



Students' Learning Obstacles in Exponential: A Case Study in Indonesian Higher Education Students

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Abstract: Exponential is an essential concept in mathematics learning. Students generally understand the idea of exponentials if the base and power are natural numbers. However, when the base and the power are a negative integer, students experience difficulties, which can create learning obstacles. This research aims to identify and analyze learning obstacles that confront students at the stage of solving exponent problems, especially in the exponent form $\llbracket -(a) \rrbracket^{-(b)}$, $\forall a \in \mathbb{N}, b \in \mathbb{Z}$. This study uses a qualitative method with a case study type. The participants of this study were nine higher education students in Bandung. Subjects are asked to take a written test of seven questions to explore their understanding of exponents with specific characteristics. The results of the research showed that all of the learning obstacles that arise are epistemological learning obstacles, including errors in defining exponents when the power is zero, when the power is a negative integer, when the base contains a negative sign, when the base contains a negative sign and the power is zero, and when the base contains a negative sign with a negative integer exponent. The research results obtained can be used to develop a hypothetical learning trajectory.

Keywords: learning obstacle, exponential, epistemological learning obstacle, negative exponential.

▪ INTRODUCTION

Mathematics lessons are mandated by the curriculum to be taught to students in Indonesia from elementary to higher education levels. According to the Merdeka Curriculum, this subject equips students with thinking, reasoning, and logic through certain mental activities that form a continuous flow of thinking and lead to an understanding of mathematics learning material. Learning mathematics aims to equip students to understand mathematics learning material, use reasoning, solve problems, communicate ideas, relate mathematics learning material, and appreciate mathematics's usefulness in life. Developing a strong mathematical understanding is very important, but more important is the ability to resolve everyday issue (Van Mier, Schleepen, & Van den Berg, 2019). The objects in mathematics learning are facts, concepts, principles, operations, relationships, problems, and mathematical solutions that are formal and universal.

Mathematical concepts are interconnected, so they must be taught according to a hierarchy in the learning process. This means that no concept should be overlooked because it might be a prerequisite for understanding the next concept to be studied. Students' incorrect prior knowledge could cause major issues while they try to grasp other mathematical subjects (Akkaya, 2015). The learning competencies that students must achieve in the Merdeka Curriculum are stated in mathematics learning outcomes for each phase

One of the learning outcomes that students must achieve at the end of phase D is that students can solve contextual problems using mathematical concepts, skills, and efficiently operate exponential numbers. Apart from being stated in learning outcomes,

this concept is also a prerequisite material for material on exponential functions and logarithms, which are learning outcomes that must be achieved in phase E. The exponent on a growth factor is the second fundamental concept that students need to understand to have a cohesive grasp of exponential, logarithmic functions, and another future concept (Kuper & Carlson, 2020; Pitta-Pantazi, Christou, & Zachariades, 2007). As a mandate from the curriculum, students should study this material with learning outcomes as minimum achievements. that students must obtain.

As material that must be taught to students, learning obstacles are still found in the exponential learning process. In general, according to Brousseau (in Suryadi, 2010), three factors cause learning obstacles, namely ontogeny obstacles, which are obstacles related to mental learning; didactic obstacles, which are obstacles related to teacher teaching and epistemological obstacles, namely obstacles related to students' knowledge has limited application.

Based on the results of interviews with mathematics teachers at the school where the research was conducted, the teacher stated that students had difficulty understanding exponential material, which hampered the further learning process. According to the interview results, in grade 12th, the mathematics lesson material for algebra elements and functions topic should have included exponential functions. However, because students experience learning obstacles, teachers are forced to repeat the teaching of exponential material to students. With this problem, of course, the learning process carried out will be ineffective because there will be a delay in introducing the next topic. It can result in disruption to the learning schedule that the teacher has prepared. This situation shows that it is crucial to carry out research that identifies the difficulties experienced by students when learning exponential material so that teachers can intervene or make adjustments to overcome these problems.

