalam memahami konsep barisan dan deret aritmetika adalah berupa representasi persamaan/ekspresi matematika dan representasi kata-kata/teks.

**Kata kunci:** Representasi, memahami konsep, barisan dan deret aritmetika.

### ▪ INTRODUCTION

Mathematics is a scientific discipline that has an important role in the world of education so that learning begins to be applied to students since sitting at the elementary level. In addition, mathematics also seems to be a difficult subject to understand because there are several abstract concepts so that there is a need for a correct understanding of the basic concepts that build these concepts (Zayyadi & Subaidi, 2018; Zayyadi & Kurniati, 2018; Lanya et al., 2020). There are five standards that can describe the relationship between mathematical understanding and mathematical competence possessed by students, one of which is representation, the mathematical representation ability used by students is a way of how they communicate mathematical ideas or ideas

in certain ways. Where students will change the ideas or mathematical ideas, from abstract to concrete by finding and developing a tool or way of thinking of students themselves so that the accepted mathematical concepts are easier to understand.

The representation can be classified into three, namely: 1) Visual representation in the form of pictures, graphic diagrams, or tables, 2) Symbolic representations in the form of mathematical statements/mathematical notation, numeric/algebraic symbols, and 3) Verbal representation in the form of written text/words (Septian et al., 2020; Sabirin, 2014). Thus, the ability to represent is one of the important things that need to be owned and mastered by students to support their success in learning mathematics (Efendi, 2012; Hutagaol, 2013; Syafri, 2017). So with the ability of representation can help students in understanding mathematical concepts in depth in order to clarify a mathematical problem.

The models or representations made can vary depending on the ability of each individual in interpreting the existing problems, because basically every student has different mathematical abilities, namely there are students with high, medium, and low abilities. Thus, the level of students' mathematical abilities will affect the variety of representations used by students in solving problems (Santia, 2015; Muhamad, 2017). From this it shows that there is a very close relationship between mathematical representation abilities and students' mathematical abilities. As a mandatory material in eleventh grade of senior high school, according to some students there are still some difficulties in determining the formula for the  $n$ th term of a sequence, this is because they do not fully understand the number pattern which is a basic competency in the material. sequence and series. Therefore, the completion of the solution obtained is not correct to be said to be the final result (Aini et al., 2021). This agrees with the statement of Hardiyanti (2016) which reveals that "some of the difficulties of students in solving series and series problems, one of which is the difficulty in determining the formula for the  $n$ th term of an arithmetic and geometritic sequence, understanding the concept of the first term of a sequence."

In this study, the researchers took the subject of students with high and low mathematical abilities, because students who have high mathematical abilities are expected both in terms of thinking and their way of thinking, researchers can find out the representations they do in understanding concepts for solving problems of sequences and series so that later Their ideas or working strategies can be used as references and alternative solutions in understanding the concept of sequences and series better. While the representational abilities of students with low mathematical abilities can be used by researchers as a tool to see the types of difficulties faced by students in understanding the concept of sequences and series.

## ▪ **METHOD**

In this study, researchers used qualitative research with a descriptive approach because this study aims to obtain actual and in-depth data or information, so that they can identify and describe students' mathematical representation abilities in understanding the concepts of arithmetic sequences and series based on students' mathematical abilities. In qualitative research, the presence of researchers in the field is very necessary as the position of researchers in qualitative research is a planner, implementer of data collection, analysis, data interpreter, and reporting on the results of research conducted. This is in accordance with the role of researchers as the main instrument in research. In addition, in this research there are also supporting instruments, namely (1) mathematics ability test, giving a mathematical ability test to obtain research subjects who will later become

students with high math abilities and students with low math abilities. The material used for this test is the material previously obtained by the students in the form of a description. (2) The mathematical representation ability test as in the form of the test used in this study is an assessment that is carried out first, then revised according to the suggestions given by the validator. (3) Interview guide, the type of interview that will be used by the researcher is a guided free interview (semi-structured).

