



## Students' Reversible Thinking Ability in Solving Quadrilateral Problems

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**Abstract:** Students' ability to engage in reversible thinking can enhance their problem-solving skills. Reversible thinking allows students to consider various perspectives, explore different options, and determine the best solution. Therefore, this study aims to describe students' abilities to solve reversible thinking problems in the context of quadrilaterals, specifically rectangles. This research uses a qualitative method. The participants in this study were three junior high school students from Jambi, Indonesia, who demonstrated sufficient mathematical abilities. This study found that students could solve forward-thinking problems effectively but faced challenges with reversible thinking problems. This difficulty stems from students' lack of familiarity with problems that require reversible thinking and their struggles with modeling mathematical scenarios from word problems. The study emphasizes the need to introduce more non-routine problems and exercises that encourage the exploration of various problem-solving approaches so that students can develop more flexible thinking skills.

**Keywords:** reversible thinking, mathematics education, problem-solving, non-routine problem.

### ▪ INTRODUCTION

Mathematics is a subject taught at every level of education, from elementary school to university. Learning mathematics involves understanding basic concepts and honing the ability to formulate and solve complex problems (Amir, 2015). Therefore, mathematical skills are essential for students (Rejeki & Rahmasari, 2022). One of these skills is problem-solving. Problem-solving is a process in which a person uses all their knowledge, skills, and understanding to find solutions to given problems (Annizar et al., 2020; Widodo et al., 2021).

The ability to solve problems is a crucial skill that students must master in learning, as it plays a vital role in developing the thinking abilities and skills needed to handle problems in a structured, comprehensive, and logical manner. This process involves students applying mathematical concepts they have learned to various situations to find solutions to difficulties that cannot be solved directly. Problem-solving involves cognitive processes and requires deep, creative, and solution-oriented thinking to overcome various challenges (Arjudin et al., 2024; Palupi & Andrijati, 2024).

Reversible thinking skills can support problem-solving skills that require students to generate solutions or strategies. This ability is one of the mathematical competencies students need to improve their problem-solving skills (Prabawanto, 2023; Simon et al., 2016). Through reversible thinking, an individual can view something from one perspective and its opposite. Students with this competence can solve complex problems and maximize their problem-solving abilities (Maf'ulah et al., 2019; Maf'ulah & D Juniati, 2019). Especially in the context of problems involving multiplicative relationships that can be reversed. These problems require problem solvers to reverse their multiplicative thinking to find a solution. For example, in solving problems involving linear equations of the form  $ax = b$ , problem solvers must be able to reverse the multiplication operation to find the value of  $x$  (Hackenberg, 2005). Thus, reversible

thinking refers to the ability to perform mathematical operations and then reverse them to verify the result (Hackenberg, 2005; Klimov & Shamir, 2003; Norton, 2016).

Reversible thinking involves opposite activities to achieve the desired result. Inlehder & Piaget (2002) stated that a reversible process means a process that can return to the initial state with opposite steps—a method in which one starts from the desired goal state and works backward toward the initial state. In other words, reversible thinking is a cognitive activity in finding solutions when the final result is known and one is asked to find the initial condition (Maf’ulah & Juniati, 2019; Olive & Steffe, 2001). Therefore, to develop students’ reversible thinking, teachers must provide extensive practice with various problems involving inverse operations (Kang, Mee-Kwang & Lee, 1999).

Generally, students solve problems that require forward-thinking, where the initial state is known, and the goal state is asked for (Prabawanto, 2023), meaning students find the final outcome given the initial state. In contrast, reversible thinking means students find solutions by producing the inverse of the operation. In this case, it means solving problems where the goal state is given, and the initial state needs to be determined (Maf’Ulah et al., 2019; Pebrianti et al., 2023). A summary can be found in Table 1 below.

