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# Strategies and Representation Employed by Primary School Students in a Functional Relationship: A Focus on Generalization Process

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**Abstract:** This investigation endeavours to investigate the strategies and representations employed by primary school students in the process of generalising patterns.. The research design employed to accomplish the objectives of this study is a case study design that explores the functional thinking of students. Data were collected from 16 students grade 5 primary school students through written tests related to generalizing pattern and interviews. Subsequent procedures involved interviewing 4 representative students to obtain comprehensive information regarding their responses to the written test. Students who used the recursive strategy focused on changing one quantity and could not make generalizations and students who used the correspondence strategy managed to build generalizations between pairs of corresponding variables and could use the generalization results appropriately. Students produce two categories of representations when generalizing a pattern. The majority of them employed verbal representation to represent the generalization, while the remainder employed pictorial representation. The research concludes that these distinctions are the result of their emphasis on pattern identification: students who observe recursive patterns are more likely to observe changes in a single variable, while correspondence patterns are associated with the corresponding pair of variables.

**Keywords:** recursive patterns, correspomdence, functional relationship, representation, generalization.

## ▪ INTRODUCTION

Algebraic thinking is a cognitive process structured around the recognition and articulation of generality (Kaput, 2017). Algebraic thinking (known as early algebra in primary and middle school) is crucial because it fosters the identification of mathematical structure and relationships rather than computational fluency and isolated arithmetic (Cai & Knuth, 2005). This is due to research indicating that the transition from arithmetic to algebra should be integrated into elementary mathematics learning.

Several subjects still need to be thoroughly examined in contemporary research. Early algebra has been implemented in the curricula of primary grades in wealthy nations, including the United States, Spain, and Japan (NCTM, 2000; Wilkie, 2016). Nevertheless, primary school mathematics in Indonesia does not incorporate early algebra, which poses challenges for students when they transition to high school and encounter algebra for the first time. Furthermore, despite the prevalence of geometric patterns in representing the cognitive development of primary school students, there has been a lack of in-depth exploration generalization of functional relationships between variables in contemporary studies.

Moreover, primary school students in Indonesia experience various challenges related to the transition from arithmetic thinking to algebraic thinking. Their ability to translate real-world problems into mathematical models (mathematization) is a significant hurdle (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014). Students often make errors in formulating equations (Jupri, Drijvers, & Van Den Heuvel-Panhuizen,

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2014), applying arithmetic operations within algebraic contexts is another area where students face difficulties (Setiawati et al., 2017) and generalizing patterns and understanding the equality concepts, which are essential for algebraic thinking (Wahyuni et al., 2020). This includes performing operations on algebraic expressions and integrating their arithmetic knowledge into algebra.

Broadly speaking, Kaput (2018) suggested that the two key components of algebraic thinking in the early grades are the symbolization of mathematical relationship and generalisation. As a result, students pay attention to analysing quantity connections and articulate their regularities. This research, in particular, focuses on functional thinking where the function is the pivotal mathematical concept. In the functional thinking approach to early algebra (Schliemann et al., 2006), students generalize using the concept of functions and articulate these generalizations through various representations. There are illuminating accounts of how primary school students can develop generalization skills and algebraic understanding (Blanton & Dartmouth, 2005; Mata-Pereira & da Ponte, 2017).

Different research has shown how students recognize, express, and generalize regularities in mathematical problem that involve function (Cooper & Warren, 2008; Schliemann et al., 2006; Utami et al., 2023). Nevertheless, several subjects have not been thoroughly examined in the current scholarly works about a functional approach to early algebra. Most studies primarily examined students' functional thinking and generally reported their consideration of the relationships between variables in various problems (Cañadas et al., 2018; Panorkou & Maloney, 2016; Syawahid et al., 2020).. Some of these studies focus on the variety of strategies used or what elements determine their use, mainly with figural pattern tasks (El Mouhayar & Jurdak, 2015; Wilkie & Clarke, 2016) and described types of reasoning related to strategies such as figural or numerical (Rivera & Becker, 2005).

Kaput (2018) defined that generalization entails the extension of the recognized regularity in a task involving the establishment of a relationship between quantities, integrating the external representation of that regularity through a general rule. Research focusing on generalization in the latter years of primary school students showed that students used a variety of strategies (El Mouhayar & Jurdak, 2015; Maula et al., 2024; Wilkie & Clarke, 2016). Students can use different strategies when progressing towards the generalization of functions (Cooper & Warren, 2008) and there are three types of relationship such as recursive patterns, correspondence, and covariation (Smith, 2017).

