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Enhancing Students' Mathematical Representation Ability Using the Concrete-Pictorial-Abstract Approach

Azmidar[1,*], Darhim[2], Jarnawi Afghani Dahlan[2]

[1]Sekolah Islam Terpadu Ibnu Sina Makassar, Indonesia
[2]Department of Mathematics Education, Postgraduate School
Universitas Pendidikan Indonesia, Indonesia

Abstract: Mathematical representation ability is basic for others' mathematical ability, such as understanding ability, communication ability, reasoning ability, and problem-solving ability. The purpose of this research was to analyze the enhance students' mathematics representation ability using the Concrete-Pictorial-Abstract Approach in the group (CPA-G) based on prior mathematical ability. The research method used the quasi-experimental with pre-response and post-response design. The population of this research was students grade VII. This research uses two classes which selected purposely, an experimental group that receives CPA-G and a control group that received regular learning (RL). Prior mathematical ability and mathematical representation ability tests were used as an instrument in this research. The result shows that the mathematical representation ability of students in high and middle of prior mathematical ability category, who received CPA-G higher than students who only received RL. This approach can enhance students' mathematical representation ability, especially on the visual and symbolic representation.

Keywords: The Concrete-Pictorial-Abstract approach in the group, mathematical representation ability, quasi experimental research

Abstrak: *Kemampuan representasi matematis merupakan dasar dalam pencapaian kemampuan-kemampuan matematika lainnya seperti kemampuan pemahaman, kemampuan komunikasi, kemampuan penalaran, dan kemampuan pemecahan masalah. Tujuan penelitian ini adalah menganalisis peningkatan kemampuan representasi matematis siswa menggunakan pendekatan Concrete-Pictorial-Abstract berkelompok (CPA-G) berdasarkan kemampuan awal matematis. Metode penelitian yang digunakan adalah kuasi eksperimen dengan desain pre-respon dan post-respon. Populasi dalam penelitian ini adalah seluruh siswa kelas VII. Penelitian ini menggunakan dua kelas yang dipilih secara purposive, yaitu kelas eksperimen yang memperoleh pembelajaran dengan pendekatan CPA-G, dan kelas kontrol yang memperoleh pembelajaran biasa. Instrumen yang digunakan adalah kemampuan awal matematis dan tes kemampuan representasi matematika siswa. Hasil penelitian menunjukkan bahwa kemampuan representasi matematis siswa pada kategori tinggi dan sedang yang memperoleh pembelajaran dengan pendekatan (CPA-G) lebih tinggi daripada siswa yang memperoleh pembelajaran tradisional. Pendekatan (CPA-G) dapat meningkatkan kemampuan representasi matematis siswa, khususnya pada aspek visual dan simbolik.*

Kata kunci: *Pendekatan Concrete-Pictorial-Abstract berkelompok, kemampuan representasi matematis, penelitian kuasi eksperimen.*

Azmidar et al.
Corresponding Email:
azmidarsafruddin.as@gmail.com

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▪ INTRODUCTION

Representation is one of the psychological concepts used in mathematics education to explain any important phenomena about students' way of thinking which are visualized in verbal, visual, and symbolic forms. The representation that the student raises is an expression of the mathematical ideas presented by the student as an effort to find a solution to the problem he is facing (NCTM, 2000). Achieving mathematics ability requires communication in verbal or written form which can be in the single or multiple representations forms arranged in the mathematical language (Dewanto, 2008). Representation ability is needed in building and fostering an understanding of the concept (Salkind, 2007). Representation used to develop student understanding are closely related to the ability to do operations with representations (Garderen et al, 2012).