**Table 1.** Types of problems given to students

<b>Problem Type</b>	<b>Forward Thinking</b>	<b>Reversible Thinking</b>
Description	Problems where the initial state is known and the goal state is sought	Problems where the goal state is known and the initial state is sought

Previous research has analyzed reversible thinking abilities in the context of functions, conducted by (Maf’ulah & Juniati, 2020) and (Ikram et al., 2020), as well as research on reversible thinking in fractions, conducted by (Dougherty et al., 2015) and (Prabawanto, 2023). However, there is still limited research analyzing reversible thinking abilities in geometry. Therefore, the researcher studied rectangles, a part of geometry. A rectangle is a parallelogram with right angles, and a square is a rectangle with all sides of equal length (Billstein et al., 1993). Based on the above explanation, this study aims to describe students’ abilities in solving reversible thinking problems related to quadrilaterals, specifically rectangles.

▪ **METHOD**

**Research Design and Procedures**

This research employed a qualitative method with a case study design to explore students' difficulties in reversible thinking to solve quadrilateral problems. The steps undertaken in this research follow the qualitative research guidelines (Creswell, 2014), which are: (1) Identification of the research problem; (2) Selection of research design; (3) Data collection; (4) Data analysis; (5) Data validation; (6) Data interpretation and reporting.

The study began with identifying the research problem and selecting the research design, where the researcher aimed to describe students’ abilities in solving reversible thinking problems in quadrilateral topics and determined that a qualitative study was the appropriate design. Next, the researcher prepared research instruments, including tests and interview guidelines, and collected data using these instruments. The researcher

systematically analyzed the data based on students' test results and interviews. Data validation was conducted using multiple research instruments, including tests and interviews, to ensure credible results. Finally, the researcher interpreted the meaning of the test and interview data and reported the findings by providing a detailed description of the research problem.

### Participants

Data were collected from an excellently accredited junior high school in Jambi Province, Indonesia. The researcher used purposive sampling techniques to select three seventh-grade students with good academic averages who had studied quadrilateral topics. Then, the researcher interviewed these three students to confirm their test answers.

### Instruments

The instruments used for data collection were tests, interview guidelines, and documentation in the form of photos and audio recordings. The test instrument consisted of four questions to identify students' reversible thinking in solving quadrilateral problems. The test included two questions requiring forward thinking and two questions requiring reversible thinking. The instruments were validated by involving lecturers from the Mathematics Education program to assess the suitability of the instruments developed by the researcher. The specific test questions given to the students are shown in Table 2.

**Table 2.** Student test instrument

Indikator Soal	Deskripsi	Nomor Soal	Soal
Forward Thinking	Problems where the initial state is known and the goal state is sought	1 dan 3	Mr. Adi owns a rectangular piece of land. The length of the land is 10 meters and the width is 7 meters. What is the area of Mr. Adi's land?
			A chessboard is a square with each side measuring 25 cm. Determine the area of the chessboard.
Reversible Thinking	Problems where the goal state is known and the initial state is sought	2 dan 4	The perimeter of a rectangular table is 44 cm. If its width is $\frac{3}{8}$ times its length, <u>what is the area of the table?</u>
			Determine the perimeter of a square card with an area of 64 cm <sup>2</sup> .

### Data Analysis

The research data, including test results and interview recordings, were collected and analyzed. The validity of the data was tested using triangulation techniques, which included comparing the test results with the interview findings to ensure consistency and validity of the information. Data analysis in this study followed the model proposed by Miles and Huberman, consisting of three main stages: (1) Data reduction: At this stage, data analysis began by filtering and refining relevant information from the test and interview results obtained from the field. Data on students' reversible thinking and forward-thinking abilities in solving quadrilateral problems were systematically classified and organized. (2) Data presentation: The reduced data were then presented in a more

structured format to facilitate the drawing of conclusions. In this study, the data were classified and described based on indicators of students' reversible thinking and forward-thinking abilities in solving quadrilateral problems. The data presentation was done using tables and descriptive narratives showcasing the findings from the students' thought processes. (3) Concluding: The final stage involved concluding the analyzed data. Conclusions were drawn based on the analysis of students' reversible thinking and forward-thinking abilities in solving quadrilateral problems, expressed in the form of descriptions of the student's thinking processes