Difficulties in learning exponential material are not only experienced by students in Indonesia but are also problems experienced by students in various countries. According to research conducted by Ulusoy (2019) in Turki, there are four obstacles in studying exponent material: obstacles to understanding exponents as repeated multiplication, understanding zero exponents, understanding negative exponents, and obstacles to operating exponents. Other research states that students better understand operations in exponentials if the base and power of the exponent are natural numbers (Iymen & Duatepe-Paksu, 2015). Students are mentally more successful in working on questions containing natural numbers (Avcu, 2014; Durkin & Rittle-Johnson, 2015; McGowen & Tall, 2013). According to Ulusoy (2019) and DeWolf & Vosniadou (2015), students often use addition and multiplication principles when working on exponent problems, which often causes overgeneralization errors. Students face many obstacles as they face new mathematical concepts, particularly those that expand or require modifications to previously held beliefs (Rabin, Fuller, & Harel, 2013). Based on several studies that have been conducted previously such as Cangelosi et al. (2013), Iymen and Duatepe-Paksu (2015), Angraini and Prahmana (2018), Ulusoy (2019), and Sumirat et al. (2023), indicates that there are still learning obstacles faced by students when studying exponents, mainly when the base contains a negative integer and the power is a negative integer.

Although many studies have explored students' learning obstacles with exponent material, particularly with zero and negative exponents, research examining the types of

learning obstacles experienced by students, as defined by Brousseau, is still limited. This research aims to identify and analyze the learning obstacles students face in solving exponential problems, especially those in exponential form $\left[-(a) \right]^b, \forall a \in \mathbb{N}, b \in \mathbb{Z}$. In this study, the hypothesis is that students find it easier to solve exponential problems when the base and exponent are natural numbers compared to when the base or exponent involves negative numbers or zero. By identifying and addressing these obstacles, teachers may be better equipped to help students develop a deeper understanding of exponents, ensuring that their mathematical progression is not hindered by foundational gaps in knowledge.

▪ METHOD

Participants

The population in this study consisted of 11th-grade students in Bandung Regency who had already learned about exponential numbers. Using purposive sampling, the participants in this study were 9 students who met the necessary criteria for the research, specifically their prior exposure to the concepts of exponential numbers. The research was conducted in November 2023, to identify learning obstacles related to exponents.

Research Design and Procedures

This research employed a qualitative approach utilizing a case study design. The case study was chosen because the researcher wanted to dig up information regarding learning obstacles experienced by participants in studying exponential material. By conducting this research, researchers gain in-depth scientific knowledge regarding the cases studied. This reason aligns with Creswell (2016), case study involves exploring a specific process, activity, or event.

The study was conducted over two weeks. In the initial phase, tests requiring exponential numbers were given to participants under strict supervision to discourage the use of calculators, textbooks, or discussions. The number of participants who answered correctly and incorrectly was recorded in a table to assist in the study. Additionally, the classifications that were applied to the incorrect answers were identified by the kinds of errors they made. By using this method, errors could be more easily identified and the precise areas in which pupils struggled could be better understood.

Upon analyzing the students' test results, it was found that questions 1c and 1f exhibited two types of student errors, while questions 1b, 1d, and 1e each demonstrated a kind of error. Subsequently, students identified and articulated the types of errors, following which 4 students were selected for in-depth interviews to serve as representatives. Selected students were interviewed one by one and asked questions regarding how students thought about answering questions and how the material had been studied.

Instruments

In data collection, the instruments used in this study included tests and semi-structured interview sheets. The researcher developed the test used in this study and validated it by a mathematics education expert to ensure its validity and relevance for assessing students' understanding of exponential numbers. Test questions on exponents were given to nine students in total. The test consists of 7 description questions. This question was designed to determine whether participants had difficulty expressing

exponential numbers into repeated multiplication when the base and exponent were no longer positive integers. Description questions were chosen because they could provide clues about the form and quality of participants' thought processes so that researchers could analyze the learning obstacles experienced by students. The 7 questions were designed to have students convert exponential expressions into repeated multiplication. These questions included problems related to exponential numbers with natural number base and power, exponential numbers with zero exponents, exponential numbers with negative integer power, exponential numbers with a base containing a negative sign, including cases where the power is an even integer and odd integer, exponential number with a base containing a negative sign and zero exponents, along with exponential numbers with a base containing a negative sign and a negative integer power. Each of these indicators is represented by one specific question. While working on questions, students are prohibited from opening books or using calculating tools in the hope that the results of the student's work are genuinely from their thinking processes.