Representation is closely linked to generalization and algebra (Kaput, 2018; Radford, 2018). It supports the explanation and justification of generalizations by embodying the meaning of operations and illustrating relationships in specific instances of a general claim (Schifter & Russell, 2022) and aids in abstracting of structural similarities across different problem situations (Sriraman, 2004), which is essential for formulating generalizations. Representations help students process and reveal mathematical ideas by transforming problems into different forms (Isyam & Hidayati, 2022). This transformation and manipulation of representations are central to mathematical activity and cognition. Research Ellis (2011) has shown multiple interrelated processes, including the use of representations support productive generalizations in classroom settings. These processes involve cyclical interactions between teachers and students, promoting generalization development and refinement.

Generalization research has distinguished differences in students' representations of generalization. Students benefitting from early algebra instruction proved able to identify inter-variable dependence and their use of tabular verbal, or symbolic representation progresses with their grade ((Blanton & Kaput, 2011; Carraher et al., 2006). The research context of functional thinking in primary school, several researchers have studied the different representations when solving generalization tasks. These showed that second graders with no prior instruction in functional tasks used numerical or verbal representations to answer questions but did not generalize the functional relationships identified (Pinto et al., 2022). In addition, Apsari et al., (2020) focuses on the generalization of geometric shapes in junior high school students and the generalization of geometric shapes in junior high school students. It shows that the students' shifting from arithmetic to algebraic thinking is highly impacted by their clarity in seeing the geometrical structure of a pattern.

Therefore, and considering the above reasons, this study's objective is to describe primary school students' functional thinking when generalize the patterns. Consequently, the research question of this study is how strategies and representations employed by primary school students in generalizing the pattern?

## ▪ METHOD

### Participants

This research occurred in one of the elementary schools in Depok City, West Java, Indonesia. The participants involved in this study were 16 grade 5 elementary school students.

### Research Design and Procedure

The research employs a qualitative approach, explicitly utilizing a case study methodology. Creswell & Poth (2018) posits that a case study is an empirical research methodology employed when several sources of evidence or data are available. This study aimed to elucidate the strategy and representation of students, with a particular emphasis on the specific work each student conducts when addressing tasks that suggest functional relationships.

We initiated the data collection process by administering an individual test to each of the sixteen fifth-grade students. The test consisted of two questions as the initial entry point to assess the students' functional reasoning concerning the strategy and representation involved in generalizing patterns. Moreover, the interview commenced after acquiring the students' exam responses. Based on a preliminary selection process that involved observing the written test responses of all students, four students were chosen to participate in extra individual interviews in this study.

The objective was to accumulate more comprehensive data regarding the functional relationships that students generated in each problem. As a result, the researcher conducted the interviews and audio-recorded for approximately 10-15 minutes per student. In the semi- structured interviews, the written responses of each student were presented, and they were encouraged to explain their reasoning. The interview was conducted according to a set of general starting questions designed to investigate students' functional thinking. These questions could be modified as needed to accommodate flexibility during the interview.

**Instruments**

The researchers were the primary instrument in this study and were directly involved in all data collection processes. The supporting instruments consisted of a specified test regarding functional thinking tasks through a table, students interview guidelines, and documentation (audio recordings). We initiated the data collection process by administering an individual test to each of the sixteen fifth-grade students. The test consisted of two questions as the initial entry point to assess the students' functional reasoning concerning the strategy and representation involved in generalizing patterns.

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**Data Analysis**

The researchers were the primary instrument in this study and were directly involved in all data collection processes. The supporting instruments consisted of a specified test regarding functional thinking tasks in a table, student interview guidelines, and documentation (audio recordings). We initiated the data collection process by administering an individual test to each of the sixteen fifth-grade students. The test consisted of two questions as the initial entry point to assess the students' functional reasoning concerning the strategy and representation involved in generalizing patterns.

All statements in the data that substantiated the research questions were assigned to their central notion. Distinct statements have the same code if they convey the same concept. All two problems coded in this research. The following phase involved validating the coded segments to determine the final initial code structure. The researchers engaged in a discussion to achieve this, which included merging codes with low frequency into a single code. As a result of the meticulous and comprehensive code labelling, 16 codes were identified, with 8 codes assigned to both Problem 1 and Problem 2.