▪ **RESULT AND DISSCUSSION**

The thinking process observed in this study focuses on students' reversible thinking ability in solving quadrilateral problems, specifically rectangles. A reversible thinking test was administered to three students selected through purposive sampling. The researcher chose students with fairly good mathematical abilities. Each student worked on four questions, consisting of both reversible thinking and forward-thinking problems for comparison, followed by an interview regarding their answers. Below are the students' responses to each type of problem.

**Table 3.** Student test results data

Indicator	Question Number	Student Answered Correctly	Student Answered Wrong
Forward Thinking	1	3	0
	3	3	0
Reversible Thinking	2	0	3
	4	1	2

The students' test results indicated that they encountered difficulties in solving problems that assessed reversible thinking abilities, where all students provided incorrect answers to question number 2, and only one student answered correctly for question number 4. Conversely, all students successfully answered the forward-thinking questions correctly. These results demonstrate a significant gap between students' reversible thinking and forward-thinking abilities, particularly in the context of solving mathematical problems involving quadrilateral concepts. These findings align with previous research showing that reversible thinking requires a higher cognitive level than forward-thinking (Inlehder & Piaget, 2002).

**Forward Thinking Problem-Solving by Students**

In questions 1 and 3, students were asked to solve forward-thinking problems, which means the initial state was known, and the goal state was asked. The students' answers to these questions are shown in Figure 1 and Figure 2.

In question 1, students were asked to find the area of a rectangular plot of land given its length and width. The students easily applied the formula for the area of a rectangle, which is  $Area = length \times width$ . The students' answers are shown in Figure 1.

<p>1. Dik: tanah berbentuk persegi panjang          = Panjang tanah 10 m          = lebar tanah 7 m</p> <p>Dit: Berapakah luas tanah Pak Adi tersebut?</p> <p>Jawaban</p> <p>1. <del>L = p x l</del>  <math>L = p \times l</math>  <math>= 10 \times 7 \text{ m}</math>  <math>= 70 \text{ m}^2</math></p>	<p>1. Tanah bentuk persegi panjang          panjang tanah 10 m          lebar tanah 7 m          Berapa Luas tanah pak Adi?</p> <p>Luas = <math>p \times l</math>          Luas = <math>10 \times 7</math>          Luas = <math>70 \text{ m}^2</math>          Luas tanah adalah <math>70 \text{ m}^2</math></p>
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1. Diketahui:  $p = 10 \text{ m}$   
 $l = 7 \text{ m}$   
 Ditanya: Berapakah luas tanah pak adi tersebut?

Jawab:

$= p \times l$   
 $= 10 \times 7$   
 $= 70 \text{ m}^2$

Figure 1. Students' answers to question number 1

<p>3. Papan catur berbentuk persegi          panjang sisi 25 cm          Tentukan luas papan catur</p> <p><math>L = s^2</math>  <math>= 25^2</math>  <math>= 625 \text{ cm}^2</math></p> <p>Luas papan catur <math>625 \text{ cm}^2</math></p>	<p>3. Diketahui: <math>s = 25 \text{ cm}</math>          Ditanya: Tentukanlah papan catur tersebut          Jawab:</p> <p><math>= s \times s</math>  <math>= 25 \times 25</math>  <math>= 625 \text{ cm}^2</math></p>
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3. Dik: Persegi dengan panjang sisinya 25 cm  
 Dit: luas papan catur tersebut  
 jawaban  
 $s = s \times s$   
 $= 25 \times 25$   
 $= 625 \text{ cm}^2$

Figure 2. Student answers to question number 3

In question 3, students were asked to find the area of a square chessboard given the side length. The students easily applied the formula for the area of a square, which is  $Area = side \times side$ . The students' answers are shown in Figure 2.

