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Analysis of Student Misconceptions in Working on Algebraic Form Counting Operations Using Certainty of Response Index

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Abstract: This study aim is to determine the misconceptions that occur in students and the factors that cause students to experience misconceptions in the material of algebraic arithmetic operations. The research which was conducted at SMP Negeri 4 Tasikmalaya class VII C with a total of 32 students was a qualitative research with a descriptive approach. The subjects of this study were taken purposively, namely 5 students of class VII C of SMP Negeri 4 Tasikmalaya. The data collection technique consisted of a misconception analysis test accompanied by the Certainty of Response Index (CRI) and interviews. Data analysis techniques consisted of data reduction, data presentation, and verification. Based on the results of data analysis, it is concluded that (1) S4 misconception of equations, S10 misconception of operations, S17 misconceptions of variables, S28 misconception of fractions, S10 misconceptions of negative signs (2) The causes of misconceptions that occur include incorrect student preconceptions, student associative thinking, humanistic thinking, incomplete reasoning, incorrect intuition, student abilities and student interest in learning.

Keywords: Certainty of response index, misconceptions, algebraic form counting operations.

Abstrak: *Penelitian ini bertujuan untuk mengetahui miskonsepsi yang terjadi pada siswa serta faktor penyebab siswa mengalami miskonsepsi pada materi operasi hitung bentuk aljabar. Penelitian yang dilaksanakan di SMP Negeri 4 Tasikmalaya kelas VII C dengan jumlah 32 peserta didik merupakan penelitian kualitatif dengan pendekatan deskriptif. Subjek dari penelitian ini diambil secara purposive yaitu 5 siswa kelas VII C SMP Negeri 4 Tasikmalaya. Teknik pengumpulan data terdiri dari tes analisis miskonsepsi yang disertai dengan Certainty of Response Index (CRI) dan wawancara. Teknik analisis data terdiri dari reduksi data, penyajian data, dan verifikasi. Berdasarkan hasil analisis data diperoleh kesimpulan bahwa (1) S4 melakukan miskonsepsi persamaan, S10 melakukan miskonsepsi operasi, S17 melakukan miskonsepsi variabel, S28 melakukan miskonsepsi pecahan, S10 melakukan miskonsepsi tanda negatif. (2) Penyebab miskonsepsi yang terjadi diantaranya prakonsepsi yang siswa yang salah, pemikiran asosiatif siswa, pemikiran humanistik, penalaran yang tidak lengkap, intuisi yang salah, kemampuan siswa dan minat belajar siswa.*

Kata kunci: *Certainty of response index, miskonsepsi, operasi hitung bentuk aljabar.*

▪ INTRODUCTION

The concepts of math are probably open to misconceptions due to abstract concepts. Ozkan & Ozkan (2013) said that misconception is the misunderstanding of the knowledge directly or wrong commending it indirectly. It means that misconceptions describe the difference between the scientific definition of a scientific concept and the concept in one's mind. Each student's thinking and understanding of a concept will be different, the difference is due to the student's stimulus in understanding the concept is also different.

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As students transition from their conception of number as natural numbers to a conception of number as including natural, rational, and real numbers, they generate misconceptions example synthetic concepts (Durkin & Rittle-Johnson, 2014) . Misconceptions will stick with students and tend to be difficult to correct if students are sure that the concept they understand is the correct concept. Of course, it has an impact on the next learning process. Therefore, further research is needed on misconceptions in algebraic form operation material.

Ulfah & Fitriyani (2017) suggest that when students are join the class, they are not with empty heads but have brought some pre-formed ideas both based on the student's experience and when they interact with the surrounding environment. It means that before the teaching and learning activities begin, students have had their thoughts, ideas, and initial concepts, then students interpret them into the learning materials. If the initial thoughts, ideas, and concepts that students have are not following the concept agreed by the experts, then this will lead to misconceptions. That is because misconceptions are personal, illogical, and stable (Driver, 1985). Each student has a different way of addressing and constructing their interpretations, ideas, and experiences. Al-Khateeb (2016) said that if learners make mistakes in learning something, their subsequent learning will based on error. The misconceptions that students have will also be different because they form an understanding of a concept by themselves. It makes series of mistakes will lasts until learners thinking become confused.

Algebraic operations include addition, subtraction, multiplication, and division operations, including simplification forms and their application in algebraic form. Booth, McGinn, Barbieri, & Young (2017) reveals the types of misconceptions that occur in algebraic material including: (1) Equation Misconceptions such as moving, deleting, or adding marks to equations, (2) Negative sign Misconceptions such as moving, removing or adding negative marks, (3) Variables Misconceptions such as combining variables that do not comply with rules, moving, deleting or adding variables, (4) Fraction Misconceptions such as summing without equating mentions, (5) Operations Misconceptions occur when students perform algebraic form addition operations but what is asked on the question is algebraic multiplication surgery. These types of misconceptions are relevant to the algebraic form material taught in grade VII and are used to group the misconceptions that occur.