## ▪ RESULT AND DISSCUSSION

This study describes the generalization strategies and representations employed by primary school students. It enhances the current literature on algebraic thinking and generalization by thoroughly analysing how students employed generalization techniques to produce and demonstrate regularities, exhibit various structures, and depict their generalizations. In contrast to tasks involving predetermined patterns, students were tasked with establishing a regularity derived from their own outputs. his strategy was demonstrated through diverse structures of functional relationships and various representations of generalization.

All students responded to strategize and represent patterns in the two problems. The proposed functional task was found to promote the use, recognition, and representation of variables and their relationships, thereby fostering algebraic thinking and simultaneously introducing students to functions. The findings validated, complemented, and reinforced the results of prior authors (Levin & Walkoe, 2022; Wilkie, 2016). The student's responses to each problem from the written test and individual interview are documented in Table 1, which summarizes the functional relationships. Furthermore, Table 1 illustrates the themes, categories, and codes that students employed in their responses.

By the data in Table 1, it was determined that each code should be assigned a distinctive designation to facilitate future explanations. For example, the code 1-SR1 represents Problem Number 1, Recursive Strategy, and first code, which corresponds to "finding the pattern without connection (10)". The reference sources derived from students' responses to the written test and interview transcripts are indicated by the number provided after each code's definition. As a result, the phrase "finding the pattern without connection" is mentioned ten times in the responses.

**Table 1.** Themes, categories, subcategories, code, and code's definition result in students' responses

| Theme | Category | Subcategory | Code | Code's Definition |
|---|---|---|---|---|
| **Problem 1** | | | | |
| Recognizing the functional relationship | Strategies | Recursive Pattern | 1-SR1 | Finding patterns without involving connections (10) |
| | | | 1-SR2 | Considering the number to be found as the sum of all previous numbers (3) |
| | | Correspondence | 1-SC1 | Finding patterns with a connection (4) |
| | | | 1-SC2 | Generalizing the formula of a pattern correctly (3) |
| | | | 1-SC3 | Generalizing the formula of a pattern incorrectly (6) |
| | Representation | Verbal | 1-RV1 | Communicating pattern Generalizations through written language (12) |
| | | Symbolic | 1-RS1 | Communicating pattern generalizations through drawn triangle or square (2) |
| **Theme** | **Category** | **Subcategory** | **Code** | **Code's Definition** |
| **Problem 2** | | | | |
| Recognizing the functional relationship | Strategies | Recursive Pattern | 1-D1 | Did not answer the question (2) |
| | | | 2-SR1 | Finding patterns without involving connections (8) |
| | | | 2-SR2 | Considering the number to be found as the sum of all previous numbers (2) |
| | | Correspondence | 2-SC1 | Finding patterns with a connection (3) |

| | | | | |
|---|---|---|---|---|
| | | | 2-SC2 | Generalizing the formula of a pattern correctly (5) |
| | | | 2-SC3 | Generalizing the formula of a pattern incorrectly (7) |
| | Representation | Verbal | 2-RV1 | Communicating pattern generalizations through written language (13) |
| | | Symbolic | 2-RS1 | Communicating pattern generalizations through drawn triangle or square (1) |
| | | Difficulty | 2-D1 | Did not answer the question |

Generally, the difficulty category is the least evidence provided by students, followed by the strategy (correspondence and recursive patterns) and representation (verbal and pictorial) in all problems. According to the student's responses, the majority of them are capable of recognizing patterns that are not explicitly conveyed in each problem. Furthermore, students appear to favor recursive patterns as their preferred approach to derive patterns from particular values. This research anticipated that learners would progressively focus on the recursive patterns present in all problems. Nevertheless, the outcome shows that Problem 1 and Problem 2 have a higher prevalence of recursively producing patterns, respectively. Typically, only a small number of students have been able to extract the generalization into formulas. Additionally, the majority of students employ verbal representations when generalizing a pattern, and two students are unable to represent patterns in any capacity.

To illustrate each functional relationship that has been identified, the subsequent section delineates the methods by which students strategize to generate patterns and develop formulas to represent the relationship between variables.