All students answered the forward-thinking questions correctly. The forward aspect is the directing aspect, which involves the mental process from the initial condition to reaching the goal destination (Mafulah & Juniati, 2020). Ramful (2015) stated that working forward starts from the given initial situation (initial state) to the desired final goal (goal state). In this thinking process, students are asked to follow problem-solving steps linearly, from understanding the problem and applying relevant concepts to finding the final solution. This approach tends to be more intuitive and easier for students to understand, as it follows a clear and structured logical flow (Jonassen, 2000). Generally,

mathematical problems are often presented as a forward-thinking process. Students who successfully solve comparison problems using forward thinking may not necessarily be successful when the thinking process is reversed (Pebrianti & Suhendra, 2023).

This proves that problems that provide the initial state at the beginning are more commonly used in class and easier for students to solve. This statement is supported by the interview results with Student A as follows:

Interviewer: How did you answer question number 1?

Student A : From the question, it's known that the shape is a rectangle, and the length is already known to be 10 m, and the width is 7 m. So, I used the formula for the area of a rectangle and multiplied them to get the result.

Interviewer: Was this question easy for you, and why?

Student A : Yes, easy. I think it's because the teacher often gives questions like these, so it's easy for me to solve.

The response from student A above represents the statements of the other two students who gave similar answers. In classroom practice, teachers often select math problems that emphasize forward thinking. This is because such an approach is easier to manage in a classroom setting and aligns with students' habitual reliance on structured problem-solving methods (Hiebert et al., 2012). This can limit the development of students' critical and creative thinking skills, especially when they are faced with problems that require non-linear thinking (Resnick, 1987) or problems that require a more complex approach, such as reversible thinking.

**Reversible Thinking Problem-Solving by Students**

Questions 2 and 4 asked students to solve reversible thinking problems. This means the goal state was known, and the initial state was asked. All students answered question number 2 incorrectly, and only one student answered question number 4 correctly. This is because the students were asked to find the initial state with the goal state provided. The provision of the goal state at the beginning made it difficult for students to determine the initial state and solve the given problem. Students tended to be confused and did not understand what steps to take to solve the problem. This difficulty arises because reversible thinking is more challenging, requiring the ability to view problems from multiple perspectives (Flanders, 2014). Below is a discussion related to the students' responses to the reversible thinking problems.

2. Dik = Persegi Panjang 44 cm  
 = lebarnya  $\frac{3}{8}$  kali panjangnya  
 Dit: luas meja tersebut  
 Jawaban  
~~2 = 44~~       $k = \frac{3}{8} * P$   
~~= 44~~

(a)

2. Keliling meja 44 cm  
 lebarnya  $\frac{3}{8}$  kali panjangnya  
 ditanya : berapa luas meja?  
 jawab :  
 $k = 2p + 2l$   
 $k = \frac{3}{8} * l + 2l$   
 $k = \frac{19 * l}{8}$

(b)

$$\begin{array}{l}
 2. \text{ Diketetahui } P = 44 \text{ cm} \\
 L = \frac{3}{8} P \\
 \text{ Ditanya: Maka berapakah luas meja tersebut} \\
 \text{ Jawab:} \\
 K = 2P + 2L \\
 = 2P + 2 \cdot \frac{3}{8} P
 \end{array}$$

(c)

**Figure 3.** Student's answer for question number 2

Based on Figure 3, the students' responses to question number 2, which required reversible thinking ability, are shown. In question 2, students were asked to find the area of a rectangular table, but the length and width were not provided, and only the perimeter was given. Therefore, students were required to determine the length and width before finding the area of the table.