Identifying students who experience misconceptions can be identified with the Certainty of Response Index (CRI) developed by Hasan, Bagayoko, & Kelley (1999) with the aim is to identify misconceptions through the degree of confidence of students in answering questions. The Certainty of Response Index (CRI) is one of the methods used to identify the occurrence of misconceptions, as well as distinguish students who understand concepts, misconceptions, and do not understand concepts at all. Sadhu et al. (2017) revealed that the Certainty of Response Index (CRI) is a measure of the certainty of students' answers to a given question. The honesty of students in filling their confidence level on the answer sheet is indispensable so that the results obtained maximum. CRI method based on research conducted by Hasan et al., has several functions, including assessing whether or not the emphasis of a concept is appropriate; as a diagnostic tool for teachers in modifying their teaching methods; assess how effective the teaching is; and as a tool to compare the effectiveness of a method, strategy, learning approach used in the process of teaching and learning activities in improving students' understanding. The following is a table of criteria for the CRI scale in question:

Table 1. CRI scale criteria (Hasan et al., 1999)

CRI	Criteria
0	*Totally guessed answer*
1	*Almost a guess*
2	*Not sure*
3	*Sure*
4	*Almost certain*
5	*Certain*

There are 6 scales in CRI, from 0 until 5 scales, CRI scales 0 to 2 show that students don't understand concepts while CRI scales 3 to 5 indicate that students understand concepts. If the student's answer is correct and the CRI scale is selected from 3 to 5 (high CRI scale) then the student understands the concept. Students are categorized as not understanding the concept if the student's answer is correct but the CRI scale is selected from 0 to 2 (low CRI scale), this also applies if the student's answer is wrong. When a student's answer is incorrect but the CRI scale that selected from 3 to 5 (high CRI scale) then the student experiences a misconception. The causes of misconceptions have been classified by Suparno (2013) into five groups of students, teachers, textbooks, context, and teaching methods. However, this study only discusses the causes of misconceptions in terms of students who are grouped into 8 categories, namely students' preconceptions or initial concepts, associative thinking, humanistic thinking, incomplete (wrong) reasoning, inappropriate intuition, student abilities, and students interest to learn. The purpose of this misconception analysis is to minimize misconceptions so that other misconceptions do not arise due to previous misconceptions so that students do not experience difficulties in understanding the material related to the previous material.

▪ METHOD

This research is qualitative research with a descriptive approach that aims to reveal and describe the misconceptions of students in working on the problem of algebraic form counting operations and also the causes of misconceptions of students.

Research Subjects

The subject of this study is 5 students in 7th grade in SMPN 4 Tasikmalaya based on the consideration that the student performed misconceptions by the types of Booth misconceptions. The research subjects were selected purposively with the consideration is based on misconceptions made by the student is a type of Booth misconception. Another consideration is that the student is communicative and able to provide information that can reveal the misconceptions experienced by the subject.

Data Collection

Data collection is done by written essay test accompanied by validated CRI and interview. The problem of written essay test used based on indicators of achievement of competency of algebraic form counting operation material, namely: (a) determining the completion of the summation and subtraction operation of algebraic form, (b) solving contextual problems in algebraic form multiplication operations, (c) determining the completion of algebraic form division operations.

Data Validity Test

For this study, researchers conducted a credibility test of data or trust in the data of the results of the study by triangulation so that the data obtained by the researchers did not produce different data. Moleong (2014) states that data validity techniques are needed to check whether data has been presented valid or not so that the research results can be accounted for answers. Data were analyzed using content analysis technique, which initially included straight coding. Once the open coding process was completed, axial coding was performed. The same data were coded twice by the researcher with a view to enhancing validity and reliability of the results. Expert opinions were obtained in order to check the consistency of the codes obtained between each other and among the categories. Coding consistency was performed using the reliability formula proposed by Miles and Huberman (1994)\

Data Analysis

Data analysis techniques are performed in a way: grouping data, presenting data in narrative text and inferring misconceptions and causes of misconceptions in algebraic counting operation materials.

▪ RESULT AND DISCUSSION

The results obtained in this study are data on the results of student misconception analysis tests on algebraic form counting operations accompanied by Certainty of Response Index (CRI) and interview results. Based on the results of the study, from 32 students involved in the study obtained 5 students who met the consideration of being the subject of the study, namely subjects who experienced misconceptions based on the type of misconception Booth et al. and provide clear information orally and in writing.

Table 2. List of research subjects

Research Subject	Misconception	CRI Scale
S4	Equation Misconceptions	5
S10	Operation Misconceptions	5
S17	Variable Misconceptions	4
S28	Fraction Misconceptions	5
S31	Negative Sign Misconception	4

Results of Research on Subject 4

Based on the results of S4 misconception analysis test accompanied by CRI and interview, S4 experienced a misconception in question number 2 which contains indicators of competency achievement, namely solving contextual problems in algebraic multiplication operations. S4 already understands the steps that must be taken to solve the problem, such as creating a mathematical model of the problem. However, the answer given by the subject is incorrect and the CRI scale given by the subject is 5. The scale shows that S4 is confident. Hasan et al. (1999) suggest that if the subject's confidence level is high (CRI scale 3-5) means students understand the concepts, principles, and laws used in answering questions, and students are confident.