Data Analysis

For data analysis, a triangulation method was employed to ensure the robustness and validity of the findings. This approach involved integrating multiple data sources, including test results and semi-structured interview responses. Initially, students' answers to the test questions were analyzed to identify patterns and difficulties in their understanding of exponential numbers. These results were then cross-referenced with insights gained from the interviews, which provided deeper context and elaboration on the students' thought processes and challenges. By comparing and contrasting data from these different sources, the researchers were able to achieve a more comprehensive and accurate understanding of the learning obstacles faced by the students. This triangulation method enhanced the credibility of the findings and allowed for a thorough investigation of the issues related to exponential numbers.

▪ RESULT AND DISSCUSSION

Following the objectives of this research, this research is expected to provide an overview of the learning obstacles experienced by students in learning exponentials, especially in the exponential form $-(a)^{-b}, \forall a \in N, b \in Z$. The objectives of this research can be achieved through analysis of student test results and in-depth interviews. Students are asked to express exponential numbers using repeated multiplication through written tests. Based on this test, we can obtain possible learning obstacles in exponential material, which can be seen from how students solve the problems given and the student's test results. Based on the test results, the results are shown in Table 1 below.

Table 1. Accumulation of student answers

Questions Number	Questions	Number of Students Answered Correctly	Number of Students Answered Incorrectly
1a	2^4	9	0
1b	3^0	4	5
1c	4^{-3}	5	4
1d	$-(2)^2$	6	3
1e	$-(2)^3$	7	2
1f	$-(2)^{-2}$	3	6

lg	$-(2)^0$	2	7
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Based on Table 1, students had no difficulty expressing exponential numbers if the power and base of the number were positive integers. The absence of students who answered incorrectly in question 1a indicated that students could express exponential numbers through repeated multiplication and carried out the calculation process well when the base and power of the exponent number were positive integers. When faced with exponential number problems, especially in the exponential form $-(a)^{-b}, \forall a \in N, b \in Z$ some students experienced learning obstacles expressing these numbers in the form of repeated multiplication. In several cases, students could answer the questions correctly but had difficulty explaining the reasons and justification for the answers.

After analyzing the student test results, 4 students were selected for in-depth interviews. Based on the analysis of test results and interviews that have been carried out, there are four learning obstacles experienced by students in exponent material when the exponent form is $-(a)^{-b}, \forall a \in N, b \in Z$

Learning Obstacle Related to the Zero Exponent Rule

The first learning obstacle discovered was related to the zero exponent rule. There were 4 out of 9 students who were able to answer the questions correctly while 5 students gave wrong answers. From the analysis of student test results, five students answered incorrectly and students made one type of error. In the question given, five students answered $3^0=0$. Based on this, students need to fully understand that the zero exponent always has the value of one unless the base is zero. One of the student's answers is presented in Figure 1 below.



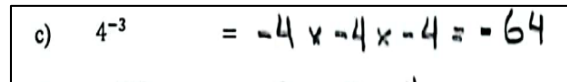
The image shows a handwritten student answer for question b). It consists of a rectangular box containing the text "b) 3^0 = 0". The "0" in the exponent and the "0" in the result are handwritten.

Figure 1. The answers to the questions about the zero exponent rule S1

The idea that every non-zero number raised to the power of zero equals one is one that many students find difficult to understand. This misconception frequently results from a failure to apply fundamental principles and an incapacity to generalize exponent properties. The idea that a number may be reduced to one just by having a zero exponent may seem strange to students, which could cause misunderstanding and calculation mistakes. Students' difficulty in understanding exponents with a power of zero is also one of the most commonly encountered challenges (Cangelosi et al., 2013; Kontorovich, 2016; Levenson, 2012).