From the students' answers in Figures 3(a) and 3(b), it is clear that students had difficulty creating a mathematical model, as evidenced by their use of the same sentences as in the problem statement in the given section they wrote. This led to mistakes when using the rectangle's perimeter formula with the known information. The students' failure to create a mathematical model made it difficult for them to apply the formula and obtain the correct solution (Melani et al., 2023). This difficulty indicates that students struggled with word problem types. From the answer in Figure 3(c), it is evident that the student was able to create a mathematical model but was unable to perform the operations correctly. Below is an interview with Student B, who struggled with the reversible thinking problem.

Interviewer: How did you answer question number 2?

Student B : I did not answer it well. Based on the question, it is known that the perimeter of the rectangle is given, and it is also known that the width is <sup>3</sup> of the length. However, the length value is not provided. So, I was confused about finding the length and width values.

Interviewer: Was this question difficult? Has the teacher ever given you a question like this?

Student B : Yes, difficult. I don't think the teacher has ever given a question like this.

According to the interview, student B found it difficult and could not engage in reversible thinking. Thus, the student perceived problems that provided the goal state at the beginning as challenging. The student acknowledged that such problems are rarely given in classroom instruction. Students tend to be more accustomed to routine and linear problems, where they can follow the teacher's established steps. This makes them less trained in flexible and critical thinking when faced with reversible problems that require backward thinking and the integration of various mathematical concepts (Sweller, 1988).

In Figure 4, students were asked to find the perimeter of a square card, where the side length was unknown, but the area was provided. Therefore, students had to determine the side length first to find the perimeter using the given information. However, two students could not answer the problem correctly. Figure 4(a) shows that the student used the known area information to find the perimeter. This indicates the student's difficulty in creating a mathematical model from the word problem, resulting in obstacles in solving

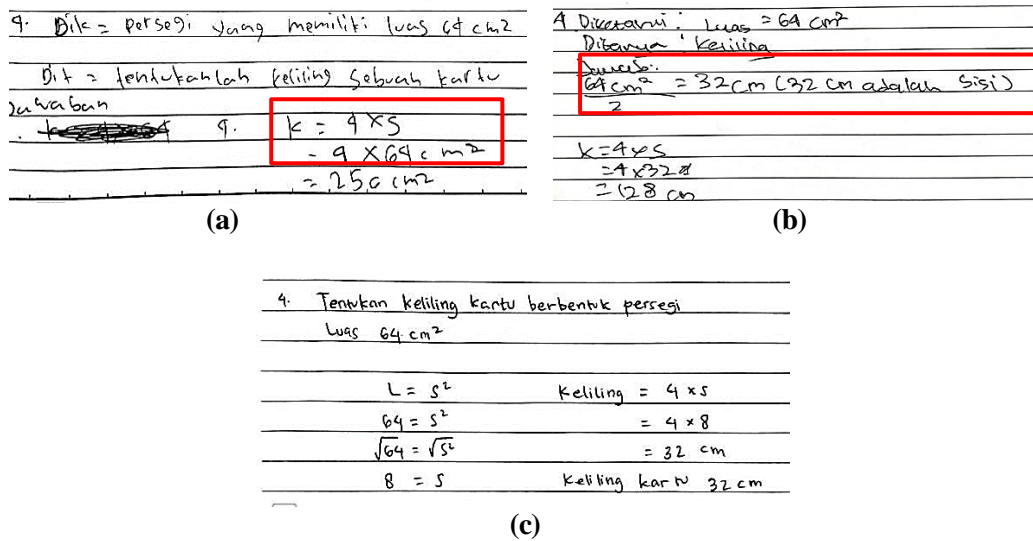


Figure 4. Student’s answer for question number 4

the problem. The challenges students face in solving word problems are not new. Students’ difficulties can occur due to several factors. Factors influencing students’ difficulties in solving mathematical word problems include difficulty understanding and translating the problem, understanding the concept, skill difficulties, and problem-solving difficulties (Emanuel et al., 2021; Utari et al., 2019). Therefore, students’ ability to solve word problems needs attention.