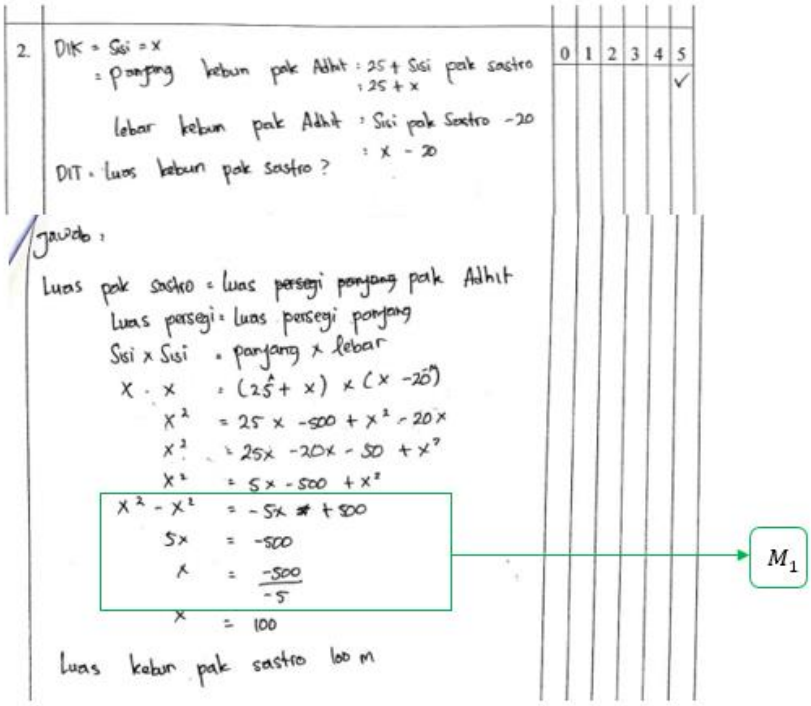

Figure 1. Answer sheet of subject 4

Description:
M_1 = Equation Misconceptions

This was confirmed in the following interview:

P : *So, it means that the result of the multiplication is* $5x - 500 + x^2$. *Why in this section *points to* $x^2 - x^2$* *there is* $-x^2$*?*

S4 : *It's both* x^2 *miss, so put it together. Such as move field (pindah ruas).* → A^4

P : *What does move field (pindah ruas) like?*

S4 : *If there were the same form so we can put it together, for example in the right side and left side is the same, so we can move it to the right side or left side.*

P : *Okay next, why it becomes* $-5x + 500$*?*

S4 : *Because of the move field, so the sign must be changing too.* → B^4

P : *Why it must be changing?*

S4 : *I only remember like that, miss* → C^4

P : *So the sign from* $5x - 500$ *must be changing? Not only the sign of* x^2 *is changing ?*

S4 : *Yes miss, moving this *points to* x^2 *on the* 7^{th} *line* to the left side. The sign for the right side must be changing too because the sign on this part *points to* x^2 *on the* 7^{th} *line* is changing to negative sign.*

P : *How about the sign on* 500*, is it changing too?*

S4 : *Of course, i remember when we wanna move the same thing to the right or left side or we wanna put it together, we have to change the sign too.* → D^4

P : *So if there is a question like this or move fields, it means the sign on the left and the right field must change?*

S4 : *I think so* → E^4

P : *How can you conclude that Pak Sastro's garden is 100 meters?*

S4 : *From this, miss *points to* $x = 100$*

P : *What is x refers to?*

S4 : *x is the area of the garden, miss* → F^4

P : *Previously you took the x as the side right? So now why is x become an example of the area of the garden?*

*S4 : At first, i think we have to find the area, but when i get this *points to $x = 100$*, so that's the answer miss.* → G^4

Information:
S4 Keywords: $A^4, B^4, C^4, D^4, E^4, F^4, G^4$

When eliminating variables, S4 also changes the mark in the algebraic tribe (A^4, B^4, D^4). In that case, S4 has also tried to recall how to eliminate the correct variables and has previously been studied (C^4). Byrd, C. E., McNeil, N. M., Chesney, D. L et al. (2015) suggest that equal sign understanding matters for equation-solving success (p. 2). Unfortunately, the concept understood by S4 is that when going to eliminate a variable then the sign (such as a negative sign or a positive sign) in the algebraic form must be changed so that it is no longer the same as the previous sign. The concept understood by S4 is that when going to eliminate a variable then the sign (such as a negative sign or a positive sign) in the algebraic form must be changed so that it is no longer the same as the previous sign. This indicates that S4 relies on its intuition in eliminating variables in that form of algebra (E^4).