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**Kerjakan Soal Berikut dengan Disertai Prosesnya**

1. Jawablah pertanyaan berikut ini:
  - a. Coba anda jelaskan apa yang dimaksud dengan barisan dan deret aritmetika!
  - b. Sebutkan syarat suatu barisan dapat dikatakan barisan aritmetika!
  - c. Buatlah contoh dan bukan contoh barisan aritmetika!
  - d. Apa yang dimaksud dengan simbol  $a$ ,  $b$ ,  $U_n$ , dan  $S_n$  pada barisan dan deret aritmetika? Amati contoh yang anda buat di c! Tunjukkan nilai  $a$ ,  $b$ , beserta contoh dari  $U_n$  dan  $S_n$  nya!
2. Diketahui jumlah 5 suku pertama deret aritmetika adalah 55 dan jumlah 9 suku pertamanya adalah 171. Tentukan:
  - a. Rumus suku ke- $n$  dari deret aritmetika!
  - b. Rumus jumlah  $n$  suku pertama deret aritmetika!
3. Angka kelahiran disuatu tempat setiap tahun selalu tetap jika angka kelahiran pada tahun 2008 mencapai 100 jiwa dan pada tahun 2018 mencapai 700 jiwa tentukan angka kelahiran pada tahun 2022!

**Figure 1.** The mathematical representation ability test

The subjects who became the source of data in this study were students of class XI MA Putri Sumber Payung for the academic year 2020/2021. The technique of determining the subject used is purposive sampling. Data collection techniques are ways of obtaining data in research activities. In this study the techniques that will be used are (1) test questions and (2) interviews. In addition to bringing the instrument as an interview guide, the researcher also used audio-visual tools. It aims to record making it easier to collect data.

The data analysis technique was carried out with the following steps: Data reduction is an activity to summarize the data obtained from the results of tests and interviews into written form with a neat language arrangement then presented a data related to the presentation of student work and interviews to draw conclusions in the form of data findings in order to answer the formulation of the problem in this study.

## ▪ RESULT AND DISCUSSION

Data on students' representational abilities in understanding the concept of arithmetic sequences and series are taken from the results of tests that have been carried out by students. As for the mathematical representation ability test, there are three questions containing the seven indicators of the ability to understand mathematical

concepts. The following are the results of tests that have been carried out by two students, namely one student with high math ability and one student with low math ability.

**Representation of Students with High Mathematics Ability**

a. 2. Barisan aritmetika adalah barisan bilangan dimana diantara dua suku yang berurutan memiliki selisih/beda yang tetap  
 Deret aritmetika adalah penjumlahan berurut dari suku-suku barisan aritmetika  
 Misal : 5, 8, 11, 14, ... selisih/bedanya tetap yaitu 3  
 $5 + 8 + 11 + 14 + \dots \rightarrow$  Deret aritmetika  
 b. harus memiliki beda/selisih yang tetap pada masing-masing sukunya  
 c. 5, 8, 11, 14, ... selisih/bedanya tetap. 1, 3, 6, 10, ... selisih/bedanya tidak tetap  
 d.  $a = 5, b = 3,$   
 $U_6 = 5 + (6-1)3 = 5 + 15 = 20$   
 $S_6 = \frac{6}{2}(5 + 20) = 3(25) = 75$   
 Ket :  $a =$  suku pertama  
 $b =$  beda  
 $U_n =$  suku ke-n  
 $S_n =$  jumlah n suku pertama