Based on Figure 4(b), the student correctly identified the first step: finding the side length and then calculating the perimeter. However, the student encountered difficulties in finding the side length. To find the side length, the student divided the given area value by two. This approach was incorrect; the student should have used the square root operation on the area value to obtain the side length. The formula for the area of a square is  $Area = l^2$ , so to find the side length, the formula  $l = \sqrt{A}$  should be used, where  $A$  is the area of the square, and  $l$  is the side length.

In Figure 4(c), the student’s correct answer for question 4 demonstrates the ability to solve the reversible thinking problem. This student could successfully reverse the problem-solving process, showing a strong understanding of the involved concepts and the ability to view the problem from different perspectives (Flanders, 2014). This success highlights the importance of providing students with opportunities to practice with problems that challenge them to think non-linearly and flexibly (Resnick, 1987).

From the above discussion, it is evident that most students struggled with solving problems requiring reversible thinking. Analyzing the students’ answers to these reversible thinking questions reveals that only one student could correctly solve one of the two reversible thinking problems. It can be concluded that students have not yet effectively developed their reversible thinking abilities. This is in contrast to the forward-thinking problems that students can easily solve. This is because students have a limited context when solving different types of problems (Suryadi, 2019). Students are limited in solving these problems because they are rarely exposed to such problems during classroom instruction. This statement is supported by the interview results with Student Babove, as well as an interview with Student C as follows:



Interviewer : How did you answer question number 4?

Student C : I was a bit confused. From the question, the area is known, so I thought that the side length could be obtained from the area value, so I divided the area value by two. I forgot that it shouldn't be divided by two, but the area should be square-rooted. So, I didn't answer the question correctly.

Interviewer : Was this question difficult? Has the teacher ever given you a question like this?

Student C : Quite difficult. I think it's been given, but very rarely.

According to the interview, the student mentioned that problems requiring reversible thinking are difficult and rarely given by teachers in class. In classroom instruction, the practice problems teachers provide for quadrilateral topics are typically routine types that require students to find the area and perimeter of the shape. However, quadrilateral topics can serve as an option for enhancing students' problem-solving skills through reversible thinking problems. This way, students can better understand the concepts of quadrilaterals rather than just memorizing the steps to solve them.

Teachers need to provide non-routine problems, such as those involving reversible thinking, to enhance students' problem-solving skills and challenge them to think critically and creatively and apply mathematical concepts. Teachers' infrequent use of non-routine problems can hinder students' problem-solving abilities (Waty, 2017). Without regular exposure to such problems, students tend to rely on memorization and routine procedures, limiting their capacity to tackle more difficult and unexpected challenges. Therefore, teachers must introduce and integrate non-routine problems involving reversible thinking into classroom instruction. This will help students develop more profound thinking skills and better prepare them to face various more complex mathematical problems (Kilpatrick, 2010).

#### ▪ CONCLUSION

Based on the analysis in this study, it can be concluded that students demonstrate difficulties in reversible thinking when solving quadrilateral problems, particularly rectangles, due to a lack of familiarity and challenges in modeling the mathematical concepts presented in the word problems. Although students could solve forward-thinking problems effectively, they encountered significant challenges in reversible thinking problems. This indicates that students' understanding of the fundamental mathematical concepts involved is not strong enough to be applied in situations requiring non-linear thinking. Therefore, more targeted instructional strategies are needed to enhance students' reversible thinking abilities, including introducing more non-routine problems and exercises that encourage the exploration of various problem-solving approaches. This will enable students to develop more flexible thinking skills and better prepare them to face various more complex mathematical challenges.