Eichhorn, Perry and Brombacher (2018) found that without a conceptual understanding of the equal sign, students will likely solve equations based on rote knowledge and procedures (e.g., if a value is added to one side of an equation, it must be added to the other side) which may lead to frequent errors since students do not know why the procedures they are using work (p. 656). S4 considers that the result of Pak Sastro's garden area is 100 (F^4, G^4), it is due to S4's thinking that the final result found is the answer sought from the problem. S4 reasoning is incomplete when understanding the problem of making the information obtained incomplete and resulting in S4 drawing incorrect conclusions. The causes of misconceptions experienced by S4 include humanistic thinking, wrong intuition, and incomplete reasoning.

Results of Research on Subject 10

Based on the answer sheet, S10 suffered a misconception in question number 1.

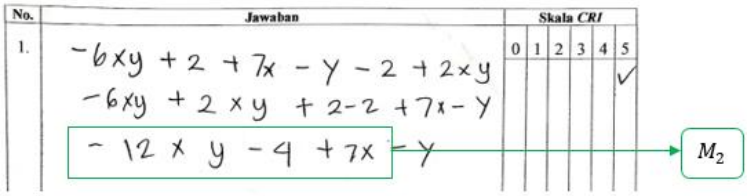

Figure 2. Answer sheet of subject 10

Description:
M_2: Operation Misconception
Here is some of the interview conversation with S10:

P : Why is -6xy multiplied by 2xy?
S10 : Because the variables are the same, miss
P : Please read the question again.
S10 : -6 times y plus 2 plus 7 times negative y less 2 plus 2 times y → H^{10}
*P : Is this *pointing variable x* multiplication operation?*
S10 : Yes miss, it's multiplication → I^{10}
P : Are you sure? Do you understand with this question?

S10 : *Yes miss, i understand.* → J^{10}
P : *Well, now how do you do this one? (pointing -6xy+2xy)*
S10 : *Because the variables are the same, which I remember if the variables were the same it could be directly multiplied, miss.* → K^{10}
P : *From $-6xy + 2xy$, which one is a variable?*
S10 : *y, that's the variable* → L^{10}
P : *Well, you said this *pointing to the addition operation at -6xy+2xy* is a plus sign, then how do you think to solve it?*
S10 : *It should be added ma'am*
P : *Okay, why it *menunjuk $-12xy$* becomes 12?*
S10 : *Because 6 times 2 is 12, miss* → M^{10}
P : *How about 6 times y, the result is?*
S10 : *Eum *mumbled**
P : *How about this * pointing to the student's final answer * how do you do it?*
S10 : *I just multiplied it, ma'am, because not all of them could be operated, so I just multiplied with the same variables.*
P : *And then?*
S10 : *As we can see variables 6 and 2 are the same, ma'am, so I multiplied 6 and 2, then this * points 2-2 * the same, so I just multiplied it, well, the rest is because the variables are different so it can't be done.* → N^{10}
P : *How do you know that? Self-study or what?*
S10 : *I'm self study and learn from school*
P : *Are you happy learning math?*
S10 : *Not really, miss* → O^{10}

Description:
S10 Keywords: $H^{10}, I^{10}, J^{10}, K^{10}, L^{10}, M^{10}, N^{10}, O^{10}$

Based on the answer and interview, S10 experienced an operation misconception. The misconception that S10 performs occurs when S10 operates the algebraic form, S10 assumes that the variable x at $-6xy + 2xy$ is a multiplication operation, so the misconception of the operation performed by S10 is performing a multiplication operation that should be an algebraic form addition and subtraction operation (K^{10}, M^{10}, N^{10}). The subject considers that to operate a similar tribe is by performing multiplication operations on the coefficient of the tribe. However, this does not apply to tribes that are not of the same kind. The concept understood by S10 is that the operation of similar tribes is carried out by multiplying the coefficients in the same tribe. Taş (2017) was found several important results such as providing incorrect explanations of the critical attributes on the examples, making incorrect classifications of the examples/not giving correct examples, under-generalization and over-generalization, inability to explain the critical attributes on the examples, and inability to distinguish the coordinate examples (p. 116). It is found that S10 is incorrect in defining a variable, S10 assumes that the definition of that variable is a multiplication operation.

According to Ojose (2015) this is an execution type of error because the student knows something but the implementation process breaks down somewhere along the line (p. 115). It suggests that the reasoning possessed by the S10 is incomplete. In the question, S10 considers that variable x is a multiplication operation (H^{10}, I^{10}) and according to S10 the variable belongs to the algebraic form -6xy i.e. only variable y (L^{10}). The concept owned by S4 is in line with Suparno's opinion that misconception is a concept that belongs to a person but the concept is not following the concept recognized by experts (2013, p.