b. ② Diket :  $S_5 = 55$   
 $S_9 = 171$   
 $S_5 = \frac{5}{2}(2a + (5-1)b) = \frac{5}{2}(2a + 4b) = \frac{10a}{2} + \frac{20b}{2} \Rightarrow 5a + 10b = 55$   
 $S_9 = \frac{9}{2}(2a + (9-1)b) = \frac{9}{2}(2a + 8b) = \frac{18a}{2} + \frac{72b}{2} \Rightarrow 9a + 36b = 171$   
 Eliminasi a  
 $5a + 10b = 55 \quad | \times 9 | \quad 45a + 90b = 495$   
 $9a + 36b = 171 \quad | \times 5 | \quad 45a + 180b = 855$   
 $-90b = -360$   
 $b = \frac{-360}{-90} = 4$   
 Substitusi  $b = 4$   
 $5a + 10(4) = 55$   
 $5a + 40 = 55$   
 $5a = 55 - 40$   
 $5a = 15$   
 $a = \frac{15}{5} = 3$   
 a.  $U_n = a + (n-1)b = 3 + (n-1)4 = 3 + 4n - 4 = -1 + 4n$   
 b.  $S_n = \frac{n}{2}(2a + (n-1)b) = \frac{n}{2}(2(3) + (n-1)4) = \frac{n}{2}(6 + 4n - 4) = \frac{n}{2}(2 + 4n) = \frac{2n}{2} + \frac{4n^2}{2} = n + 2n^2$

c. ③ Diket : 2008 = 100  
 2013 = 700  
 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022  
 $U_1 = 100$   
 $U_{11} = 700$   
 $U_1 = 100 \Leftrightarrow a + 0b = 100$   
 $U_{11} = 700 \Leftrightarrow a + 10b = 700$   
 $-10b = -600$   
 $b = \frac{-600}{-10} = 60$   
 $a = 100$   
 $b = 60$   
 $U_n = a + (n-1)b = 100 + (n-1)60 = 100 + 60n - 60 = 40 + 60n \rightarrow$  Rumus Suku  
 $U_{15} = 40 + 60(15) = 40 + 900 = 940$   
 Jadi pada tahun 2022 angka kelahiran mencapai 940 jiwa.

**Figure 2.** Student with high mathematics ability answers on question number (a) 1, (b) 2, (c) 3

It can be seen in Figure 2(a), student with high mathematical abilities can mention the definition of the concept, the essential nature of the concept, and can make examples and non-examples of the concept. It can be seen that in understanding the basic concepts of arithmetic sequences and series in question number 1, student with high math abilities use representations of words/written texts, namely writing an interpretation or a representation in the form of compiling a story that is in accordance with a representation

presented and answering questions using words, written word or text. In addition, he also uses the representation of mathematical equations/expressions, namely making mathematical equations or models from other representations given and solving problems by involving mathematical expressions.

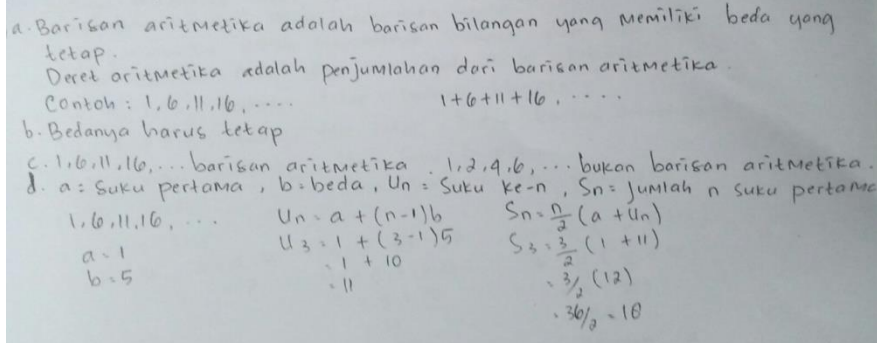
Based on Figure 2(b), student with high mathematical abilities can find a relationship or link between a concept and other concepts. So that it is known that in determining the formula for the  $n$ th term and the sum of the first  $n$  terms in the arithmetic sequence and series in problem number 2, student with high math abilities use mathematical equations/expressions, namely making equations or mathematical models from other representations given and solving problems by involving expressions. Mathematical.

Figure 2(c) shows that question number 3 has questions in the form of story questions in order to reveal students understanding of concepts in determining the value of the  $n$ th term and the value of the sum of the first  $n$  terms of arithmetic sequences and series. It is known that in understanding this concept, student with high mathematical abilities use representations of mathematical equations/expressions, namely making mathematical equations or models from other representations given and solving problems by involving mathematical expressions.