Learning Obstacle When the Power of the Exponent is a Negative Integer

The second learning obstacle experienced by students is a learning obstacle when the power of the exponent is a negative integer. When faced with exponential numbers where the bases are positive integers but the exponents are negative integers, 4 out of 9 students answered incorrectly. From the analysis of student test results, 4 students answered incorrectly, and two types of errors were made; the following is an example of student 1 (S1) test results, presented in Figure 2 below.



A rectangular box containing the handwritten text: c) $4^{-3} = -4 \times -4 \times -4 = -64$

Figure 2. The answers to the questions about exponential numbers with negative integer power S1 and S2

Based on the student's answers in Figure 2, students understand 4^{-3} as repeating multiplication -4 three times, so the final result is -64 . The following is an interview conducted with S1.

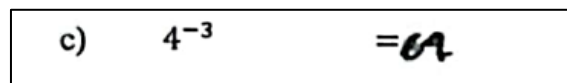
Researcher : "For question number 1c, how do you interpret it?"

S1 : "I interpret it as repeated multiplication of -4 , so the result is -64 "

Researcher : "This means that if the exponent is negative then the negative becomes attached to the base, right?"

S1 : "I think so."

Meanwhile, 2 other students answered questions 4^{-3} by repeating the multiplication of 4 three times, resulting in the final answer being 64 . Here is Figure 3, which represents one of participant's test result.



A rectangular box containing the handwritten text: c) $4^{-3} = 64$

Figure 3. The answers to the questions about exponential numbers with negative integer power S3 and S4

In interviews conducted by researchers and S4, the following results were obtained.

Researcher : "For question number 1c, how do you interpret it?"

S4 : "I understand the question as repeating multiplication 4 three times to produce 64 ".

Researcher : "Does that mean if the power is a negative integer, you can just ignore it?"

S4 : "Yes ma'am"

From the student's test results and the reinforcement of the student's answer in the interview, the student ignored the negative sign in the power and assumed that $4^{-3} = 4^3 = 4 \times 4 \times 4 = 64$. This indicates that students cannot represent exponent numbers when the base is a positive integer and the power is a negative integer in the form of repeated multiplication.

Interviews were also conducted with S5 students who were able to answer this question correctly. The following is the interview that was conducted.

Researcher : "For question number 1c, how do you interpret it?"

S5 : "I was taught that if the power is a negative integer, then the exponent number changes to one over the power of exponent, with the power of exponent converted to a positive integer."

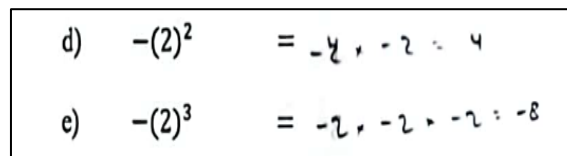
Researcher : "Can you explain why, when the power is a negative integer, the result becomes one over the power of exponent, with the power of exponent converted to a positive integer?"

S5 : "I forgot ma'am"

Based on the test results and interview, students had learning obstacles when the power of the exponent is a negative integer. Previous research also stated that students experienced mathematical errors when converting negative powers into fractions (Sumirat et al., 2023). This indicates that students cannot build a relationship between the meaning of the negative sign and the meaning of the multiplicative inverse of negative in exponential (Ulusoy, 2019). Students know that $1/a$ is the multiplicative inverse of a , but students often need help understanding that $a^{(-b)}$ is the multiplicative inverse of a^b (Cangelosi et al., 2013). From interviews with students who answered this question correctly, it was found that they based their answers on the information given by the teacher, rather than on their own understanding of the reason for the change in form. According to Mayasari & Habeahan (2021), during the learning process, students frequently rush to record every concept and piece of material presented without fully comprehending the concepts.

Learning Obstacle When the Base of the Exponential Number Contained a Negative Sign

The third learning obstacle was when the base of the exponential number contained a negative sign. The students' test results on the tests given showed that students experienced learning obstacles when the base, which was initially a positive integer, was changed to a positive integer but contained a negative sign. Based on the test results for question 1d, 3 out of 9 students made a mistake where the negative sign presented in the base of the exponential number was also raised to the power of 2. Figure 4 is an example of a student's answer for this type of error.