8). Deviation of understanding a concept that is done usually occurs because S10 feels the concept that it understands corresponding to the actual concept. The causes of S10 misconceptions include associative thinking, incomplete reasoning, and low interest in studying maths lessons.

Results of Research on Subject 17

The following is the result of S17's work on question number 1:

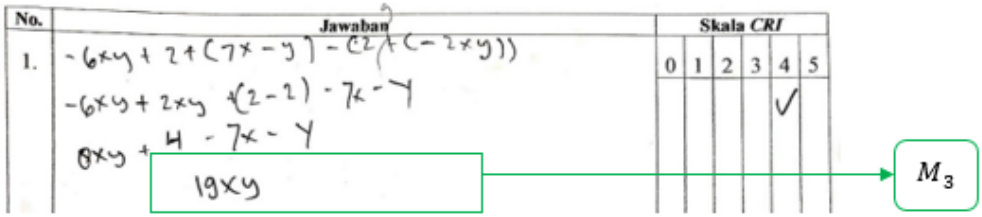

Figure 3. Answer sheet of subject 17

Description:

M_3: Variable Misconceptions

In the picture, it can be known that S17 misconceptions when operating all forms of algebra (M_3) to simplify the algebraic form of the problem. This was confirmed in the following interview:

P : *Now, please explain to me about this one *points to the 2nd line at the S2 answer**

S17 : *Eum* $-6xy + 2xy$ *becomes* $8xy$ *and then* $-2-(-2)$ *is* 4*, this *points to* $-7x - y$ *on the 3rd line* is the rest* → P^{17}

P : *You write* $-7x$ *in here*pointing second line*, but you write* $7x$ *in first line, so?*

S17 : *I think the negative sign have moved too and becomes positif*

P : *What do you mean?*

S17 : $-7x$ *from above, this* $7x$ *is positif and y is negative, so* $7x$ *times negative sign from y variable*. → Q^{17}

P : *Now how do you get the third step?*

S17 : $-6xy + 2xy$ *is* $8xy$ → R^{17}

P : *Where did you get 4 from?*

S17 : *From here *pointing 2-2 on the second line**

P : *Okay, so it's from 2-2 or anything else?*

S17 : *This one is positif *points to 2 on (**2**-2) second line*, this one is negative *points to –2 on (2 **-2**) second line* becomes 4*

P : *Can you explain more?*

S17 : *+ plus – is positive, so the answer is 4* → S^{17}

P : *How about this *points to fourth line 4*how do you solve it?*

S17 : *I summed it all,* $8xy + 4$ *is* 12 *plus* 7 *is* 19 *and put all together* $(-y)$ *becomes* xy → T^{17}

P : *Why do you do that? Have you ever solved a question like this before?*

S17 : *The question is said to be the simplest result, so I added it all up so there is only one answer, ma'am. Ever, miss* → U^{17}

P : *Are you sure?*

S17 : *Maybe, i;m not really sure* → V^{17}

P : *Do you usually in a good mood or not when studying maths?*

S17 : *I'm usually not in a good mood.* → W^{17}

Description:

S17 Keywords: $P^{17}, Q^{17}, R^{17}, S^{17}, T^{17}, U^{17}, V^{17}, W^{17}$

Based on the interview and the coding, the misconception experienced by S17 is a variable misconception. According to Booth et al. (2017), the classification of misconceptions on variables includes combining variables that do not comply with the rules, moving, deleting, or adding variables. The ability of S17 in algebraic form counting operation material especially in similar tribal counting operations is still lacking, it can be seen from the answer of S17 that $-6xy + 2xy$ the result is 8xy (P^{17}, R^{17}). S17 also argues that signs in a form of algebra should follow the negative sign (Q^{17}). S17 do the misconceptions when operating algebraic forms regardless of the rules that need to be considered such as similar tribes and non-similar tribes. S17 thinks that the algebraic form can be operated even though the tribes are not similar tribes. According to Şahin and Soylu (2011) variable is an important concept and there are various forms of representation and these are generally represented by letter symbols because letter symbols are not only more economical but also more practical and easier to write, read and understand compared to other symbols (p. 3323). But, S17 ignores variables and focuses only on summing and subtraction operations. Letters, which represent variables, may gain different meanings according to the content, and this condition causes difficulties and conceptual misconceptions in teaching the concept of variable.

Misconceptions experienced by S17 are also contained in the results of research put forward by Herutomo (2017) that misconceptions that occur in variables include conjoining summing and multiplication operations. The interesting thing that researchers found in the subject's answer was that the marks on a form of algebra would always follow the negative signs found in similar tribes as well as those in parentheses. In the interview, it is known that S17 relies on his intuition in answering question number 1 (S^{17}, V^{17}). Then S17 is also wrong in interpreting the word "simple" in the question, S17 considers that the word has the meaning that the final result of the question should be one answer only (T^{17}, U^{17}). This thought makes S17 operate the form of algebra regardless of the rules or rules contained in the form of algebra. Therefore, even a simple misconception in mathematics may cause some misconceptions in the subjects which are related to previous one (Mehmetlioğlu, 2014). The lack of S17 skills and improper thinking in working on the question can be influenced by the lack of interest in learning S17 (W^{17}). The causes of S17 misconceptions include associative thinking, incorrect intuition, lack of ability, low interest in studying maths lessons.