The description above is in accordance with statement of Komala & Suryadi (2018) that students with high mathematics ability have a positive mathematical representation. Each student has a distinctive way of thinking. Habits of thinking affect the process of students' representation. This students' thinking habits influence the students' success in solving mathematical problems in his own way from the abilities they possesses in their mind (Komala & Suryadi, 2018).

### Representation of Students with Low Mathematics Ability

a.



a. Barisan aritmetika adalah barisan bilangan yang memiliki beda yang tetap.  
Deret aritmetika adalah penjumlahan dari barisan aritmetika.  
Contoh:  $1, 6, 11, 16, \dots$        $1+6+11+16, \dots$

b. Bedanya harus tetap

c.  $1, 6, 11, 16, \dots$  barisan aritmetika.  $1, 2, 4, 6, \dots$  bukan barisan aritmetika.

d.  $a$  = suku pertama,  $b$  = beda,  $U_n$  = suku ke- $n$ ,  $S_n$  = jumlah  $n$  suku pertama

$$1, 6, 11, 16, \dots$$

$$a = 1$$

$$b = 5$$

$$U_n = a + (n-1)b$$

$$U_3 = 1 + (3-1)5$$

$$= 1 + 10$$

$$= 11$$

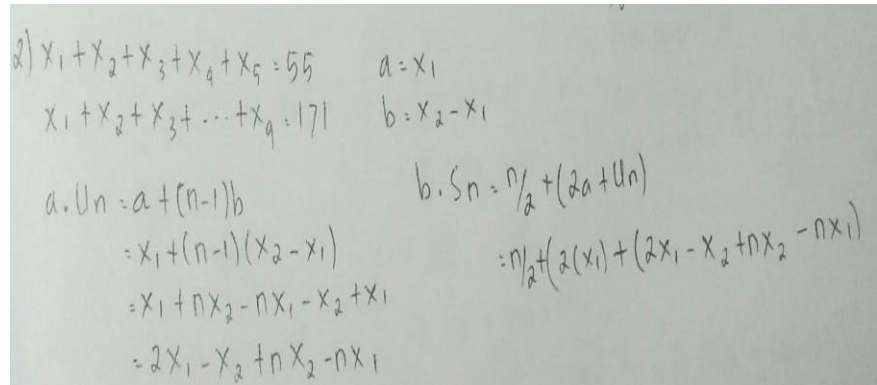
$$S_n = \frac{n}{2}(a + U_n)$$

$$S_3 = \frac{3}{2}(1 + 11)$$

$$= \frac{3}{2}(12)$$

$$= \frac{36}{2} = 18$$

b.



$$2) \begin{cases} X_1 + X_2 + X_3 + X_4 + X_5 = 55 \\ X_1 + X_2 + X_3 + \dots + X_9 = 171 \end{cases}$$