**Strategies Employed by Students to Identify Functional Relationships**

The first problem required students to input a single value for both the independent (square) and dependent (triangle) variables. In this scenario, students were obligated to complete the void in the table provided and ascertain the relationship between the square and triangle on the table. The following diagram shows the strategies used by students in generalizing patterns in Problem 1 and 2.

Based on the diagram in Figure 1, most primary school students use the recursive pattern rather than the correspondence strategy to generalize a pattern. The recursive strategy is more intuitive and straightforward for younger students (El Mouhayar & Jurdak, 2015), making it suitable for near-generalizing tasks (Amit & Neria, 2008). However, it is less efficient for far-generalizing tasks and does not easily lead to the derivation of a general formula. In contrast, while more complex, the correspondence strategy is efficient for any term in the sequence. It supports the development of algebraic thinking, making it more suitable for higher grades and far-generalizing tasks (Sterner, 2024). Three of the sixteen students demonstrate proficiency in generalizing patterns through the use of correspondence strategies in this problem. In contrast, the remaining variables were addressed through a recursive strategy in their response, and the relationship between them was not generalized.

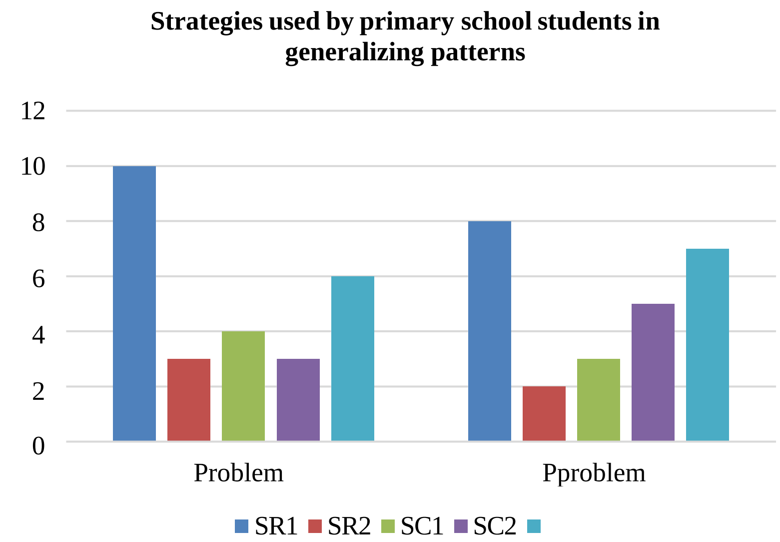



**Figure 1.** Strategies employed by primary school students in generalizing patterns

After their written response, we interviewed two students, one from correspondence and the other from recursive patterns, to obtain comprehensive information regarding their strategic approach to generalizing the pattern.

**Table 2.** Example of students' interview transcripts in generating the pattern from problem 1

| Question | Students | Students' Answer |
|---|---|---|
| 1. How do you determine the value of the two vacant spaces in the table? | S14 | I merely attempted to peruse the pairings in the square and triangle columns. Given that $1 \times 4 = 4, 2 \times 4 = 8$, it follows that $6 \times 4 = 24$. Consequently, 6 will be paired with 24. Similarly, the number 36 in the triangle column will be paired with 9, which is produced by the equation $\ldots \times 4 = 36$, which is 9. In conclusion, if the square is 1, then the triangle is 4. If the square is 2, then the triangle is 8, and so on. |
| 2. What can be deduced from the relationship between square and triangle on the table? | S02 | If the number in the square column is multiplied by 4, the number in the triangle column will be obtained from the table in this problem. As an illustration, multiplying 4 by 1 yields 4. Similarly, multiplying 4 by 2 yields 8, by 3 yields 12, and by 6 yields 24. Therefore, 6 will be paired with 24 and $\ldots \times 4 = 36$ *t*he outcome of this multiplication is 9, so 9 is paired with 36. I have concluded that a triangle results from multiplying a square by 4. |

During the interview, S14 clarified that he only attempted to multiply each number in the square column by 4 until the third column, which is 3×4=12. S14 observed this pattern without a correct connection (2-SR1). S14 was not able to determine how the change in square coloumn (independent variable) could relate to the change in triangle coloumn (dependent variable) in local instances. Nevertheless, S02 recognized the pattern in table. He clarified that the number in the triangle column will be generated by multiplying the square column by 4 (2-SC1). For instance, 1 be paired to 4, 2 be paired to 8, 3 be paired to 12, 6 be paired to 24, and 9 be paired to 36.