#### ▪ REFERENCES

- Amir, A. (2015). *Pemahaman konsep dan pemecahan masalah dalam pembelajaran matematika*. Logaritma, 3(1), 13–28.
- Annizar, A. M., Maulyda, M. A., Khairunnisa, G. F., & Hijriani, L. (2020). *Kemampuan pemecahan masalah matematis siswa dalam menyelesaikan soal pisa pada topik geometri*. Jurnal Elemen, 6(1), 39–55. <https://doi.org/10.29408/jel.v6i1.1688>

- Arjudin, Turmuzy, M., Kurniati, N., & Wulandari, N. P. (2024). Problem solving skills of mathematics education students with lack number sense ability. *Jurnal Pendidikan MIPA, 25(1)*, 103–115.
- Billstein, R., Libeskind, S., & Lott, J. W. (1993). *A problem solving approach to mathematics for elementary school teachers* (5th ed.). Addison-Wesley.
- Creswell, J. W. (2014). *Research design: qualitative, quantitative, and mixed methods approaches* (4th ed.). Sage.
- Dougherty, B. J., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: the reversibility, flexibility, and generalization approach. *Intervention in School and Clinic, 50(5)*, 273–281. <https://doi.org/10.1177/1053451214560892>
- Emanuel, E. P. L., Kirana, A., & Chamidah, A. (2021). Enhancing students' ability to solve word problems in Mathematics. *Journal of Physics: Conference Series, 1832(1)*. <https://doi.org/10.1088/1742-6596/1832/1/012056>
- Flanders, S. T. (2014). *Investigating flexibility, reversibility, and multiple representation in a calculus environment*. University of Pittsburgh.
- Hackenberg, A. (2005). *Construction of algebraic reasoning and mathematical caring relations*. In Dissertation. University of Georgia.
- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., Wearne, D., Hiebert, J., & Carpenter, T. P. (2012). *Solving*.
- Ikram, M., Purwanto, Parta, I. N., & Susanto, H. (2020). Exploring the potential role of reversible reasoning: Cognitive research on inverse function problems in mathematics. *Journal for the Education of Gifted Young Scientists, 8(1)*, 591–611. <https://doi.org/10.17478/jegys.665836>
- Inleher, B., & Piaget, J. (2002). *The growth of logical thinking from childhood to adolescence* (5th ed.). Routledge.
- Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development, 48(4)*, 63–85. <https://doi.org/10.1007/BF02300500>
- Kang, Mee -Kwang & Lee, B.-S. (1999). On fuzzified representation of piagetian reversible thinking. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education, 3(2)*, 99–112. [http://www.koreascience.or.kr/search/articlepdf\\_ocean.jsp?admNo=SHGHEN\\_1999\\_v3n2\\_99](http://www.koreascience.or.kr/search/articlepdf_ocean.jsp?admNo=SHGHEN_1999_v3n2_99)
- Kilpatrick, J. (2010). Helping children learn mathematics. In *Academic Emergency Medicine* (Vol. 17, Issue 12). <ftp://129.132.148.131/EMIS/journals/ZDM/zdm026r1.pdf>
- Klimov, A., & Shamir, A. (2003). A new class of invertible mappings. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 2523*, 470–483. [https://doi.org/10.1007/3-540-36400-5\\_34](https://doi.org/10.1007/3-540-36400-5_34)
- Maf'Ulah, S., Fitriyani, H., Yudianto, E., Fiantika, F. R., & Hariastuti, R. M. (2019). Identifying the reversible thinking skill of students in solving function problems. *Journal of Physics: Conference Series, 1188(1)*. <https://doi.org/10.1088/1742-6596/1188/1/012033>