Mathematical representations are also closely related to problem-solving abilities Hwang (2007) found that students' multiple representation abilities were the key to success in solving problem. A good problem solver usually builds a representation of a problem to facilitate understanding (Garderen & Montague, 2003). Sajadi et al. (2013) stated that success in problem-solving was impossible without starting with an accurate representation of the problem. Furthermore, it is said that students who have difficulty in representing mathematical problems will have difficulty solving them. Chen et.al (2015) stated that the greatest difficulty in the problem-solving process occurred at the representation stage. On the other hand, Zhe (2012) and Mielicki (2016) found that the successful solution in mathematical problems is by combining the ability of representation and the ability to manipulate symbols. If students can understand various forms of the mathematical representation conversion process, they will be able to understand mathematical concepts used in solving a problem.

Based on the expert's explanation, it can be concluded that the mathematical representation ability is very important for a student because it is the basis for other mathematical abilities such as concept understanding skills, communication skills, problem-solving abilities, reasoning abilities, and so on.

The facts show that most students have mathematical representation skills which are still classified as middle and even low (Khaerunnisa, 2015), (Minarni, et.al 2016), (Rahmawati 2014). This was also proven by the researcher when conducting preliminary research at the school to be researched.

The low representation ability of junior high school students is because they are not allowed to construct and present their representations related to the material being studied (Hutagaol, 2008). Meanwhile, junior high school students who are in the range 11-14 years old, based on Piaget's theory of development, are at the initial formal operation stage. At this stage, students experience a transition from the concrete thinking stage to the abstract thinking stage. This shift in mindset can be bridged with the help of representations because students are not yet fully able to think abstractly. This representation ability will lead students to more abstract thinking skills. Also, according to Dahlan (2011), the low representation ability of students is due to the mathematics learning process designed by the teacher which tends to be deductive, where the delivery of mathematical formulas, rules, or propositions is carried out directly without providing context related to the material being taught.

Therefore, students need a learning approach that can provide the opportunity to explore their abilities, construct and present their representation related to the material being studied. Also, students need learning that begins with providing context related to the material before leading the abstract stage. Thus, students can achieve the standards of mathematical representation ability.

The approach chosen should be adjusted to the student's thought process development stages so that learning is more meaningful, enjoyable and makes students actively involved in reconstructing their understanding and knowledge. Piaget's theory of cognitive development (Santrock, 2012) suggests four stages of thinking for each individual in receiving knowledge, namely (1) the sensorimotor stage (2) the pre-operation stage (3) the concrete operational stage (4) the formal operational stage. One of the alternative learning forms that refer to the student's thinking stage and reflect the active involvement of students in reconstructing understanding and knowledge is learning with the Concrete-Pictorial-Abstract (CPA) approach in groups.

The Concrete-Pictorial-Abstract (CPA) is a learning approach based on Bruner's heuristic concept of “enactive-iconic-symbol”. Representation was introduced in Singapore in 1980 (Hoong, Kin & Pien, 2015). Sousa (2007) states that learning with the CPA approach is very beneficial for students who have difficulty learning mathematics because this approach begins with using real objects, through pictures then ended with the using symbols. The CPA approach (Witzell, 2005) consists of three learning process stages, namely: the concrete stage (doing), where students learn through the manipulation of concrete objects; the pictorial stage (seeing), where students learn to transform concrete objects into the form of a drawing or painting model; and the last stage, namely abstract (symbolic), students learn to solve problems using abstract symbols. Manipulatives are effective because they are concrete, so they probably mean objects that students can grasp with their hands. This sensory nature ostensibly makes manipulatives ‘real,’ connected with one’s intuitively meaningful personal self, and therefore helpful (Clemen, 1999).

Learning mathematics through the CPA approach in groups facilitates students making meaningful relationships between concrete, pictorial, and more abstract level of understanding and thinking. This is because students start learning with visual, real, and kinesthetic experiences to build basic understanding. Then students can expand their knowledge through pictorial representations (pictures, diagrams, or sketches) and finally move to the abstract thinking level, where students can exclusively use mathematical symbols to represent and model problems related to the material being studied.