According to the findings above, students who could identify the relationship between variables (independent and dependent) in general are categorized as having functional thinking ability at the correspondence level (Smith, 2017). The result corroborates (Doorman et al., 2012)'s assertion that students at the correspondence level can comprehend functions as mathematical objects that can be represented in various representations. In this study, tasks given provides the function in a table, and each student must transform it another representation. Typical correspondence-level students could determine the function formula for any given problem by addressing the phases of pattern generalization. The stage, according to (Radford, 2010) belongs to the symbolic generalization, namely that students can express the generalization with alphanumeric symbols.

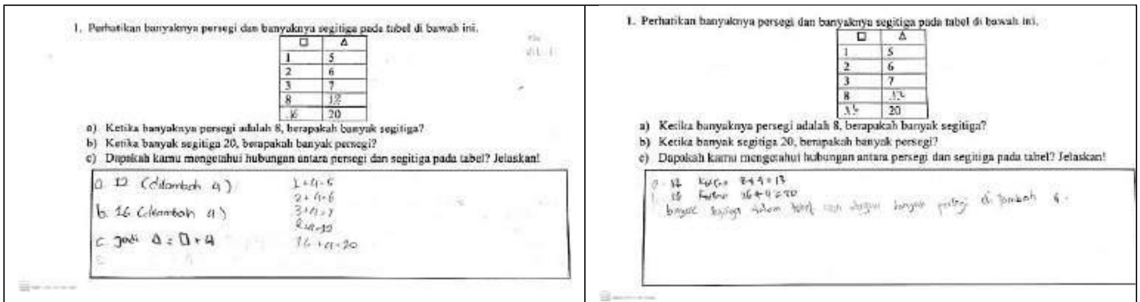
Additionally, most students who could generate patterns with correspondence demonstrated no difficulty in abstracting the general value of the dependent variable. They utilised a local rule to describe the relationship between two variables, allowing for the calculation of the corresponding value of the dependent variable based on a specific independent variable value (Wilkie & Ayalon, 2018). They could generalise the rule to be applied to any value by identifying a local rule. According to (Usiskin, 1988), the correspondence strategy to functions is a critical component of school algebra, as it necessitates students to perceive a function as an object that can be transformed into other representations (Lichti & Roth, 2019).

One of the contributions of this study is the differentiation between two categories of functional strategy, which are contingent upon the degree of regularity referenced in the structure. The identification of various functional relationship structures influenced how students interpreted and proposed regularities and established relationships. Functional strategy was more frequently linked to full regularity; however, there were instances where the structures were represented incompletely.

In contrast to students at the correspondence level, (Smith, 2017) considers students who can identify variations in a restricted number of sequence patterns to have functional reasoning ability at the recursive patterns level. At this level, students perceive functions as a request for calculation or as a process of input-output assignment. It could be seen from the student's responses in the individual interview that focuses solely on the relationship between the independent variable and the dependent variable in the columns that have been paired. The findings are consistent with prior (Bajo-Benito et al., 2023; El Mouhayar & Jurdak, 2015; Wahyuni et al., 2020) that has demonstrated that secondary graders frequently identify patterns by calculating the difference between two consecutive terms and subsequently incorporating it into a given quantity to generate the subsequent term's quantity. In this instance, the student's comprehension of variables is restricted to the particular unknown and does not extend beyond "variability" as it relates to the function (Usiskin, 1988).

**Representations Employed by Students to Generalizing Functional Relationship**

Students provided broad responses to each question. They articulated the generalization of a regularity verbally, symbolically, or failed to provide any representation due to difficulties. At the Problem 1, students identified to see the functional relationship between corresponding pairs of variables (square and triangle) and also managed to determine its representation. There are two types of representation that students wrote such as verbal representation and pictorial representation. Students who recursively identified the functional relationship between variables could not write down the representation correctly. Some students wrote: “if the value of 1 represents a square, then the corresponding value for a triangle is 5. Similarly, if the value of a square is 2, then the corresponding value for a triangle is 6, and this pattern continues” (1-SR2). They struggled to understand and represents the relationship between two variables (1-SR1). In contrast, students that managed to find the functional relationship between corresponding pairs of variables also managed to determine the representation for the relationship between square and triangle (1-SC2). There are two types of representation that students wrote symbolically as triangle = square+4 (1-RS1) and verbally as “the number of triangles in the table is equal to the number of squares add 4” (1-RV1).