- Maf'ulah, S., & Juniati, D. (2019). Students' strategies to solve reversible problems of function: the part of reversible thinking. *Journal of Physics: Conference Series*, 1417(1). <https://doi.org/10.1088/1742-6596/1417/1/012051>
- Maf'ulah, S., & Juniati, D. (2020). Exploring reversible thinking of preservice mathematics teacher students through problem-solving task in algebra. *Journal of Physics: Conference Series*, 1663(1). <https://doi.org/10.1088/1742-6596/1663/1/012003>
- Melani, R., Herman, T., Hasanah, A., Mefiana, S. A., & Samosir, C. M. (2023). *Kemampuan membuat model matematika dan daya juang produktif siswa SMP dalam menyelesaikan soal pemecahan masalah*. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, 7(3), 2391–2404. <https://doi.org/10.31004/cendekia.v7i3.2545>
- Norton, A. (2016). (IR) reversability in mathematics. *International Group for the Psychology of Mathematics Education*, 8.
- Olive, J., & Steffe, L. P. (2001). The construction of an iterative fractional scheme: The case of Joe. *Journal of Mathematical Behavior*, 20(4), 413–437. [https://doi.org/10.1016/S0732-3123\(02\)00086-X](https://doi.org/10.1016/S0732-3123(02)00086-X)
- Palupi, S. R. E., & Andrijati, N. (2024). Discovery learning model assisted by geogebra-based napier bones on students' division problem solving ability. *Jurnal Pendidikan MIPA*, 25(1), 223–235.
- Pebrianti, A., Prabawanto, S., & Nurlaelah, E. (2023). How do students solve reversible thinking problems in mathematics? *Jurnal Elemen*, 9(2), 630–643. <https://doi.org/10.29408/jel.v9i2.17821>
- Pebrianti, A., & Suhendra, S. (2023). Ways of thinking senior high school student to solve geometri van hiele problem use reversible thinking ability. *Al-Jabar : Jurnal Pendidikan Matematika*, 14(2), 401. <https://doi.org/10.24042/ajpm.v14i2.18116>
- Prabawanto, S. (2023). Improving prospective mathematics teachers' reversible thinking ability through a metacognitive-approach teaching. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(6). <https://doi.org/10.29333/ejmste/13201>
- Ramful, A. (2015). Reversible reasoning and the working backwards problem solving strategy. *The Australian Mathematics Teacher*, 71(4), 28–32.
- Rejeki, S., & Rahmasari, L. (2022). Students' problem-solving ability in number patterns topic viewed from cognitive styles. *Jurnal Elemen*, 8(2), 587–604. <https://doi.org/10.29408/jel.v8i2.5699>
- Resnick, L. B. (1987). Education and learning to think. in education and learning to think (issue september). <https://doi.org/10.17226/1032>
- Simon, M. A., Kara, M., Placa, N., & Sandir, H. (2016). Categorizing and promoting reversibility of mathematical concepts. *Educational Studies in Mathematics*, 93(2), 137–153. <https://doi.org/10.1007/s10649-016-9697-4>
- Suryadi, D. (2019). *Landasan filosofis penelitian desain didaktis (DDR)*. Gapura Press.
- Sweller. (1988). Cognitive Load During Problem Solving: Effects on Learning – Sweller - 2010 - Cognitive Science - Wiley Online Library. *Cognitive Science*, 285, 257–285. [https://doi.org/10.1016/0364-0213\(88\)90023-7](https://doi.org/10.1016/0364-0213(88)90023-7)

- Utari, D. rizky, Wardana, M. Y. S., & Damayani, A. T. (2019). *Analisis kesulitan belajar matematika dalam menyelesaikan soal cerita*. *Jurnal Ilmiah Sekolah Dasar*, 3(4), 534–540.
- Waty, E. R. K. (2017). *Menelaah kualitas soal ujian sekolah buatan guru pada pencapaian kelulusan siswa*. *Jurnal Pendidikan Dan Pemberdayaan Masyarakat*, 4(2), 11–17.
- Widodo, S. A., Ibrahim, I., Hidayat, W., Maarif, S., & Sulistyowati, F. (2021). Development of mathematical problem solving tests on geometry for junior high school students. *Jurnal Elemen*, 7(1), 221–231. <https://doi.org/10.29408/jel.v7i1.2973>