Results of Research on Subject 28

The CRI scale chosen by S28 is a scale of 5 which means that the subject is confident with the answer. Unfortunately, the answer given by S28 is the wrong answer.

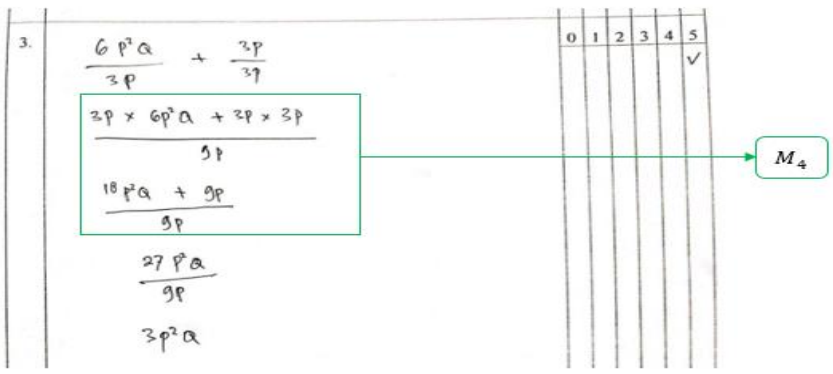

Figure 4. Answer sheet of subject 28

Description:
M4: Fractional Misconceptions

Here is a piece of conversation from the interview conducted with S28:

P	:	*Okay, please explain to me the second line.*	
S28	:	*First step is equalizing the denominator* $3p \times 3p$ *is* $9p$*. After that* $9p$ *divided by* $3p$ *is* $3p$ *and then* $3p$ *times* $6p^2q$ *is* $18p^2q$	X^{28}
P	:	*Look at the problem carefully, Do you think the denominator is same?*	
S28	:	*Ye, the denominator is same*	
P	:	*Okay, if the denominator is same, is it need to be equated again?*	
S28	:	*Yes, I remember the denominator need to be equated*	Y^{28}
P	:	*The result of* $3p \times 3p$ *is* $9p$*?*	
S28	:	*Yes, because* 3×3 *is* 9 *bu*	
P	:	*So multiplied is only for the coefficients? Variables are not multiplied too?*	
S28	:	*Yes, only for the coefficients, variables is still the same*	Z^{28}
P	:	*Now, look at this, why* $18p^2q + 9p$ *is* $27p^2q$*?*	
S28	:	*18+9 is 27*	AA^{28}
P	:	*If the sign changing becomes substraction, so the result is?*	
S28	:	$18 - 9$ *is* 9	AB^{28}
P	:	*Do you know the type of this tribe?*	
S28	:	*If I'm not mistaken, that tribe doesn't same.*	AC^{28}
P	:	*Do you think, we can adding or subtracting if the tribe are not the same?*	
S28	:	*I think it can because its only adding or subtracting the coefficient*	AD^{28}

Description:
S28 Keyword: $X^{28}, Y^{28}, Z^{28}, AA^{28}, AB^{28}, AC^{28}, AD^{28}$

Based on the results, it was obtained that S28 suffered a fraction misconception (M_4). Classification of misconceptions on fractions according to Booth et al. (2017) among them do the addition without equating the denominator, then combine the numerator with the denominator. Alghazo & Alghazo (2017) found that most students find addition and subtraction of fraction tricky because of having to find a common denominator. Fraction misconceptions performed by S28 occur when subject re-equates the denominator of the algebraic form fraction. S28 uses the concept of fractions to solve the question, but the fractional concept is incorrect because the subject still re-equates the denominator of the algebraic form fraction even though the denominator is same. Kara & Incikabi (2018) found that the mistakes that students make during the steps of fulfilling the operations and determining the numerator are also more prevalent in the fraction operation. S28 considers that fractions even though the denominator is the same must still be equalized again (X^{28}, Y^{28}). It shows that S28 still does not understand the concept of fractions that have been studied before.