(a) (b)

**Figure 2**. Example of students' representation in problem 1

Furthermore, students that provided a correspondence way in relating two variables and not merely focused on the change in the dependent variable in problem 2 managed to state the representation triangle = 4×square. Meanwhile, students who recursively identified the functional relationship between variables could not write down the representation correctly. Students were unable to accurately put down the representation when they recursively determined the functional connection between variables. A few students wrote “if square is 1 then triangle is 4, how many times so that the triangle is 4” (2-SR1). They encountered difficulties in comprehending and producing representations of the link between the triangle (dependent variable) and the square (independent variable).

A further contribution is the analysis of the representations employed by primary school students to depict generalization in relation to the functional strategy, the identified regularities, and the utilized structures. The findings illustrate the diverse and adaptable

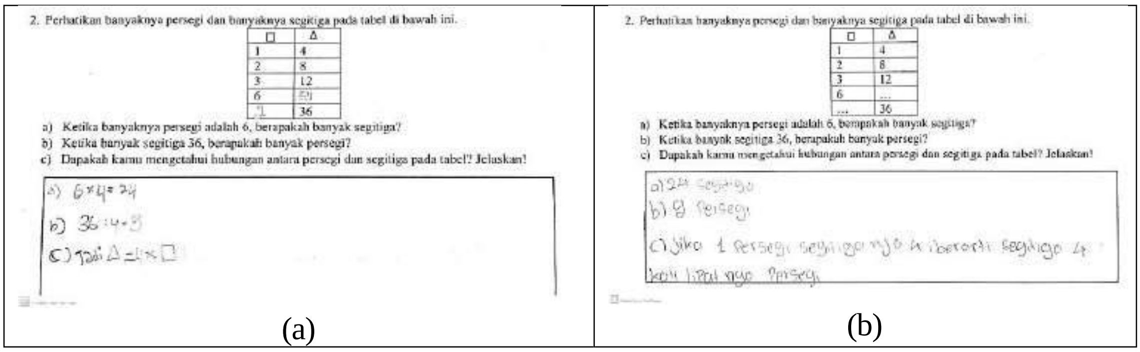


**Figure 3**. Example of students' representation in problem 2

ways in which students utilize various forms of representation, beyond traditional symbolic methods, to structure and express their understanding of algebraic concepts.

The findings of this paper present a reorganization of the representations of generalization, differentiating between students who successfully represented generalization and those who, despite not achieving this, demonstrated indications of recognizing a regularity through numerical calculations. The recognition is significant as it indicates that students could identify commonalities among specific cases and engage with functional relationships implicitly, demonstrating methods of expressing regularities beyond algebraic symbolism, thereby complementing findings from other studies (Wilkie, 2016). The verbal representation of generalization was more frequently utilized by primary school students. The widespread use of verbal representation revealed notable inconsistencies in its description of the structures of functional relationships. The outcome may stem from the ambiguity inherent in this representation (Molina, 2014) where the variables and functional relationships are clearly defined. The prominence of this representation in elementary school aligns with findings from prior studies (Pinto et al., 2022) which suggest that students' familiarity contributes to its prevalence (Merino et al., 2013) and few opportunities with varied representations as contrasted by results in early algebra research (Blanton et al., 2017).

A limited number of students identified symbolic representation. Students utilized symbolic representation to articulate a variety of complex structures, thereby demonstrating underlying regularities. Multiple representations utilized words as variables in quasi-algebraic expressions, serving as a precursor to the symbolic representation of generalization or semi-symbolic representation (Amit & Neria, 2008). Many students implementing that procedure substituted the verbal representation of the variable with the letter n when instructed.

In another study (Ramírez, Cañadas, et al., 2022) pictorial representations supported the generalization of some students. Nonetheless, only a limited number of students utilized drawings. The underlying structure of the pattern could have been extracted and generalized, potentially employing a figural term as a generic example, as inferred from (Küchemann, 2010). The findings indicate that students have limited familiarity with generalization tasks, highlighting the challenges these tasks present for primary school students and their experiences with diverse representations.