$$a = X_1$$

$$b = X_2 - X_1$$

a.  $U_n = a + (n-1)b$

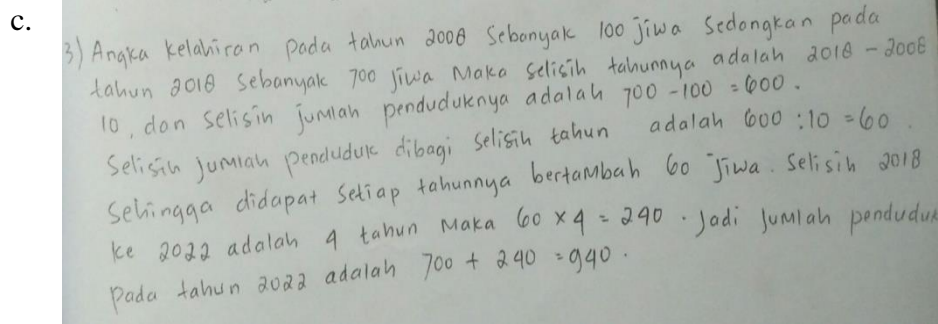
$$= X_1 + (n-1)(X_2 - X_1)$$

$$= X_1 + nX_2 - nX_1 - X_2 + X_1$$

$$= 2X_1 - X_2 + nX_2 - nX_1$$

b.  $S_n = \frac{n}{2}(2a + U_n)$

$$= \frac{n}{2}(2(X_1) + (2X_1 - X_2 + nX_2 - nX_1))$$



**Figure 3.** Students with low mathematics ability answers on question number (a) 1, (b) 2, (c) 3

It can be seen in Figure 3(a), students with low mathematical abilities can mention the definition of a concept but the definition given is incomplete when explaining the meaning of arithmetic sequences and series. Student can create examples and non-examples of the concept. It can be seen that in understanding the basic concepts of arithmetic sequences and series in question number 1, students with low math abilities use representations of words/written texts, namely writing an interpretation or a representation in the form of compiling a story that is in accordance with a representation presented and answering questions using words, written word or text. In addition, he also uses the representation of mathematical equations/expressions, namely making mathematical equations or models from other representations given and solving problems by involving mathematical expressions.

Based on Figure 3(b), student with low math skills cannot find a relationship or link between a concept and another concept. So that in determining the formula for the  $n$ th term and the sum of the first  $n$  terms in the arithmetic sequence and series in question number 2, student with low math abilities do not get the desired answer to the question. In this case, he solves the problem using a mathematical equation/expression representation, namely making a mathematical equation or model from another given representation and solving the problem by involving a mathematical expression but does not get an answer.

Figure 3(c) shows that question number 3 has questions in the form of story questions in order to reveal the understanding of students' concepts in determining the value of the  $n$ -th term and the value of the sum of the first  $n$  terms of an arithmetic sequence and series. It is known that in understanding the concept, student with low math abilities use representations of words/written texts, namely creating problem situations based on the data or representations provided by compiling stories that match the representations presented.

The description above is in accordance with statement of Zhe (2012) that students often represent their ideas incoherently, incompletely, which shows that the process of students' thinking is not smooth; language processing is incorrect. According to Komala & Suryadi (2018), students with low mathematics ability have a negative mathematical representation. In fact, there are still many students who have low representation ability (Widakdo, 2017).

Based on the data analysis of the research results that have been described, it can be seen that students with high mathematical abilities do not involve visual representations in understanding the concept of arithmetic sequences and series, because



students do not involve images or other visuals at all in the completion process to get the answers requested in the questions. The research from Rahmawati & Anwar (2020) finds that there are two characteristics of unpacking the source, which are drawing (schema) and verbal (paraphrasing). According to Tong et al., (2021), students begin organizing and formulating ideas by using various visual and verbal representations to get their ideas off the ground. Students are perplexed visual representation problems because they usually have only been exposed to arithmetic representations.

The explanation of the mathematical representation ability of students with high mathematics abilities states that students with high levels of ability have the ability to represent mathematical equations/expressions (symbolic) and represent words. written (verbal) words/texts look more fluent than using visual representations, this shows that for representations of mathematical equations/expressions and representations of words/written texts these students are in high criteria (Mulyaningsih and Haerudin, 2019). Solving problems by making pictures based on problems shows that these students for visual representation indicators are in low criteria.

Based on the data analysis of the research results that have been described, it can be seen that students with low math abilities do not involve visual representations in understanding the concept of arithmetic sequences and series, because students do not involve images or other visuals at all in the completion process to get the answers requested in the questions. In the process of solving test questions and interviews that have been carried out by students with low math abilities, they do not really understand some of the concepts of arithmetic sequences and series so that there are some answers that have not been completed on test questions number 1 and 2 in the completion process, so they cannot get the final solution correctly.

#### ▪ CONCLUSION

Based on the results of the study, it can be concluded that in understanding the concept of arithmetic sequences and series students tend to use representations of words/written texts and mathematical equations/expressions, the reason for not using visual representations is that teachers often use verbal and symbolic explanations in teaching arithmetic sequences and series material. so this keeps them from getting answers using visual representations. Some suggestions that can be submitted based on the results of research that have been carried out are as follows: 1) it can be used as reference material to develop or conduct research related to students' mathematical representation abilities, and 2) the representation made can be developed so that it can bring up the three representations..

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