The figure shows two handwritten equations in a box. The first equation is labeled 'd)' and shows $-(2)^2 = -4, -2 \cdot -4$. The second equation is labeled 'e)' and shows $-(2)^3 = -2, -2 \cdot -2 \cdot -8$. Both equations show a misunderstanding of the order of operations, where the negative sign is incorrectly treated as part of the base being raised to the power.

Figure 4. The answers to the questions about exponential numbers with the base containing negative signs S5

Next, an interview was conducted with S5 regarding his interpretation of the questions given. The following are the results of interviews with researchers and S5.

Researcher : "How do you interpret question 1d?"

S5 : "If the base of the exponentiation contains a negative sign, then the negative sign is also raised to the power. So, for 1d, the answer is $-2 \times -2 = 4$."

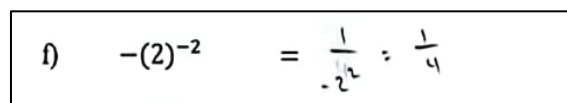
Researcher : "So, if the base is negative, does it mean the negative sign is also raised to the power of 2?"

S5 : "Yes, that's how I see it."

Based on the analysis of student test results, students have difficulty determining whether the negative sign in the base is attached to the base or not. These learning obstacles led to the misconception if the power was an even number, the result would be positive, but if the power is an odd number, the result would be negative. Furthermore, this indicates that students need help understanding that $-(a)^b$ is the additive inverse of a^b (Cangelosi et al., 2013).

Learning Obstacle When the Base Contained a Negative Sign and The Power was a Negative Integer

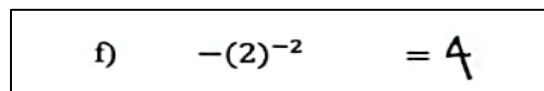
The fourth learning obstacle was when the base contained a negative sign and the power was a negative integer. According to the student test results, only 3 out of 9 students were able to solve exponential problems when the base contained a negative sign and the power of the exponent number was a negative integer. Based on question 1f, students make two types of errors. First, students understand that if an exponential number has an opposing integer power, its form will be changed to a fractional form, but the negative sign in the base is also raised to the power. The second type of error is when students multiply negatives in powers and bases. The Figure 5 was an example of a student's answer with the first type of error.



$$f) \quad -(2)^{-2} = \frac{1}{-2^2} = \frac{1}{4}$$

Figure 5. The answer of an exponential number with the base containing negative signs and the power is a negative integer S5

Through interviews conducted with S5, a student with the first type of error made this error because the student did not understand that the negative sign on the base is not attached to the base, so it should not be raised to the power. Figure 5 was an example of a student's answer for the second type of error.



$$f) \quad -(2)^{-2} = 4$$

Figure 6. The answer of an exponential number with a base containing negative signs and the power is a negative integer S4

The repeated multiplication form was not written for question number 1f on the student answer sheets. Below are the results of an interview conducted with S4.

Researcher : "How does you interpret question 1f?"

S4 : "I multiply the negative sign in the exponent with the negative sign in the base."

This learning obstacle could have occurred as a result of the second and third learning obstacles. Those two learning obstacles were discussed previously. However, there are interesting findings from this learning obstacle, where some students didn't experience the second and third learning obstacles but experienced the fourth. This may be attributed to students' insufficient understanding of exponential concepts, especially in the exponential form $-(a)^{-b}, \forall a \in N, b \in Z$

Learning Obstacle Related to Zero Exponent Rule and Base Contained a Negative Sign

In question number 1g, only 2 students answered correctly, while 7 students gave incorrect answers. The correct answer to this problem should be -1 because $2^0=1$

according to the zero exponent rule, and the negative sign in front meant the final result was -1 . Figure 7 and Figure 8 below show two types of errors students make.

$$\text{g) } -(2)^0 = 0$$

Figure 7. The answer of an exponential number with a base containing negative signs and the power is zero S1

$$\text{g) } -(2)^0 = 1$$

Figure 8. The answer of an exponential number with a base containing negative signs and the power is zero S5