In a study of Loc, Tong, & Chau (2017), it was also stated that the students solve the division operations based on rules without considering the underlying reasoning behind the operations. Then S28 lacked an understanding of multiplication operations on algebraic forms. It can be seen in the answer which is also confirmed in the conversation section of the interview that the multiplication operation on the algebraic form is operated only at its coefficient and the variable is not operated (Z^{28}). According to S28 the opinion also applies to adding operations as well as subtracting operations (AA^{28}, AB^{28}). According to S28 tribes that are not the same type can be operated both adding and subtracting operations (AD^{28}). S28 also uses his intuition when answering interview

questions (AC^{28}). The opinion expressed by S28 is incorrect, it makes researchers dig deeper related to the understanding of S28 to the form of algebra:

P : *You said that p is variable, so do you know what is variable mean?*
S28 : *I don't know the definition, but if you ask me to show it, I can show it.* → AE^{28}
P : *How about this *points to p on 3p* what is the name?*
S28 : *Variable*
P : *There is number 3 in front of a variable, in algebra topic, 3 is called?*
S28 : *Coefficient*
P : *Now look at number 1, there is a 2, what's it called?*
S28 : *Yes, it's Constants*
P : *Do you enjoy learning mathematics?*
S28 : *Not really* → AF^{28}
P : *Why?*
S28 : *It's difficult because we need to memorized the formula and the step.* → AG^{28}

Description:
Keywords S28: $AE^{28}, AF^{28}, AG^{28}$

Besides fraction misconceptions, S28 also did variable misconceptions. This is because S28 cannot answer when the researcher asks about the definition of a variable, but S28 can indicate what the researcher asked. It indicates that the ability of S28 is still lacking (AE^{28}). S28 only sums the coefficient and ignores the variables on the fractional form and directly uses the concept of fractions to answer the question regardless of the concept in the algebraic form. S28 is also less capable of defining algebraic forms such as variables. It is in accordance with the results of research submitted by Ningrum & Budiarto (2016) that the subject is still less able to define but able to show what the researchers asked. S28 felt that math lessons were very difficult and many formulas and ways to be memorized (AF^{28}, AG^{28}). This was supported by Deringöl (2019) that the reason students find fraction operations difficult is that they memorize formulae and algorithms instead of understanding fractions, while another is that they perceive the denominator and the numerator in fraction as two separate integers. It means the factors that cause S28 to experience fraction misconceptions include the wrong preconceptions of students on fractional materials that have previously been taught, incomplete reasoning, incorrect intuition, lack of subject ability, and low interest in learning S28 in mathematics. According to S28 at the time of summing operations, reduction, and multiplication operations in a form of algebra only the coefficient of the algebraic form is operated.

Results of Research on Subject 31

The misconception experienced by S31 lies in question number 1. The following is the answer sheet of S31:

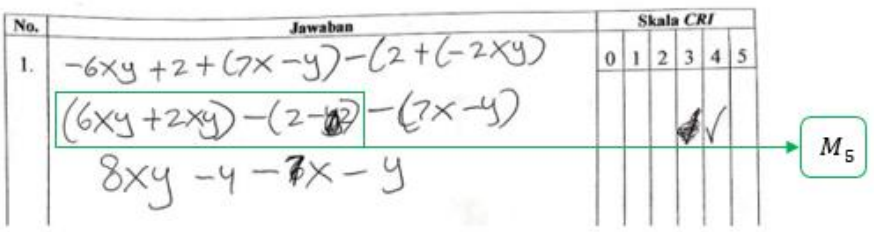

Figure 5. Answer sheet of subject 31

Description:
M_5: Negative Sign Misconceptions

In the figure, it is known that S31 misconceptions when operating a negative sign and also assumes that the sign in an algebraic form will follow the negative sign that is in the algebraic form that has like terms (M_5). Here is some conversation with S31:

P : *There are $-6xy$, why it becomes $6xy$? Can you explain it more detail?*
S31 : *This $-2xy$ times this negative sign *points to $-(2+(-2xy))$ on the first line*, $-6xy$ times with the negative sign too* → AH^{31}
P : *Where the negative sign in front of parentheses comes from? *points to $-(2-2)$**
S31 : *That's from 2*
P : *Which 2? This one? *points to 2 besides $-6xy$**
S31 : *No, this positive sign *points to 2 besides $-6xy$* times negative *points to negative sign in front of $(2+(-2xy))$ becomes negative**
P : *Wait, where is this 2 *points to 2 besides $-6xy$* placed?*
S31 : *In here *points to $-(\mathbf{2}-2)$** → AI^{31}
P : *For number 2 in here *points to $-(\mathbf{2}+(-2xy))$ where is it placed?*
S31 : *In here *points to $-(2-\mathbf{2})$** → AJ^{31}
P : *How come? Please explain it to me.*
S31 : *I remembered, this *points to 2 besides $-6xy$* times with *points negative sign in front of $(2+(-2xy))$ is negative** → AK^{31}
P : *I see. Where do you get this negative sign *points to negative sign in front of $-(7x-y)$*?*
S31 : *Here *points to $(7x-y)$ on the first line**
P : *Where is the negative sign? In here?*points to negative sign in front of $-(7x-y)$**
S31 : *Oh, that sign is equalized by me, so it's multiplication operation *points to $7x-y$ on the first line** → AL^{31}
P : *What?*
S31 : *This one is positive *points to $7x$ on the first line*, this one is negative *points to $-y$ on the first line*. So the sign must be equalized because the y sign is negative, so this part *points to adding operation in front of $7x-y$) on the first line* becomes negative sign too.*
P : *Please explain clearer*
S31 : *So it is equalized becomes this *points to $-y$ on the first line* and after that times with this *points to $7x$ on the first line**. → AM^{31}