Primary school students often face significant challenges when transitioning from verbal to symbolic representations. This struggle can be attributed to several interrelated factors, including cognitive development (Veraksa & Veraksa, 2016) and lack of early intervention (Adamuz-Povedano et al., 2021). The process of translating verbal information into symbolic form requires significant cognitive effort. Students must maintain semantic congruence between the source (verbal) and target (symbolic) representations, which can be challenging due to misinterpretation (Nurrahmawati et al., 2021). Early interventions that focus on developing both arithmetical and algebraic thinking can help mitigate future difficulties. However, such interventions are not always implemented, leading to challenges when students encounter symbolic representations later in their education (Adamuz-Povedano et al., 2021)

Difficulties and issues were identified in relation to other studies, with implications for teaching (Sari et al., 2024; Warren, 2005; Wilkie & Clarke, 2016). The efficient and appropriate use of multiple representations, the identification of regularities linked to variability, the transition between different types of representation, the accurate formulation and representation of the structure reflecting the identified regularity, the coordination of various reasoning modes (e.g., numerical, visual), and the processes of validating and justifying generalizations are included. (Kilhamn et al., 2022) argue that students possess the potential for algebraic thinking and the ability to generalize and articulate generalizations; however, these skills require cultivation and development. Research in early algebra (M. Blanton et al., 2018; Carraher et al., 2006) indicates that students can comprehend and manipulate variables represented by letters as indeterminate quantities or to express and generalize functional relationships upon exposure to these concepts. This research indicates the importance of exposing students to generalization contexts while also incorporating approaches that integrate various components of algebraic thinking and mathematical skills.

In summary, this study reveals that the students' approach to problem-solving is significantly influenced by the specific function rule allocated to each problem. Initially evidenced for the correspondence level, students gradually transitioned to the recursive patterns level as the function's rule became more intricate, from $y = ax + b$ and $y = ax$. Likewise, pupils at the recursive patterns level gradually could not recognize existing patterns. Consequently, the results of this study extend those of previous research (Blanton et al., 2019; Pinto et al., 2022; Stephens et al., 2017) by demonstrating that the diversity of representations employed by students and the diversity of levels of students' functional thinking abilities are both influenced by various types of functions.

Functional thinking should be introduced early and developed progressively throughout elementary school. This gradual approach helps students build a strong foundation in understanding relationships between variables (Ding et al., 2022; Tanışlı, 2011). One way is to design a task that incorporates functional relationships that can enhance students' algebraic thinking. For instance, tasks that involve linear function tables or pattern generalizations can help students understand and generalize correspondence relationships (Ding et al., 2023; Syawahid et al., 2020). Instructional sequences that scaffold learning from simple to more complex tasks can support students in developing sophisticated algebraic thinking (Chimoni et al., 2021; Stephens et al., 2017).

Based on the findings from this study, teachers can apply the CPA approach (Bouck et al., 2017), encourage pattern formation and generalization (M. L. Blanton & Kaput, 2005), and use word problems with explicit functional relationships (Ramírez, Brizuela, et al., 2022). Implement the CPA approach, which moves from concrete manipulatives to representational drawings and finally to abstract symbols. This method has been effective in teaching functional-based mathematics (Bouck et al., 2017). In addition, arithmetic content should be extended to include opportunities for pattern-building, conjecturing, and generalizing mathematical relationships. This approach helps students see the broader connections between quantities and establish classroom norms that encourage students to justify and discuss their mathematical reasoning, fostering a deeper understanding of functional relationships (M. L. Blanton & Kaput, 2005). Moreover, design word problems that explicitly involve functional relationships, such as additive functions, to prompt functional thinking. These problems should encourage students to generalize relationships verbally or with generic examples rather than symbolic representation (Ramírez, Brizuela, et al., 2022). Include tasks that require students to generalize from specific to general cases. This helps them transition from operational strategies to more abstract functional strategies.

## ▪ CONCLUSION

Students who used the recursive strategy focused on changing one quantity and could not make generalizations and students who used the correspondence strategy managed to build generalizations between pairs of corresponding variables and could use the generalization results appropriately. Students produce two categories of representations when generalizing a pattern. The majority of them employed verbal representation to represent the generalization, while the remainder employed pictorial representation. The research concludes that these distinctions are the result of their emphasis on pattern identification: students who observe recursive patterns are more likely to observe changes in a single variable, while correspondence patterns are associated with the corresponding pair of variables.

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