This learning obstacle was the most common mistake students made in this study. The difficulties experienced by students in answering questions, as shown in Figure 7 and Figure 8, are the accumulation of students' learning obstacles in understanding zero exponents or learning obstacles when the base of the exponential number contains a negative sign or both. As many as 5 out of 7 students who answered incorrectly assumed that $-(2)^0 = 0$. This indicated that students had difficulty understanding zero exponents. Meanwhile, 2 out of 7 other students answered $-(2)^0 = 1$, which indicated that students understood zero exponents but experienced learning obstacles when the base of the exponential number contained a negative sign.

The Analysis of Learning Obstacles

This research shows that students still experience learning obstacles when studying exponential material. In this research, students experienced at least six learning obstacles. The results of student tests and interviews indicated that the learning obstacles encountered by students were caused by the questions presented, which were not only the exponential questions with powers and bases of positive integers. This finding aligns with previous research results by various scholars (Avcu, 2014; Iymen & Duatepe-Paksu, 2015). From several previous studies and research, students can better determine exponential number problems when the base and exponent are natural numbers.

Another finding in this research is that some students still struggle with understanding the sequence of negative numbers, possibly due to insufficient number sense. Iymen & Duatepe-Paksu (2015) stated that the difficulties faced by students are due to their low sense of exponential numbers, which is also supported by several studies conducted in different countries. This aligns with the findings of Senay (in Ulusoy, 2019), which mentions that students perceive exponential material as challenging, illogical, and unrelated to everyday life, partly due to a lack of exponential number sense. The basis for this lack of exponential number sense is found in students' inability to understand integers and rational numbers (Iymen & Duatepe-Paksu, 2015). Someone with number sense will find it easier to learn various mathematics topics since they will be flexible in applying their numerical knowledge and more confident in their ability to solve issues (Byrnes & Wasik, 2009; Cirino et al., 2015; Clements et al., 2016; Maloney & Beilock, 2012; Wulandari et al., 2020). Number sense affects students' learning outcomes and their

success in mathematics at school (Akkaya, 2015; Yang, 2019; Yang & Lin, 2015; Yang & Sianturi, 2021).

Based on the analysis of errors made by students, all learning obstacles found in this research are epistemological learning obstacles because these errors are related to difficulties in understanding the concept of exponential numbers. These epistemological obstacles include students' incapacity to apply basic exponential laws, difficulties in recognizing exponential patterns, and mistakes in exponent-related operations like multiplying and dividing exponential numbers. These mistakes suggest that an established understanding of basic concepts is necessary to get beyond these epistemological obstacles and assist students in developing a better knowledge of exponential numbers. As a result, educators must recognize these epistemic obstacles and take steps to overcome them through the use of learning tools and more efficient teaching strategies that might improve students' comprehension of the idea of exponential numbers. Early diagnosis of the misconceptions is necessary to create instructional strategies and instructions that will help students acquire stronger conceptual knowledge (Cheung & Yang, 2020).

▪ CONCLUSION

Based on the results and discussions previously outlined, it can be concluded that there are four types of learning obstacles faced by 11th-grade students at a high school in Bandung when dealing with exponent problems. Through an analysis of student tests and interviews, it was revealed that students often struggle with the zero exponent rule. Additionally, the concept of negative exponents presents significant challenges, as students commonly fail to grasp the relationship between a negative exponent and its corresponding fractional form. When the base of an exponential number contains a negative sign, students often make errors, either by incorrectly attaching the negative sign to the base. The most pervasive difficulties arose when the base contained a negative sign and zero as well as when the exponent both the base and the exponent were negative, indicating that these learning obstacles are interrelated and compound each other. Furthermore, a lack of number sense complicates students' understanding and application of exponential concepts. These findings suggest that the identified obstacles are primarily epistemological, rooted in fundamental misunderstandings of exponential laws and operations. To address these challenges, educators should focus on early identification of misconceptions and take steps to overcome them through the use of learning tools and more efficient teaching strategies that might improve students' comprehension of the idea of exponential numbers.

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