Constructivism learning theory states that learning is an active process of constructing knowledge and understanding through social interaction. This social interaction is in the form of communication between students and teachers, peers, or with the environment. Abdurrahim (2015) states, one of the learning models suggested by experts to facilitate student activities in the learning process are the cooperative learning model. Using group learning can be optimal if the members are heterogeneous both in terms of abilities and characteristics, resulting in good collaboration between students who have high, medium, and low abilities (Suherman et.al, 2003).

This study will combine the CPA approach with cooperative learning so that it becomes CPA in groups approach. cooperative learning present characteristics which enabled it to contribute effectively to activate the role of the educational process more than other traditional instructional methods (Gubbad, 2010). Learning through the CPA in group approach which is implemented stage by stage can improve students' mathematical representation ability because learning starts from the simplest stage, the concrete stage. In this stage, the teacher presents context problems in the manipulative form related to the material being studied. The CPA group approach provides many opportunities for students to have and improve their mathematical representation skills.

▪ METHOD

Research design

The research was quasi-experimental with pre-response and post-response. This research used a nonequivalent control-group design, where the experimental group and the control group were not randomly selected. This design has an experimental class that learns by the concrete-pictorial-abstract in groups approach, and the control class that learns by regular learning. The two classes are given pre-response first before given treatment, and at the end given a post-response..

Population and sample

This study was conducted at SMPN 7 Alla, Enrekang Regency in 2017. The population in this study was students in the VII grades which divide into four classes. The sample was selected using a purposive sampling method. There were two classes were selected, class A as the experimental class that received learning using the concrete-pictorial-abstract in groups (CPA-G) approach and class C as the control class that received regular learning (RL). Both classes were given pre-response before given treatment and given post-response at the end of the meeting.

Instruments

The study instrument includes data of prior mathematical ability and tests of mathematical representation ability on flat shape. The mathematical representation ability test consists of five items questions that represent aspects of the mathematical representation ability, which are visual, verbal, and symbolic.

The values that are used as the basis for grouping prior mathematical ability (PMA) are the average of the scores for the odd semester tests, daily tests, and even semester of the two groups. The table below shows the student grouping data for the experimental and control classes based on PMA.

Table 1. Grouping of PMA

PMA	Class		Total
	Experimental (CPA-G)	Control (RL)	
High	6	6	12
Middle	18	14	32
Low	5	5	10
Total students	29	25	54

Data Analysis

In analysing the data, the Statistical Package for Social Sciences (SPSS) was utilized. Mean and the standard deviation were used to quantify the scores of the participants in their pre-response and post-response. To determine if there was a significant difference between the scores of the two groups before and after using the concrete-pictorial-abstract in groups approach and the regular learning, a paired sample t-test and independent-sample t-test were employed.

▪ RESULT AND DISCUSSION

The mathematical representation ability test (MRA) was given in two steps, before and after learning treatment. The purpose of giving the MRA test before learning was to see the similarity in early representation ability between the research classes so that they could be given different treatments. If the two classes have the same early representation ability, it is expected that the difference in increasing mathematical representation ability will be the result of differences in the treatment given.

The table 2 shows that the experimental class and the control class have relatively the same minimum score, maximum score, average, and standard deviation (not too different). If written in a percentage, the average response score for the experimental class is 11.64% of the ideal score and the control class is 12.9% of the ideal score, so it can be said that the response scores of the experimental class and control class students are still relatively low.