Description:
S31 Keywords: $AH^{31}, AI^{31}, AJ^{31}, AK^{31}, AL^{31}, AM^{31}$

Based on the conversation, S31 assumes that the sign of an algebraic tribe will follow an algebraic tribe that has the same variables. As the negative sign of $6xy$ in the question turns into a positive sign, it happens because according to S31 the negative sign of $6xy$ must follow the positive sign of $2xy$ obtained from the distributive multiplication result (AH^{31}). Khalid & Embong (2020) found that students just remember rules such as negative and negative becomes positive and apply it indiscriminately, sometimes they apply this rule when adding or subtracting integers although this rule is used when multiplying or dividing number. S31 also had a misconception of negative signs that are in constants and in terms that are not similar, it can be seen that students add a negative

sign because there are other terms that have a negative sign ($AI^{31}, AJ^{31}, AL^{31}$). S31 argues that although the form of algebra is different, when there is a similar tribe and one of the signs of the same tribe is a negative sign, there is a multiplication of signs in the form of algebra. This indicates that the reasoning of S31 at the negative sign is still lacking (AK^{31}, AM^{31}). Then the researchers continued the interview to explore the factors that cause S31 to experience misconceptions as well as the properties of algebraic surgery and the third line completion step on the S31 answer. Here is the conversation:

P	:	*Please explain to me how can you get this result is* $8xy - 4 - 7x - y$*?*	
S31	:	$8x$ *is from this plus this *points to* $6xy + 2xy$**. And* -2 *substract by* -2 *is* -4*. Its different *points to* $-(7x - y)$** so just continue the step.*	AN^{31}
P	:	*Do you know the terms of addition and substraction in algebraic?*	
S31	:	*I don't understand miss.*	
P	:	*For example, if the variable from algebraic expression is different, can it you add it up or do multiplication operations?*	
S31	:	*As far as I can remember, they cannot because the variable is different.*	AO^{31}
P	:	*So, when the variable is not same, it can't to operated?*	
S31	:	*Maybe*	AP^{31}
P	:	*Do you confidence with your answer?*	
S31	:	*Sure*	AQ^{31}
P	:	*Do you enjoy mathematics learning*	
S31	:	*I don't really like it*	AR^{31}
P	:	*Why?*	
S31	:	*There are so much things to memorize*	AS^{31}

Description:
Keyword S31: $AN^{31}, AO^{31}, AP^{31}, AQ^{31}, AR^{31}, AS^{31}$

Based on the conversation, S31 made a misconception when working on a reduction operation with a term with a negative sign, namely at $-2 - (-2)$. The result of S31's answer is -4, it shows that the concept of integer counting operations, especially the multiplication distributive properties owned by S31 is not perfect and S31 does not understand the properties of integer arithmetic operations (AN^{31}). This is in line with Cangelosi, Madrid, Cooper et al. (2013) that the dual usage of the plus and minus signs as both binary and unary operators is lead to difficulties for students (p. 71). When researchers ask about the terms of the addition of algebraic forms, S31 has understood that dissimilar terms cannot be operated. This applies to all operations whether for subtraction operations, multiplication operations and division operations (AO^{31}, AP^{31}). This shows that S31 uses his intuition in answering the researcher's questions regarding the terms of the addition of algebraic forms, but this intuition is a wrong intuition. S31 considers that mathematics is a subject that requires to memorize and in subject opinion, memorization in mathematics is too much. This large amount of memorization made the interest in studying S31 low and this made S31 not really like mathematics (AR^{31}, AS^{31}).

Based on the interview, student answer and coding, S31 experienced a negative sign misconception. S31 changes the negative sign that is in a variable when S31 performs a subtraction operation with a term that is negative and S31 always assumes that the sign of a variable will follow algebraic terms that have similar variables. Factors that cause S31 to experience misconceptions on a negative sign include wrong preconception of S31 on a negative sign, incomplete reasoning, wrong intuition, and low interest in learning S31 in mathematics lessons.

▪ CONCLUSION

Misconception of equations is changing the signs in the algebraic form when eliminating variables. The misconception of the operation carried out is operating an algebraic form that is incompatible with the operation that should be performed. The variable misconception carried out is operating the algebraic form without paying attention to the rules or rules that must be considered in the algebraic form. The misconception of fractions is that they equalize the denominators in the algebraic fraction. The misconception of a negative sign is changing the negative sign that is in a variable when the subject performs a subtraction operation with a term that is negative and the subject always assumes that the sign on a variable will follow the algebraic term that has the same variable. The factors that cause misconceptions experienced by students include: wrong preconceptions, students' associative thinking, humanistic thinking, incomplete reasoning, wrong intuition, student abilities and student interest in learning.

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