Table 2. Descriptive statistics of students' mathematical representation ability

PMA	Data	CPA-G			RL		
		Pre-Response	Post-response	N_gain	Pre-response	Post-response	N_gain
High	N	6	6	6	6	6	6
	Min.	4	26	0.55	6	19	0.34
	Max.	9	36	0.88	11	26	0.58
	$\bar{x}$	7.33	30.5	0.71	8.5	22.83	0.45
	S	2.06	3.73	0.12	2.07	2.93	0.09
Middle	N	18	18	18	14	14	14
	Min.	2	16	0.37	1	9	0.16
	Max.	8	30	0.74	7	21	0.50
	$\bar{x}$	3.56	22.44	0.52	4.14	16.64	0.34
	S	1.72	3.71	0.10	1.96	3.67	0.11
Low	N	5	5	5	5	5	5
	Min.	2	15	0.26	2	11	0.15
	Max.	7	22	0.45	6	18	0.39
	$\bar{x}$	5.4	18	0.36	4	14.6	0.29
	S	2.07	3.00	0.09	1.58	3.21	0.09
Total	N	29	29	29	25	25	25
	Min.	2	15	0.26	1	9	0.16
	Max.	9	36	0.88	11	26	0.58
	$\bar{x}$	4.65	23.34	0.53	5.16	17.7	0.36
	S	2.36	5.36	0.15	2.66	4.47	0.1
Ideal Maximum Score = 40							

Ideally, learning by using the CPA-G has not been able to provide a high contribution to students' mathematical representation ability. This can be proven by the students who still get a score of 15. On the other hand, some students get a score of 36 from the maximum ideal score, 40. Besides, it is also seen from an average of 23.34, so that if generalized only 58.35% of students achieve mathematical representation ability.

In the N_gain score, we can see that two students in the experimental class only achieved an increase of 0.26 with a low increase classification. From the above results, the Concrete-Pictorial-Abstract approach has not been able to encourage low-ability students to be more optimal.

The table below shows that increasing the mathematical representation ability of the experimental class for the indicator: refresenting data or information from one representation to a visual representation is fairly high. This happens because students have been able to express representational ideas that exist in their thinking. It's agrees with Flores et.al (2015) stated that the ability to solve complex problems and transfer skills to new situations are related to how well students' procedural and conceptual knowledge has developed. Meanwhile, the control class is still in the middle level. In the other indicators, such as: using visual representations to solve problems, making mathematical equations or models from other given representations, and solving problems involving mathematical expressions, both the two classes are in the middle level, but the experimental class score was higher than the control class on that indicators. In the verbal aspect where the indicators include: compiling a story by a presented representation and answering questions using words or written text, the two research classes are also in the middle level.

Table 3. N-gain of students' mathematical representation ability based on indicators

No	Indicator	CPA-G				RL			
		$\overline{x_1}$	$\overline{x_2}$	$\overline{x_3}$	Category	$\overline{x_1}$	$\overline{x_2}$	$\overline{x_3}$	Category
1	Representing data or information from one representation to visual representation	1.28	3.90	0.96	High	1.80	3.32	0.69	middle
2	Use visual representations to solve problems	0.86	2.66	0.57	middle	0.92	2.08	0.38	middle
3	Create mathematical equations or models from other given representations	0.22	2.66	0.64	middle	0.06	1.6	0.39	middle
4	Solve problems by involving mathematical expressions	0.09	1.50	0.36	middle	0.1	0.36	0.07	middle
5	Compile a story or create a mathematical situation following a presented representation	0.67	2.71	0.61	middle	0.66	2.36	0.51	middle
6	Answer questions using words or written text	0.27	1.53	0.34	middle	0.40	1.84	0.4	middle

Note:

$\overline{x_1}$: Average of pre-response score

$\overline{x_2}$: Average of post-response score

$\overline{x_3}$: Average of N_gain score

Overall, increasing the mathematical representation ability of the experimental class is higher than the control class. An indicator of problem-solving involving mathematical expressions is the highest difference in improvement between the two classes. On this indicator, the experimental class has 0.29 higher than the control class. This is This is like the results of research conducted by Wessel, Jolles, and Schoot

(2014) which state that students who make accurate visual representations will increase their chances of solving problems correctly. The second-highest difference is in the indicator of restating data or information from a given representation to another. On this indicator, the experimental class has 0.27 higher than the control class. Meanwhile, on the indicator of answering questions using words or written text, the control class was 0.06 higher than the control class.

Cognitive representational systems function partly but very importantly by evoking affect and the information it encodes. This applies specifically to the internal verbal/syntactic systems, imagistic systems, formal notational systems, and strategic/ heuristic systems of representation discussed elsewhere in formulating a model for mathematical problem-solving competence. That is related to Chang (2015) who stated that representations of learning objects are the essential objects for individual reasoning and cognizing during the learning of mathematics. The different representations of the concepts will maintain different knowledge structures and algorithms to show specific directions for information recognition and acquisition.

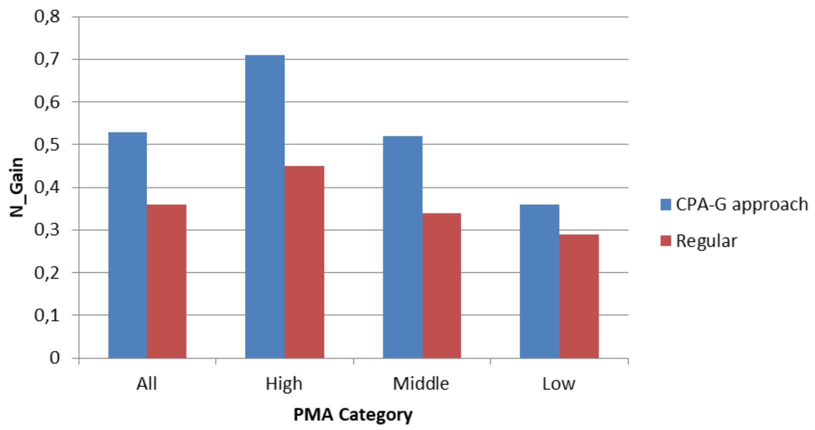

Figure 2. N-gain of students' mathematical representation ability based PMA

Learning by the Concrete-Pictorial-Abstract approach in groups is one of constructivism-based learning, where learn starts with using the manipulative objects related to the material being studied to seek and find a concept. Before learning started, the researcher divides students into heterogeneous groups based on prior mathematical ability and also conducts discussions with the teacher who teaches in class to support this learning activity.

Although this learning is new to students, learning with the Concrete-Pictorial-Abstract approach in groups works well. It can be seen from the results of statistical analysis which states that increasing the mathematical representation ability of students who learn by the Concrete-Pictorial-Abstract approach in groups is higher than students who get regular learning. This is prove that the CPA-G approach increasing the interesting student to learn. According to Abarantes (2007), Lin and Huang (2016) who found that learning approach very influence in learning process. The right learning approach can increase mathematical ability students (Ittel, 2021) and (Heinze, 2005). This implies that learning by the Concrete-Pictorial-Abstract (CPA) approach in groups can provide different results in improving students' mathematical representation ability.

Learning using the Concrete-Pictorial-Abstract (CPA) approach in groups is carried out by providing the manipulative objects related to the material studied along with worksheets to guide students to carry out activities independently from the beginning to make conclusions. It’s related to Zaskis (2004) who state that, an important role of representation in mathematics is as a tool for thinking ang gaining insight. Students are asked to find their own concepts who learn before so that learning is more meaningful. Students are grouped before the implementation of the CPA in groups approach in the experimental class and regular learning in the control class. Each group consists of 4 or 5 students with heterogeneous abilities and gender. It's intended that students can help each other and work together in completing the worksheets given. Related to Gubbad (2010) who stated that the cooperative work compared with competitive work and individual work increases the achievement and productivity of students' performance, emphasizes the positive relations among the members, and improves mental health and self-esteem.

▪ CONCLUSION

The mathematical representation ability of students who learn by the Concrete-Pictorial-Abstract approach in the group is higher than students who learn by regular learning. Based on the prior mathematical ability, students' mathematical representation ability in the high and medium categories who learn by the Concrete-Pictorial-Abstrac approach in the group is higher than students who learn by regular learning in the same category, especially in visual and symbolic representation.

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